Intrinsic Spin and the Kaluza-Klein Fifth Dimension

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Abstract:

Kaluza-Klein Theories provide a compelling unification of classical gravitation with classical electrodynamics, but have long been plagued by the perceived absence of physical evidence of a compactified, hypercylindrical, spacelike fifth dimension. We examine the possibility that this fifth dimension may in fact exist physically, and be fundamentally responsible for the quantized "intrinsic" spins which, with the exception of the hypothesized Higgs boson, are exhibited by all of the known elementary particles in nature.

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1. Introduction

The possibility of employing a fifth spacetime dimension to unite classical gravitation and electrodynamics has intrigued physicists for almost a century. [1], [2] Indeed, the feature of Kaluza-Klein theories which most firmly commends their serious consideration, is their ability to seamlessly unite Einstein's gravitation and Maxwell's electrodynamics. This includes their ability to represent the Lorentz Force motion of charged particles in an electromagnetic field as geodesic motion, and to accommodate both of Maxwell's equations and the Maxwell Stress-Energy tensor (See [2], pp. 71-73.) But the main perceived shortcoming of these theories, quite simply, is that nobody to date has been able to point to physical evidence of a fifth dimension. [3], [4] Especially for theories in which the fifth dimension is taken to be a compactified hypercylindrical spatial dimension, see, e.g., [5], Figure 1, the clear benefits of this gravitational and electrodynamic union has often been outweighed by skepticism about a curled-up fifth dimension which appears to have no observed (or even observable) physical manifestation.

We demonstrate here, that the compactified fifth dimension may be quite real physically, and that it many well manifest itself directly in the so-called "intrinsic spin" exhibited by all of the known elementary particles, with the exception of the hypothesized scalar Higgs. In short, it is provisionally suggested herein, subject to further consideration and scrutiny, that the compactified fifth dimension may well be the "intrinsic spin dimension."

2. Geodesic Motion in Five Dimensions

The foundation of Kaluza-Klein theory is a five-dimensional Riemannian geometry, often without any changes or enhancements, which merely extends the entire apparatus of gravitational theory into one more dimension. In five dimensions, $g_{\mathrm{MN}} \equiv g_{\mathrm{NM}}$ with uppercase Greek indexes $\mathrm{M},\mathrm{N} = 0,1,2,3,5$ may be used to denote the metric tensor, so $g_{\mu\nu}$ with lowercase $\mu,\nu = 0,1,2,3$ is the ordinary metric tensor in the spacetime subspace. Inverses are defined in the usual manner according to $g^{\mathrm{M}\Sigma} g_{\Sigma\mathrm{N}} = \delta^{\mathrm{M}}{}_{\mathrm{N}}$ and so $g^{\mathrm{M}\Sigma}$ and $g_{\Sigma\mathrm{N}}$ raise and lower indexes in the customary manner, but must be applied over all five dimensions to achieve proper five-covariance.

Kaluza-Klein theories typically maintain the usual interval in the 4-dimensional spacetime subspace, using $d\tau^2 = g_{\mu\nu} dx^\mu dx^\nu$, and define a five-space interval as:

$$
\begin{aligned}
dT^2 \equiv g_{MN} dx^M dx^N &= g_{\mu\nu} dx^\mu dx^\nu + g_{5\nu} dx^5 dx^\nu + g_{\mu 5} dx^\mu dx^5 + g_{55} dx^5 dx^5 \\
&= d\tau^2 + 2 g_{5\sigma} dx^5 dx^\sigma + g_{55} dx^5 dx^5
\end{aligned} \quad . \tag{2.1}
$$

The above is independent of whether the weak field $g_{55} \to \eta_{55} = \pm 1$, i.e., of whether the fifth dimension is timelike or spacelike, and is generally model-independent.

