

Intrinsic Spin and the Kaluza-Klein Fifth Dimension

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Abstract:

Kaluza-Klein Theories provide a compelling unification of classical gravitation with classical electrodynamics, but have long been plagued by the perceived absence of physical evidence of a compactified, hypercylindrical, spacelike fifth dimension. We examine the possibility that this fifth dimension may in fact exist physically, and be fundamentally responsible for the quantized “intrinsic” spins which, with the exception of the hypothesized Higgs boson, are exhibited by all of the known elementary particles in nature.

PACS: 04.50.Cd

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1. Introduction

The possibility of employing a fifth spacetime dimension to unite classical gravitation and electrodynamics has intrigued physicists for almost a century. [1], [2] Indeed, the feature of Kaluza-Klein theories which most firmly commends their serious consideration, is their ability to seamlessly unite Einstein's gravitation and Maxwell's electrodynamics. This includes their ability to represent the Lorentz force motion of charged particles in an electromagnetic field as geodesic motion, and to accommodate both of Maxwell's equations and the Maxwell Stress-Energy tensor (See [2], pp. 71-73.) But the main perceived shortcoming of these theories, quite simply, is that nobody to date has been able to point to physical evidence of a fifth dimension. [3], [4] Especially for theories in which the fifth dimension is taken to be a compactified hypercylindrical spatial dimension, see, e.g., [5], Figure 1, the clear benefits of this gravitational and electrodynamic union have often been outweighed by skepticism about a curled-up fifth dimension which appears to have no observed (or even observable) physical manifestation.

We demonstrate here, that the compactified fifth dimension may be quite real physically, and that it may well manifest itself directly in the so-called "intrinsic spin" exhibited by all of the known elementary particles, with the exception of the hypothesized scalar Higgs. In short, it is provisionally suggested herein, subject to further consideration, development, and scrutiny, that the compactified fifth dimension may well be the "intrinsic spin dimension."

2. Geodesic Motion in Five Dimensions

The foundation of Kaluza-Klein theory is a five-dimensional Riemannian geometry, often without any changes or enhancements, which merely extends the entire apparatus of gravitational theory into one more dimension. In five dimensions, $g_{MN} \equiv g_{NM}$ with uppercase Greek indexes $M, N = 0, 1, 2, 3, 5$ may be used to denote the metric tensor, so $g_{\mu\nu}$ with lowercase $\mu, \nu = 0, 1, 2, 3$ is the ordinary metric tensor in the spacetime subspace. Inverses are defined in the usual manner according to $g^{M\Sigma} g_{\Sigma N} = \delta^M_N$ and so $g^{M\Sigma}$ and $g_{\Sigma N}$ raise and lower indexes in the customary manner, but must be applied over all five dimensions to achieve proper five-covariance.

Kaluza-Klein theories typically maintain the usual interval in the 4-dimensional spacetime subspace, using $d\tau^2 = g_{\mu\nu} dx^\mu dx^\nu$, and define a five-space interval as:

$$\begin{aligned}
d\Gamma^2 &\equiv g_{MN} dx^M dx^N = g_{\mu\nu} dx^\mu dx^\nu + g_{5\nu} dx^5 dx^\nu + g_{\mu 5} dx^\mu dx^5 + g_{55} dx^5 dx^5 \\
&= d\tau^2 + 2g_{5\sigma} dx^5 dx^\sigma + g_{55} dx^5 dx^5 .
\end{aligned} \tag{2.1}$$

The above is independent of whether in the weak field linear approximation, $g_{55} \rightarrow \eta_{55} = \pm 1$, i.e., of whether the fifth dimension is timelike or spacelike, and is generally model-independent.

Like any metric equation, (2.1) can be algebraically-manipulated into:

$$1 = g_{MN} \frac{dx^M}{dT} \frac{dx^N}{dT}, \tag{2.2}$$

which is the first integral of the equation of motion.

