Lab Note 2 Intermezzo: Change of View to a Spacelike Fifth Dimension, as the Geometric Foundation of Intrinsic Spin

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Those who have followed the development of this lab note know that I have been working with a Kaluza-Klein theory which regards the fifth dimension as timelike, rather than spacelike. After reviewing some key literature in the field including a Sundrum Lecture recommended by Martin Bauer and several articles by Paul Wesson linked over at The 5-D Spacetime-Matter Consortium, I have *undergone a conversion to the view that the fifth dimension needs to be spacelike* – not timelike – and specifically, that it needs to be a compact, spacelike hypercylinder. In this conversion, I am motivated by the following reasoning, which gives a geometric foundation to intrinsic spin:

I have shown in earlier parts of this Lab Note 2 that $dx^5/d\tau \propto q/m$, which is derived from requiring that the Lorentz force be a form of purely geodesic motion in 5-Dimensional spacetime. However, this result is *independent* of whether one chooses a timelike or spacelike fifth dimension. With all dimensional constants restored, and considering a bi-directional dx^5, this proportionality is given by the equivalence:

$$\frac{dx^5}{d\tau} \equiv \pm \frac{\sqrt{\hbar c \alpha}}{\sqrt{Gm}} = \pm \frac{1}{\sqrt{4\pi G}} \frac{q}{m}, \tag{1}$$

where, $\alpha = q^2/4\pi\hbar c$ is the dimensionless electromagnetic (running) coupling which approaches $\alpha \to 1/137.036$ at low energy. It will be appreciated that $\alpha = q^2/4\pi\hbar c$ specifies the strength of a unit charge (such as that of the charged leptons, e.g., electron), and that the equivalence between the first two terms is (importantly) independent of the system of units but the final term is in Heaviside-Lorentz units.

For a *timelike* fifth dimension, as I have discussed previously, x^5 may be drawn as an "axial time" axis orthogonal to x^0, and the physics ratio q/m (which, by the way, results in the q/m material body in an electromagnetic field actually "feeling" a Newtonian force in the sense of $F = ma$) measures the "angle" at which the material body moves through the x^5, x^0 "time plane."

But, for a *spacelike* fifth dimension, where a compactified, hyper-cylindrical $x^5 \equiv R\phi$ (see , the Sundrum Lecture Figure 1)) and R is a constant radius, $dx^5 \equiv Rd\phi$. Substituting this into (1), and inserting c into the first term to maintain a dimensionless equation given dx^5 now regarded as spacelike, then yields:

$$\frac{Rd\phi}{cd\tau} = \pm\frac{\sqrt{\hbar c\alpha}}{\sqrt{G}m} = \pm\frac{1}{\sqrt{4\pi G}}\frac{q}{m}. \qquad (2)$$

We see that here, the physics ratio q/m measures an "angular frequency" of fifth-dimensional rotation. Interestingly, *this frequency runs inversely to the mass*, and by classical principles, this means that the angular momentum is independent of the mass, i.e., constant. If one doubles the mass, one halves the tangential velocity, while the radius stays constant. Together with the $\pm$ factor, one might suspect that this constant angular momentum is related to intrinsic spin. In fact, following this hunch, one can arrive at an exact expression for the compactification radius R, in the following manner:

Assume that x^5 is spacelike, casting one's lot with the preponderance of those who study Kaluza-Klein theory. Move the c away from the first term and move the m over to the first term. Then, multiply all terms by another R. Everything is now dimensioned as an angular momentum, which we have just ascertained is constant irrespective of mass. So, set this all to $\pm\frac{1}{2}n\hbar$, which for $n=1$, represents intrinsic spin. The result is as follows:

$$m\frac{Rd\phi}{d\tau}R = \pm\sqrt{\frac{\hbar c^3\alpha}{G}}R = \pm\frac{c}{\sqrt{4\pi G}}qR = \pm\frac{1}{2}n\hbar. \qquad (3)$$

Now, take the second and fourth terms, and solve for R with $n=1$, to yield:

$$R = \frac{1}{2\sqrt{\alpha}}\sqrt{\frac{G\hbar}{c^3}} = \frac{1}{2\sqrt{\alpha}}L_P, \qquad (4)$$

where $L_P = \sqrt{G\hbar/c^3}$ is the Planck length. *This gives a definitive size for the compactification radius, and it is very close to the Planck length.* What is of interest, is that α is a *running* coupling. At low probe energies, where $\alpha \to 1/137.036$, $R = 5.853 \cdot L_P$. However, this is just the *apparent* radius from low energy. If one were to probe to a regime where α becomes large, say, of order unity, then $R = \frac{1}{2} L_P$, and is actually inside the black hole radius of the geometrodynamic vacuum "foam." [Misner, C. W., Wheeler, J. A., and Thorne, K. S., *Gravitation*, Freeman (1973)] at §43.4, [Wheeler, J. A., *On the Nature of Quantum Geometrodynamics*, Annals of Physics: 2, 604-614 (1957)] Since we have based the foregoing on a unit charge with spin ½, and since this is independent of the mass, the foregoing would appear to characterize the compactification radius R for all of the charged leptons, and to provide a geometric foundation for intrinsic spin!

I have changed my view over to the fifth dimension being spacelike, because of the foregoing reasoning, and specifically, because of the geometric foundation which this gives to intrinsic spin and the insight it gives into Plank-length physics.