Like any metric equation, (2.1) can be algebraically-manipulated into:

$$
1 = g_{MN} \frac{dx^M}{dT} \frac{dx^N}{dT}, \tag{2.2}
$$

which is the first integral of the equation of motion. In five dimensions, the Christoffel connections may also be specified as $\Gamma^M{}_{\Sigma T} = \frac{1}{2} g^{MA} \left(g_{A\Sigma,T} + g_{TA,\Sigma} - g_{\Sigma T,A} \right)$ in the usual manner, hence $\Gamma^M{}_{\Sigma T} = \Gamma^M{}_{T\Sigma}$. Similarly, $g_{MN;\Sigma} = 0$ as usual, with the usual first rank covariant derivative $A^M{}_{;\Sigma} = A^M{}_{,\Sigma} + \Gamma^M{}_{A\Sigma} A^A$.

Thus, one may take the covariant derivative of each side of (2.2) above, and after the usual reductions employed in four dimensions, and multiplying the result through by $dT^2 / d\tau^2$, may arrive at a five-dimensional geodesic equation which bears an exact resemblance to the four-dimensional gravitational equation:

$$
\frac{d^2 x^M}{d\tau^2} + \Gamma^M{}_{\Sigma T} \frac{dx^\Sigma}{d\tau} \frac{dx^T}{d\tau} = 0. \tag{2.3}
$$

The above contains five independent equations. The four equations for which $M = \mu$, which specify motion in ordinary spacetime, are given by:

$$
\frac{d^2 x^\mu}{d\tau^2} + \Gamma^\mu{}_{\Sigma T} \frac{dx^\Sigma}{d\tau} \frac{dx^T}{d\tau} = 0. \tag{2.4}
$$

which expands, using the metric tensor symmetry $g_{MN} = g_{NM}$, to:

$$
\frac{d^2 x^\mu}{d\tau^2} + \Gamma^\mu{}_{\sigma\tau} \frac{dx^\sigma}{d\tau} \frac{dx^\tau}{d\tau} + 2\Gamma^\mu{}_{5\sigma} \frac{dx^5}{d\tau} \frac{dx^\sigma}{d\tau} + \Gamma^\mu{}_{55} \frac{dx^5}{d\tau} \frac{dx^5}{d\tau} = 0. \tag{2.5}
$$

Now, let us contrast (2.5) above to the gravitational geodesic equation which includes the Lorentz force law, namely, equation (20.41) of [6]:

$$\frac{d^2x^\mu}{d\tau^2}+\Gamma^\mu{}_{\sigma\tau}\frac{dx^\sigma}{d\tau}\frac{dx^\tau}{d\tau}-\frac{q}{m}F^\mu{}_\sigma\frac{dx^\sigma}{d\tau}=0\,. \tag{2.6}$$

The first two terms in (2.5) and (2.6) are identical, and they specify geodesic motion in an ordinary gravitational field absent any electrodynamic fields or sources. The absence of any mass or charge in the first two terms captures the Galilean principle of equivalence, and further expresses Newtonian inertial motion in a gravitational field via the Christoffel connections $\Gamma^\mu{}_{\sigma\tau}$. The way in which the Lorentz force becomes geodesic motion in Kaluza-Klein theories, is by the effective equivalence between the third terms in (2.5) and (2.6), that is, by virtue of the relationship:

$$2\Gamma^\mu{}_{5\sigma}\frac{dx^5}{d\tau}\frac{dx^\sigma}{d\tau}\equiv-\frac{q}{m}F^\mu{}_\sigma\frac{dx^\sigma}{d\tau}\,. \tag{2.7}$$

Whether one starts with the Lorentz force and derives other aspects of Kaluza-Klein, or starts elsewhere and derives the Lorentz force, is irrelevant. No matter where one starts, somewhere along the line, Kaluza-Klein will demonstrate that one of its relationships, is the one given by (2.7) above.

3. The Spacetime Metric and Electromagnetic Field Strength Tensors.

The equation in Klein's [2] between (6) and (7) establishes the connection between the metric tensor and the electromagnetic field strength tensor, and its upshot may be represented in the notation employed here as $F_{\mathrm{MM}} \propto g_{5\mathrm{M,N}} - g_{5\mathrm{N,M}}$.