In five dimensions, the Christoffel connections may also be specified in the usual manner as $\Gamma^M_{\Sigma T} = \frac{1}{2} g^{MA} (g_{A\Sigma, T} + g_{TA, \Sigma} - g_{\Sigma T, A})$, hence $\Gamma^M_{\Sigma T} = \Gamma^M_{T\Sigma}$. Similarly, $g_{MN, \Sigma} = 0$ as usual, with the usual first rank covariant derivative $A^M_{, \Sigma} = A^M_{, \Sigma} + \Gamma^M_{A\Sigma} A^A$. Thus, one may take the covariant derivative of each side of (2.2) above, and after the usual reductions employed in four dimensions, and multiplying the result through by $d\Gamma^2 / d\tau^2$, may arrive at a five-dimensional geodesic equation:

$$\frac{d^2 x^M}{d\tau^2} + \Gamma^M_{\Sigma T} \frac{dx^\Sigma}{d\tau} \frac{dx^T}{d\tau} = 0, \tag{2.3}$$

which bears an exact resemblance to the four-dimensional gravitational equation.

The above contains five independent equations. The four equations for which $M = \mu$, which specify motion in ordinary spacetime, are given by:

$$\frac{d^2 x^\mu}{d\tau^2} + \Gamma^\mu_{\Sigma T} \frac{dx^\Sigma}{d\tau} \frac{dx^T}{d\tau} = 0. \tag{2.4}$$

which expands, using the metric tensor symmetry $g_{MN} = g_{NM}$, to:

$$\frac{d^2 x^\mu}{d\tau^2} + \Gamma^\mu_{\sigma\tau} \frac{dx^\sigma}{d\tau} \frac{dx^\tau}{d\tau} + 2\Gamma^\mu_{5\sigma} \frac{dx^5}{d\tau} \frac{dx^\sigma}{d\tau} + \Gamma^\mu_{55} \frac{dx^5}{d\tau} \frac{dx^5}{d\tau} = 0. \tag{2.5}$$

The $M = 5$ equation, specifies fifth-dimensional ‘‘acceleration,’’ $d^2 x^5 / d\tau^2$.

Now, let us contrast (2.5) above to the gravitational geodesic equation which includes the Lorentz force law, namely, equation (20.41) of [6]:

$$\frac{d^2 x^\mu}{d\tau^2} + \Gamma^\mu_{\sigma\tau} \frac{dx^\sigma}{d\tau} \frac{dx^\tau}{d\tau} - \frac{q}{m} F^\mu_{\sigma} \frac{dx^\sigma}{d\tau} = 0. \quad (2.6)$$

The first two terms in (2.5) and (2.6) are identical, and they specify geodesic motion in an ordinary gravitational field absent any electrodynamic fields or sources. The absence of any mass or charge in the first two terms captures the Galilean principle of equivalence, and further expresses Newtonian inertial motion in a gravitational field via the Christoffel connections $\Gamma^\mu_{\sigma\tau}$. The way in which the Lorentz force becomes geodesic motion in Kaluza-Klein theories, is by the effective equivalence between the third terms in (2.5) and (2.6), that is, by virtue of the relationship: (see, e.g., [7], eqs. (13) versus (14), [8])

$$2\Gamma^\mu_{5\sigma} \frac{dx^5}{d\tau} \frac{dx^\sigma}{d\tau} = -\frac{q}{m} F^\mu_{\sigma} \frac{dx^\sigma}{d\tau}. \quad (2.7)$$

Whether one starts with the Lorentz force and derives other aspects of Kaluza-Klein, or starts elsewhere and derives the Lorentz force, is irrelevant. No matter where one starts, somewhere along the line, if the Lorentz force is to constitute geodesic motion, then Kaluza-Klein theories must have as one of their relationships, the one given by (2.7) above.

3. The Spacetime Metric and Electromagnetic Field Strength Tensors

The equation in Klein's original paper [2] between (6) and (7) establishes the relationship between the metric tensor and the five-dimensional electromagnetic field strength tensor, and may be represented in the notation employed here as $F_{MN} \propto g_{5M,N} - g_{5N,M}$.

Let us formalize the above relationship of Klein's a bit further. If we define:

$$\Gamma^M_{5\Sigma} \equiv -\frac{1}{4} b \bar{\kappa} F^M_{\Sigma}, \quad (3.1)$$

where b is a dimensionless numeric constant of proportionality and $\bar{\kappa} = \sqrt{16\pi G/c^4}$ is the constant employed, for example, in the expression $g_{MN} = \eta_{MN} + \bar{\kappa} h_{MN}$ from gravitational theory, then it is easy to show that (3.1) above is but an equivalent restatement of Klein's relationship set forth above.

