Hello to all:

In a recent thread, I asked about wavefunctions, and am especially inclined toward something of the form $\psi(x) \propto e^{-\frac{1}{2}Ax^2 - V(x) + Bx}$ because a) this is very general, b) this is similar to what Zee (*QFT in an Nutshell*) refers to as the "Central Identity of Quantum Field Theory," and c) the $V(x)$ takes this wavefunction off-Gaussian, that is, after Fourier transforming to $\psi(p)$ and obtaining variances, it leads to an uncertainty $\Delta x \Delta p > \hbar / 2$ that stems from the $V(x)$ term. As you may glean from my other posts, I am hoping to pin this upward deviation from $\hbar / 2$ to the magnetic moment anomaly.

If I use this wavefunction $\psi(x) \propto e^{-\frac{1}{2}Ax^2 - V(x) + Bx}$, then I am of course working with the integral which underlies the "Central Identity of Quantum Field Theory," given on Zee page 460:

$$\int d\varphi\, e^{-\frac{1}{2}\varphi \cdot K \cdot \varphi - V(\varphi) + J\varphi} = e^{-V\left(\frac{\delta}{\delta J}\right)} e^{\frac{1}{2} J \cdot K^{-1} \cdot J} \quad . \quad (1)$$

It is noted that Zee leaves out the numerical factors which in Gaussian integration generally stem from the form $\sqrt{2\pi / K}$ of the basic Gaussian integral.

Am I correct to believe that the underlying Gaussian integral for (1) is:

$$\int e^{-\frac{1}{2}Ax^2 - V(x) + Bx} dx = \int e^{-V(x)} e^{-\frac{1}{2}Ax^2 + Bx} dx = \sqrt{\frac{2\pi}{A}} e^{-V\left(\frac{d}{dB}\right)} e^{\frac{B^2}{2A}} \,? \quad (2)$$

I note, in particular, that:

$$x = \frac{d}{dB} \ln e^{-\frac{1}{2}Ax^2 + Bx} \qquad (3)$$

which accounts for $V(x) \Rightarrow V\left(\frac{d}{dB} \ln e^{-\frac{1}{2}Ax^2 + Bx}\right)$, though I am trying to pinpoint in the simplest terms possible how the $\ln e^{-\frac{1}{2}Ax^2 + Bx}$ factor gets stripped off so that we get from $V(x) \Rightarrow V(d / dB)$, where d / dB is simply an operator without an operand. It seems to me like the key to this is in the term $e^{-V(x)} e^{-\frac{1}{2}Ax^2 + Bx} = e^{-V\left(\frac{d}{dB} \ln e^{-\frac{1}{2}Ax^2 + Bx}\right)} e^{-\frac{1}{2}Ax^2 + Bx}$ in (2), though I am not sure how to nail this firmly to the right hand side of (2).

Is there some simple, direct calculation starting with the above to illustrate the precise origin of $V(d / dB)$ better? Any help is appreciated.

Thanks, Jay