Let us formalize this connection a bit further. If we define:

$$\Gamma^{\mathrm{M}}{}_{5\Sigma}\equiv\tfrac{1}{4}b\overline{\kappa}F^{\mathrm{M}}{}_{\Sigma}\,, \tag{3.1}$$

where b is a dimensionless numeric constant of proportionality and $\overline{\kappa}=\sqrt{16\pi G/c^4}$ is the constant employed, for example, in the expression $g_{\mathrm{MN}}\equiv\eta_{\mathrm{MN}}+\overline{\kappa}h_{\mathrm{MN}}$ from gravitational theory, then it is easy to show that (3.1) above is but an equivalent restatement of Klein's relationship set forth above.

To demonstrate this equivalence, we simply require the field strength tensor to be antisymmetric $F^{\mathrm{MN}}\equiv-F^{\mathrm{NM}}$ in the usual manner, including its fifth dimensional components.

Then, we can employ the Christoffels $\Gamma^{\rm M}{}_{5\rm T} = \frac{1}{2} g^{\rm MA}\left(g_{\rm A5,T} + g_{\rm TA,5} - g_{\rm 5T,A}\right)$ to write $F^{\rm MN} = -F^{\rm NM}$ completely in terms of the metric tensor $g_{\rm MN}$ and its first derivatives, as:

$$\tfrac{1}{4} b\bar{\kappa} F^{\rm MN} = -\tfrac{1}{4} b\bar{\kappa} F^{\rm NM} = g^{\rm MA} g^{\Sigma \rm N}\left(g_{\rm A5,\Sigma} + g_{\Sigma \rm A,5} - g_{5\Sigma,\rm A}\right) = -g^{\rm NA} g^{\Sigma \rm M}\left(g_{\rm A5,\Sigma} + g_{\Sigma \rm A,5} - g_{5\Sigma,\rm A}\right). \quad (3.2)$$

Renaming indexes, and using the symmetry of the metric tensor, this is readily reduced to:

$$g^{\rm M\Sigma} g^{\rm TN} g_{\rm T\Sigma,5} = 0 . \quad (3.3)$$

Then, using the inverse relationship $g^{\rm TN} g_{\rm T\Sigma} = \delta^{\rm N}{}_{\Sigma}$, which we can differentiate to obtain $\left(g^{\rm TN} g_{\rm T\Sigma}\right)_{,\rm A} = g^{\rm TN}{}_{,\rm A} g_{\rm T\Sigma} + g^{\rm TN} g_{\rm T\Sigma,A} = 0$, i.e., $g^{\rm TN} g_{\rm T\Sigma,A} = -g^{\rm TN}{}_{,\rm A} g_{\rm T\Sigma}$, with $\rm A = 5$, we may reduce (3.3) to the very simple expressions, for both the covariant and contravariant metric tensor:

$$g^{\rm MN}{}_{,5} = 0 ; \; g_{\rm MN,5} = 0 . \quad (3.4)$$

This is the second of Klein's "special assumptions" on page 68 of [2], in the notation presently employed, that "the quantities $g_{\rm MN}$ must not depend on the fifth coordinate x^5." In other words, Klein's second "special assumption" is, simply, the perfectly-reasonable $F^{\rm MN} \equiv -F^{\rm NM}$.

But, returning to the main point, this means that with (3.4), (3.1) expands to:

$$\Gamma^{\rm M}{}_{5\Sigma} \equiv \tfrac{1}{4} b\bar{\kappa} F^{\rm M}{}_{\Sigma} = \tfrac{1}{2} g^{\rm MA}\left(g_{\rm A5,\Sigma} + g_{\Sigma \rm A,5} - g_{5\Sigma,\rm A}\right) = \tfrac{1}{2} g^{\rm MA}\left(g_{\rm 5A,\Sigma} - g_{5\Sigma,\rm A}\right), \quad (3.5)$$

which lowers to:

$$g_{\rm TM} b\bar{\kappa} F^{\rm M}{}_{\Sigma} = \tfrac{1}{4} b\bar{\kappa} F_{\rm T\Sigma} = \tfrac{1}{2} g_{\rm TM} g^{\rm MA}\left(g_{\rm 5A,\Sigma} - g_{5\Sigma,\rm A}\right) = \tfrac{1}{2}\left(g_{\rm 5T,\Sigma} - g_{5\Sigma,\rm T}\right). \quad (3.6)$$

So, via $F^{\rm MN} \equiv -F^{\rm NM}$, this is just another way of representing Klein's $F_{\rm MM} \propto g_{\rm 5M,N} - g_{\rm 5N,M}$.