To demonstrate this equivalence, we simply require the field strength tensor to be antisymmetric $F^{MN} \equiv -F^{NM}$ in the usual manner, including its fifth dimensional components.

Then, we can employ the Christoffels $\Gamma^M_{5T} = \frac{1}{2} g^{MA} (g_{A5,T} + g_{TA,5} - g_{5T,A})$ to write $F^{MN} = -F^{NM}$ completely in terms of the metric tensor g_{MN} and its first derivatives, as:

$$\frac{1}{4} b \bar{\kappa} F^{MN} = -\frac{1}{4} b \bar{\kappa} F^{NM} = -g^{MA} g^{\Sigma N} (g_{A5,\Sigma} + g_{\Sigma A,5} - g_{5\Sigma,A}) = g^{NA} g^{\Sigma M} (g_{A5,\Sigma} + g_{\Sigma A,5} - g_{5\Sigma,A}). \quad (3.2)$$

Renaming indexes, and using the symmetry of the metric tensor, this is readily reduced to:

$$g^{M\Sigma} g^{TN} g_{T\Sigma,5} = 0. \quad (3.3)$$

Then, using the inverse relationship $g^{TN} g_{TE} = \delta^N_E$, we can differentiate to obtain $(g^{TN} g_{TE})_{,A} = g^{TN}{}_{,A} g_{TE} + g^{TN} g_{TE,A} = 0$, i.e., $g^{TN} g_{TE,A} = -g^{TN}{}_{,A} g_{TE}$. With $A=5$, we may reduce (3.3) to the very simple expressions, for both the covariant and contravariant metric tensor:

$$g^{MN}{}_{,5} = 0; g_{MN,5} = 0. \quad (3.4)$$

This is the second of Klein's "special assumptions" on page 68 of [2], in the notation presently employed, that "the quantities g_{MN} must not depend on the fifth coordinate x^5 ." In other words, Klein's second "special assumption" is deduced from the perfectly-reasonable $F^{MN} \equiv -F^{NM}$.

So, returning to the main point, this means that with (3.4), (3.1) expands to:

$$\Gamma^M_{5\Sigma} = -\frac{1}{4} b \bar{\kappa} F^M_{\Sigma} = -\frac{1}{2} g^{MA} (g_{A5,\Sigma} + g_{\Sigma A,5} - g_{5\Sigma,A}) = \frac{1}{2} g^{MA} (g_{5\Sigma,A} - g_{5A,\Sigma}), \quad (3.5)$$

which lowers to:

$$g_{TM} b \bar{\kappa} F^M_{\Sigma} = \frac{1}{4} b \bar{\kappa} F_{T\Sigma} = -\frac{1}{2} g_{TM} g^{MA} (g_{5A,\Sigma} - g_{5\Sigma,A}) = \frac{1}{2} (g_{5\Sigma,T} - g_{5T,\Sigma}). \quad (3.6)$$

So, via $F^{MN} \equiv -F^{NM}$, (3.1) is just another way of representing Klein's $F_{MN} \propto g_{5M,N} - g_{5N,M}$, with the addition of a constant of proportionality.

We pause briefly before proceeding, to examine the term $\Gamma^\mu_{55} \frac{dx^5}{d\tau} \frac{dx^5}{d\tau}$ in the geodesic equation (2.5). Using $g_{MN,5} = 0$ from (3.4), a.k.a. $F^{MN} \equiv -F^{NM}$, to reduce, this connection is $\Gamma^\mu_{55} = \frac{1}{2} g^{\mu A} (g_{A5,5} + g_{5A,5} - g_{55,A}) = -\frac{1}{2} g_{55}{}^{,\mu}$. Whether Γ^μ_{55} is zero or non-zero, thereby depends upon the constancy or not, of g_{55} , i.e., on the oft-debated question of whether or not the theory is "restricted." As Klein notes in [2] on page 68, after examining five-dimensional coordinate

transformations, “the assumption $g_{55} = \text{constant}$ is . . . allowed,” and Leibowitz and Rosen in 1973 overcame earlier perceived problems with such a restriction. [9] It can be deduced that from the foregoing that g_{55} must in fact be constant, in the following manner:

Starting with (3.1) and the Christoffel definition and $g_{MN,5} = 0$ from (3.4), write:

$$\Gamma^M_{55} = -\frac{1}{2} g^{MA} g_{55,A} = -\frac{1}{2} g_{55}^{,M} = -\frac{1}{4} \bar{b\kappa} F^M_{55}. \quad (3.7)$$

Then, lower the last two terms to $-\frac{1}{2} g_{55,M} = -\frac{1}{4} \bar{b\kappa} F_{M5}$. Then, use the covariant (lower-index) relation $F_{MN} = -F_{NM}$ to write:

$$\frac{1}{2} \bar{b\kappa} F_{M5} = g_{55,M} = -\frac{1}{2} \bar{b\kappa} F_{5M} = g_{5M,5} = 0, \text{ i.e., } g_{55,M} = 0 \quad (3.8)$$

where the final zero is again from (3.4). If $g_{55,M} = 0$, then $g_{55} = \text{constant}$, and so the condition

$F_{MN} = -F_{NM}$ does indeed “restrict” the theory. With $g_{55} = \text{constant}$, $\Gamma^\mu_{55} = -\frac{1}{2} g_{55}^{,\mu} = 0$, and the

final term $\Gamma^\mu_{55} \frac{dx^5}{d\tau} \frac{dx^5}{d\tau}$ does indeed drop out from the geodesic equation (2.5).

4. Introduction of a Compactified, Spacelike Fifth Dimension

Now, we return to (2.7), and substitute (3.1) as $\Gamma^\mu_{5\sigma} \equiv -\frac{1}{4} \bar{b\kappa} F^\mu_{\sigma}$, to obtain:

$$\frac{1}{2} \bar{b\kappa} F^\mu_{\sigma} \frac{dx^5}{d\tau} \frac{dx^\sigma}{d\tau} = \frac{q}{m} F^\mu_{\sigma} \frac{dx^\sigma}{d\tau}. \quad (4.1)$$

Reducing leaves us with:

$$\frac{dx^5}{d\tau} = \frac{2}{\bar{b\kappa}} \frac{q}{m} = \frac{2c^2}{b\sqrt{16\pi G}} \frac{q}{m} \quad (4.2)$$

Irrespective of whether the fifth dimension is timelike or spacelike, we take dx^5 to be given in dimensions of time, so $dx^5/d\tau$ is a dimensionless ratio. When taking the fifth dimension to be spacelike, one need merely divide through by c . What (4.2) clearly states, is that the charge-to-mass ratio is a measure of motion through the x^5 dimension. If x^5 is “curled up,” and spacelike, then motion through the x^5 dimension should manifest as angular motion. We will now explore the possible connection between such angular motion, and intrinsic spin.

First, we ask: can we apply (4.2) to individual particles such as the charged leptons? After all, the term $F^\mu{}_\sigma(dx^\sigma/d\tau)$ factored out from (4.1) is related to the Lorentz force equation of motion which is classical, and does not extend to individual charges other than to describe their “expected” motion. However, what is left behind in (4.2) is a q/m term which can be sensibly-considered *even for individual particles* such as the charged leptons, since these do have a definite, certain charge and a definite, certain mass. In effect, in going from (4.1) to (4.2), we have factored out that part of the Lorentz force which can only be regarded classically, and left behind that part which has precise, certain meaning even for individual charged particles, namely, the q/m ratio.

Let us therefore, consider a set of $Q = 1, 2, 3, \dots$ (already negative, by (2.6)) Coulomb charges, such as the electron, or its mu or tau partners (we disregard fractional quark charges in this analysis). In rationalized Heaviside-Lorentz units which we shall use here, with fundamental constants restored, the electric charge strength q for an individual charge is related to the dimensionless (running) electromagnetic coupling by $\alpha = Q^2 q^2 / 4\pi\hbar c$, which approaches $\alpha \rightarrow 1/137.036$ at low energy. The numerical value of α is the same in all systems of units but the numerical value of q is different. Making use of the inverse $q = \sqrt{4\pi\hbar c \alpha}$, we thereby extend (4.2) above to:

$$\frac{dx^5}{d\tau} = \frac{2}{b\kappa} \frac{Qq}{m} = \frac{2c^2}{b\sqrt{16\pi G}} \frac{Qq}{m} = \frac{c^2 Q}{b} \frac{\sqrt{\hbar c \alpha}}{\sqrt{Gm}}. \quad (4.3)$$