We pause briefly before proceeding, to examine the term $\Gamma^{\mu}{}_{55} \dfrac{dx^5}{d\tau}\dfrac{dx^5}{d\tau}$ in the geodesic equation (2.5). Using (3.4), a.k.a. $F^{\rm MN} \equiv -F^{\rm NM}$, to reduce, the Christoffel connection $\Gamma^{\mu}{}_{55} = \frac{1}{2} g^{\mu \rm A}\left(g_{\rm A5,5} + g_{\rm 5A,5} - g_{\rm 55,A}\right) = -\frac{1}{2} g_{55}{}^{,\mu}$. Whether $\Gamma^{\mu}{}_{55}$ is zero or non-zero, depends upon the constancy or not, of g_{55}. As Klein notes in [2] on page 68, after examining five-dimensional coordinate transformations, "the assumption $g_{55} = \text{contant}$ is therefore allowed." We take no position here on this question, as it is not relevant to the analysis to follow.

4. Introduction of a Compactified, Spacelike Fifth Dimension

Now, we return to (2.7), and substitute (3.1) as $\Gamma^{\mu}{}_{5\sigma} \equiv \frac{1}{4} b\bar{\kappa} F^{\mu}{}_{\sigma}$, to obtain:

$$\tfrac{1}{2} b\bar{\kappa} F^{\mu}{}_{\sigma} \frac{dx^5}{d\tau}\frac{dx^\sigma}{d\tau} = -\frac{q}{m} F^{\mu}{}_{\sigma} \frac{dx^\sigma}{d\tau} . \tag{4.1}$$

Reducing leaves us with:

$$\frac{dx^5}{d\tau} = -\frac{2}{b\bar{\kappa}}\frac{q}{m} = -\frac{2c^2}{b\sqrt{16\pi G}}\frac{q}{m} , \tag{4.2}$$

Irrespective of whether the fifth dimension is timelike or spacelike, we take dx^5 to be given in dimensions of time, so that $dx^5/d\tau$ is a dimensionless ratio. When taking the fifth dimension to be spacelike, one need merely divides through by c.

We now consider the unit charge of a single charged lepton, such as the electron, or its mu or tau partners. In rationalized Heaviside-Lorentz units which we shall use here, with fundamental constants restored, the electric charge strength q for such a unit charge is related to the dimensionless (running) coupling by $\alpha = q^2/4\pi\hbar c$ which approaches $\alpha \to 1/137.036$ at low energy. The value of α is the same in all systems of units but the numerical value of q is different. Making use of the inverse $q = \sqrt{4\pi\hbar c\alpha}$, we extend (4.2) above to:

$$\frac{dx^5}{d\tau} = -\frac{2}{b\bar{\kappa}}\frac{q}{m} = -\frac{2c^2}{b\sqrt{16\pi G}}\frac{q}{m} = -\frac{c^2}{b}\frac{\sqrt{\hbar c\alpha}}{\sqrt{G}m} . \tag{4.3}$$

Now, $dx^5/d\tau$ is dimensionless, as is $\dfrac{\sqrt{\hbar c\alpha}}{\sqrt{G}m}$. Therefore, to restore complete dimensional balance, we divide the final three terms in the above through by c^2. Thus:

$$\frac{dx^5}{d\tau} = -\frac{2}{b\bar{\kappa}c^2}\frac{q}{m} = -\frac{2}{b\sqrt{16\pi G}}\frac{q}{m} = -\frac{1}{b}\frac{\sqrt{\hbar c\alpha}}{\sqrt{G}m} . \tag{4.4}$$

is completely balanced, dimensionally. Transforming into an "at rest" frame, $dx^1 = dx^2 = dx^3 = 0$, the spacetime metric equation $d\tau^2 = g_{\mu\nu}dx^\mu dx^\nu$ reduces to $d\tau = \pm\sqrt{g_{00}}dx^0$, and (4.4) becomes:

$$\frac{dx^5}{dx^0} = \pm\sqrt{g_{00}}\,\frac{2}{b\bar{\kappa}c^2}\frac{q}{m} = \pm\frac{2}{b}\sqrt{\frac{g_{00}}{16\pi G}}\frac{q}{m} = \pm\frac{1}{b}\frac{\sqrt{\hbar c g_{00}\alpha}}{\sqrt{G}m}\,. \tag{4.5}$$