In the above, $dx^5/d\tau$ is dimensionless, as is $\sqrt{\hbar c \alpha} / \sqrt{Gm}$. Therefore, to restore complete dimensional balance, with all fundamental constants properly restored, we divide the final three terms in the above through by c^2 , to obtain:

$$\frac{dx^5}{d\tau} = \frac{2}{b\kappa c^2} \frac{Qq}{m} = \frac{2}{b\sqrt{16\pi G}} \frac{Qq}{m} = \frac{Q}{b} \frac{\sqrt{\hbar c \alpha}}{\sqrt{Gm}}. \quad (4.4)$$

Now, following many who study Kaluza-Klein, let's take the fifth dimension to be spacelike, and compactified into a hypercylinder $x^5 \equiv R\phi$ (see [5], Figure 1). Therefore, $dx^5 = R d\phi + \phi dR$, and, if one takes R to be a constant radius, then $dx^5 = R d\phi$. We then substitute the more general $dx^5 = R d\phi + \phi dR$ into the first term of (4.4), to write:

$$\frac{dx^5}{cd\tau} = \frac{Rd\phi + \phi dR}{cd\tau} = \frac{2}{b\kappa c^2} \frac{Qq}{m} = \frac{2}{b\sqrt{16\pi G}} \frac{Qq}{m} = \frac{Q}{b} \frac{\sqrt{\hbar c \alpha}}{\sqrt{Gm}}. \quad (4.5)$$

Note that we have also put an extra c into the denominator of the first term to maintain all terms as completely dimensionless, because $x^5 \equiv R\phi$ is defined to be spacelike and, as written, has spatial dimension. Now, let's study the dimensionless equation (4.5).

5. Quantization of Charge and Angular Momentum, and Derivation of the SU(2)xU(1) Rotation Group of the Four Space Dimensions

If we define the linear momentum 5-vector $p^M = m(dx^M/d\tau)$, then the $M=5$ component of this is $p^5 = m(dx^5/d\tau)$. If we multiply (4.5) through by mc into dimensions of momentum, then we may write, using the Planck mass $M_p = \sqrt{\hbar c/G}$:

$$p^5 = m \frac{dx^5}{d\tau} = m \frac{Rd\phi + \phi dR}{d\tau} = \frac{2}{b\kappa c} Qq = \frac{2c}{b\sqrt{16\pi G}} Qq = \frac{Q}{b} \sqrt{\frac{\hbar c^3 \alpha}{G}} = \frac{Q\sqrt{\alpha}}{b} M_p c. \quad (5.1)$$

Now, harking back to the old Bohr model of atomic orbits, let us specify a wavelength λ for curled-up ‘‘oscillations’’ along the x^5 dimension. If we wish to regard (5.1) as being related to an electron, then we ought make note of ‘‘orientation / entanglement’’ considerations whereby a spinor-type particle does not return to its original ‘‘version’’ until it has made a full 4π rotation (see, e.g., [6], §41.4), rather than merely a 2π rotation. Therefore, we require that an integral number of wavelengths λ be fitted around a 4π rotation through x^5 , which we write mathematically, with a first quantum number $n_1 = 1, 2, 3, \dots$ as:

$$n_1 \lambda = 4\pi \langle R \rangle, \quad (5.2)$$

where for $dR \neq 0$, $\langle R \rangle$ is the mean value of R over a 4π oscillatory cycle. If $dR = 0$, then

$\langle R \rangle = R$. Setting the frequency to $f\lambda = c$ and the energy to $E^5 = n_2 hf = n_2 \hbar c / \lambda = 2\pi n_2 \hbar c / \lambda$

(using $\hbar \equiv h/2\pi$) where $n_2 = 1, 2, 3, \dots$ is a second, independent quantum number, and writing

(5.2) as $1/\lambda = n_1/4\pi\langle R \rangle$, we then may rewrite (5.1) for the tangential linear momentum in x^5 ,

as such:

$$p^5 = \frac{E^5}{c} = \frac{2\pi\hbar}{\lambda} = \frac{1}{2\langle R \rangle} n_1 n_2 \hbar = m \frac{dx^5}{d\tau} = m \frac{Rd\phi + \phi dR}{d\tau} = \frac{2c}{b\sqrt{16\pi G}} Qq = \frac{Q}{b} \sqrt{\frac{\hbar c^3 \alpha}{G}}. \quad (5.3)$$

Further, defining the x^5 angular momentum as $J^5 \equiv p^5 \langle R \rangle$ enables us to convert (5.3) into:

$$J^5 = p^5 \langle R \rangle = m \frac{dx^5}{d\tau} \langle R \rangle = m \langle R \rangle \frac{Rd\phi + \phi dR}{d\tau} = \frac{1}{2} n_1 n_2 \hbar = \frac{2c}{b\sqrt{16\pi G}} Qq \langle R \rangle = \frac{Q \langle R \rangle}{b} \sqrt{\frac{\hbar c^3 \alpha}{G}}. \quad (5.4)$$

Now, let us extract a few relationships from (5.3) and (5.4), specifically*:

$$Q = \frac{b}{2\langle R \rangle \sqrt{\alpha}} n_1 n_2 \sqrt{\frac{G\hbar}{c^3}} = \frac{n_1 n_2}{2} b \frac{L_p}{\langle R \rangle \sqrt{\alpha}}. \quad (5.5)$$

where the Planck length $L_p = \sqrt{G\hbar/c^3}$, as well as

$$\langle R \rangle = \frac{n_1 n_2}{2} b \frac{1}{Q \sqrt{\alpha}} \sqrt{\frac{G\hbar}{c^3}} = \frac{n_1 n_2}{2} b \frac{1}{Q \sqrt{\alpha}} L_p^{**}, \text{ and} \quad (5.6)$$

$$J^5 = \frac{1}{2} n_1 n_2 \hbar. \quad (5.7)$$

Note, it is the factor of 4π rather than 2π in (5.2), to ensure proper entanglement “version” of the electron, which leads to the factor of $1/2$ in the above, especially in (5.7). For $\alpha = 1$ or on the order of unity, the compactification radius of the fifth dimension for the charged leptons appears to be very close to the Planck length of Wheeler’s geometrodynamics vacuum “foam” and the Schwarzschild radius of the vacuum. [6] at §43.4, [10] And even relative to the low energy $\alpha \rightarrow 1/137.036$, this is still very close to the Planck scale.

Most importantly, (5.7), in the “ground” state $n_1 = n_2 = 1$, tells us without further ado the central result of this paper: *that there is an intrinsic angular momentum in x^5 which is equal to $\frac{1}{2}\hbar$* . As discussed in the next section, by requiring this angular momentum – squared – to be isotropic among all four space dimensions – that is, by requiring that

* The author acknowledges and thanks D. McCullough for pointing out the essence of this calculation, but using 2π rather than 4π , on the Usenet group sci.physics.relativity.

** In a complete Kaluza-Klein analysis, which we shall not present here, it turns out that when one considers the way in which the Maxwell’s stress energy tensor embeds into Kaluza-Klein, the fifth dimension must in fact be spacelike, and $b^2 = 8$. As a result, (5.6) becomes $\langle R \rangle = (n_1 n_2 / Q) L_p \sqrt{2/\alpha}$.

$J^{1^2} = J^{2^2} = J^{3^2} = J^{5^2} = \frac{1}{4}\hbar^2$ – one can derive the non-classical two-valuedness of intrinsic spin directly from the Klauza-Klein geometry, and more globally, can establish a Riemannian geometric foundation for quantum theory.