Now, following most people who study Kaluza-Klein, let's take the fifth dimension to be spacelike, and compactified into a hypercylinder $x^5 \equiv R\phi$ (see [7], Figure 1). We further, as is customary, take R to be a constant radius, so that $dx^5 \equiv Rd\phi$. We then substitute this into (4.4), while maintaining the $\pm$ introduced in (4.5) (which can also be based on the $\pm$ charges of the electron and positron), so as to write:

$$\frac{Rd\phi}{cd\tau} = \pm\frac{2}{b\bar{\kappa}c^2}\frac{q}{m} = \pm\frac{2}{b\sqrt{16\pi G}}\frac{q}{m} = \pm\frac{1}{b}\frac{\sqrt{\hbar c\alpha}}{\sqrt{G}m}\,. \tag{4.6}$$

Note that we have also put an extra c into the denominator of the first term to maintain all terms as completely dimensionless. Now, let's study (4.6).

5. A Possible Connection Between the Fifth Dimension and Intrinsic Spin

Taking R to be constant, it is noteworthy that the physics ratio q/m measures an "angular frequency" $d\phi/d\tau$ of fifth-dimensional rotation. Of special interest, *this frequency runs inversely to the mass*. If one doubles the mass, one halves the tangential velocity, and if the radius stays constant, then so too does the angular momentum. In other words, by classical principles, this means that the angular momentum remains constant, no matter what the mass. If the charged lepton is a muon rather than an electron, then $d\phi/d\tau \propto 1/m$ will vary inversely to the mass, i.e., the fifth-dimensional rotation will be slower by that ratio of the muon mass to the electron mass. But the angular momentum will remain the same. Together with the $\pm$ factor, one might suspect that this constant angular momentum is, by virtue of its constancy independently of mass, related to intrinsic spin. In fact, following this line of thought, one can arrive at an exact expression for the compactification radius R, by the following "semi-classical" analysis:

In (4.6), move the c away from the first term and move the m over to the first term. Then, multiply all terms by another R. Everything is now dimensioned as an angular momentum $m \cdot v \cdot R$, with $v = \frac{Rd\phi}{d\tau}$, which we have just ascertained is constant irrespective of mass. Because this angular momentum is independent of the mass, let us *define* this angular momentum, by hypothesis, as the *intrinsic spin* $\pm\frac{1}{2}\hbar$ of the subject charged lepton. The result is then as follows:

$$m\frac{Rd\phi}{d\tau}R=\pm\frac{2}{b\bar{\kappa}c}qR=\pm\frac{2c}{b\sqrt{16\pi G}}qR=\pm\frac{1}{b}\frac{\sqrt{\hbar c^3\alpha}}{\sqrt{G}}R\equiv\pm\frac{1}{2}\hbar\,. \tag{5.1}$$

Now, we may take the final two terms and solve for R, which yields:

$$R=\frac{b}{2\sqrt{\alpha}}\sqrt{\frac{G\hbar}{c^3}}=\frac{b}{2\sqrt{\alpha}}L_P\,, \tag{5.2}$$

where $L_P=\sqrt{G\hbar/c^3}$ is the Planck length. *This intrinsic spin hypothesis gives a definitive size for the compactification radius, and it is very close to the Planck length.*[*] Because we have based the foregoing on a unit charge with spin ½, and since this radius R in (5.2) is *independent of the mass* of such a spin ½ unit charge, the foregoing would appear to characterize the compactification radius R for all of the charged leptons, and to provide a classical geometric foundation for the quantum mechanical phenomenon of intrinsic spin. For $\alpha=1$ or on the order of unity, the compactification radius of the fifth dimension for the charged leptons appears to be very close to the Planck length Wheeler's geometrodynamic vacuum "foam" and the Schwarzschild radius of the vacuum. [6] at §43.4, [8][**]

From (5.1), it appears at first sight that for a neutral body, $q=0$, such as the neutrino, we have $d\phi/d\tau=0$, and so there is no fifth-dimensional rotation. One might take this to suggest that the neutrino has no intrinsic spin, which is, of course, contradicted by empirical knowledge. However, in electroweak theory, we know that $Q\equiv q=Y+I^3$, so there are in fact other charge generators implicit in (5.1). Following this, it is plausible that this question about the neutrino

[*] In a complete Kaluza-Klein analysis, which we shall not do here, it turns out that when one considers the way in which the Maxwell's stress energy tensor embeds into Kaluza-Klein, the fifth dimension must in fact be spacelike, g_{55} must be constant, and $b^2=8$, so that so (5.2) becomes $R=L_P\sqrt{2/\alpha}$.

[**] By way of review, the Planck mass, defined from the term atop Newton's law as a mass for which $GM_P{}^2=\hbar c$, is thus $M_P=\sqrt{\hbar c/G}$. In the geometrodynamic vacuum, the negative gravitational energy between Planck masses separated by the Planck length $L_P=\sqrt{G\hbar/c^3}$ precisely counterbalances and cancels the positive energy of the Planck masses themselves. The Schwarzschild radius of a Plank mass $R_S=2GM_P/c^2=2\sqrt{G\hbar/c^3}=2L_P$.

may be resolved if one considers Kaluza-Klein in a non-Abelian (Yang-Mills) $SU(2)_W xU(1)_Y$ rather than the present abelian $U(1)_{em}$ context, because the neutrino will then have a non-zero weak isospin $I^3 = +\frac{1}{2}$ to lay a geodesic foundation for its intrinsic spin, similarly to q above for the charged leptons. It should also be recognized that in the context of a Kaluza-Klein theory based on $U(1)_{em}$, one really cannot speak anyway, about any particles other than charged leptons and photons. It should be observed also, that the intrinsic spin interpretation (5.1) suggests conversely, that any elementary scalar particle which has no intrinsic spin, must be electrically neutral. This is, in fact, true of the hypothesized Higgs boson.

So, is it possible that the compactified fifth dimension of Kaluza-Klein, which has long begged for a physical foundation, might in fact the Riemannian geometric foundation of intrinsic spin? If one is scrupulous about the use of language, it seems that use of the term "intrinsic" to describe an inherent quantized angular momentum of elementary particles, covers up what is actually a deep ignorance of what "intrinsic spin" really means, geometrically. Why? For a material body to have an angular momentum, there must implicitly be a radius R with which that body circles about a rotational origin. Even the smallest objects, if they have an angular momentum, must be rotating or spinning – at some finite spatial radius – about an origin. At the same time, nobody believes that intrinsic spin represents an angular momentum about a radius R in the three usual spatial dimensions. By associating intrinsic spin with motion through a fourth, compactified, hyper-cylindrical spatial dimension, one simultaneously makes sense of intrinsic spin and of a compact fourth spatial dimension. The material body now has a spatial radius R of rotation through a spatial dimension other than the usual three spatial dimensions to give meaning to its "intrinsic" spin, and the compactified fourth dimension now takes on real, physical meaning as something which is physically observed, via the phenomenon of intrinsic spin, and not merely a fictional idea that gives people pause about Kaluza-Klein theories specifically, and dimensional compactification in general.

In sum, the understanding of intrinsic spin as cyclical motion through a fourth dimension of space which is curled up into a radius on the order of the Planck length, if this can be developed further and sustained, may be useful to overcome one of the most nagging objections about Kaluza-Klein theories, and would underscore a clearly-observed, physical manifestation of the fourth space dimension, rather than requiring one to reply, with some disingenuity, that the extra space dimension is too small so nobody will ever see it anyway. Thus, we conclude with the provisional hypothesis, to be studied in other contexts and for other elementary particles, that the fourth spatial dimension may best be thought of as the “intrinsic spin dimension” of a real, physical, five-dimensional spacetime.

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