Equation (5.5) also leads to charge quantization, but if and only if $\langle R \rangle \sqrt{\alpha} = \text{constant}$. We know that the running coupling $\alpha(\mu)$ is a running function relative to the probe energy μ with which we conduct experimental measurements, which means that the mean radius $\langle R \rangle$ must also run, $\langle R \rangle(\mu)$, depending upon the energy relative to which we probe, and must vary inversely with the running coupling. Therefore, to ensure charge quantization, we do proceed to require that $\langle R \rangle \sqrt{\alpha} = \text{constant}$, thereby making $\langle R \rangle(\mu)$ something which is observed differently depending on μ , and which grows *smaller* in its observed appearance as our observations probe into higher energy regimes. (In this manner, $\langle R \rangle(\mu)/L_p$ can be thought of to be a “dual” dimensionless running coupling strength in relation to $\alpha(\mu)$, which then yields a possible connection to Dirac’s quantization condition $q \cdot g = 2\pi\hbar c$ utilizing magnetic charges g . This will not be explored here, but in the end, may be the best fundamental foundation for $\langle R \rangle \sqrt{\alpha} = \text{constant}$, and it gives a geometric foundation to running couplings as a length measured in relation to Planck’s length.)

Now, let’s turn to (5.7) and to $E^5 = n_2 h f$, in order to determine the physical meaning of each of the quantum numbers n_1 and n_2 . **Author’s note: this is a first draft attempt to associate these quantum numbers with known quantum numbers for the electron. Even if there are errors in this first attempt, the main result is that $J^5 = \frac{1}{2}n_1 n_2 \hbar$ in (5.7), and that in the ground state, $J^5 = \frac{1}{2}\hbar$. Thus, much of the following discussion could be bypassed, but is presented in the hopes of being able to associate n_1, n_2 with known quantum numbers.** We consider a single electron for which negative $Q = 1$. The quantum number n_2 arose from $E^5 = n_2 h f$, and so is an energy quantum number. Because a larger n_2 is associated with energy amplitude (versus frequency), let us, tentatively, associate this with the principal electron quantum number n . Now, we consider the lowest energy state $n = 1$. In this state, (5.7) reduces to:

$$J^5 = \frac{1}{2}n_1\hbar, \quad (5.8)$$

which says that the angular momentum in x^5 is given by $J^5 = (\frac{1}{2}, 1, \frac{3}{2}, 2, \dots)\hbar$. Of course, the angular momentum quantum number $j = l + s$, which is the sum of the spin $s = \frac{1}{2}$ and orbital l angular momentum quantum numbers, also can, *in general*, take on the values $j = \frac{1}{2}, 1, \frac{3}{2}, 2, \dots$. So, also tentatively, we define $\frac{1}{2}n_1 \equiv j = l + s$ as the total angular momentum quantum number. Now, (5.8) becomes:

$$J^5 = (l + s)\hbar, \quad (5.9)$$

where $l = 0, 1, 2, \dots, n-1$ and $s = \frac{1}{2}$. (We know, of course, that we should really have $s = \pm\frac{1}{2}$. We will be able to correct this in section 8, when we consider the positron as well.) However, because (5.8) is for $n = 1$, and because $l = 0, 1, 2, \dots, n-1$, this means that (5.9), reduces simply to:

$$J^5 = s\hbar = \frac{1}{2}\hbar. \quad (5.10)$$

(Again, this will become $s = \pm\frac{1}{2}$ when we consider to positron in section 8.) Treated as an operator equation rather than classically, we write out (5.10) fully as:

$$J^5|n=1, l=0, m=0, s=\frac{1}{2}\rangle = s\hbar|n=1, l=0, m=0, s=\frac{1}{2}\rangle = \frac{1}{2}\hbar|n=1, l=0, m=0, s=\frac{1}{2}\rangle. \quad (5.11)$$

This means that more generally, our tentative association with known quantum numbers is $\frac{1}{2}n_1n_2 = n(l + s)$, so that (5.5)-(5.7) now read:

$$Q = n(l + s)b \frac{L_p}{\langle R \rangle \sqrt{\alpha}}, \quad (5.12)$$

$$\langle R \rangle = n(l + s)b \frac{1}{Q\sqrt{\alpha}} L_p. \quad (5.13)$$

$$J^5 = n(l + s)\hbar. \quad (5.14)$$

For a given charge Q , (5.13) tells us that the mean radius $\langle R \rangle$ varies with n, l, s . The x^5 hypercylinder thus appears to have some flexibility to vary its radius, not only by oscillating with a wavelength $n_1\lambda = 4\pi\langle R \rangle$ which is now $(l + s)\lambda = 2\pi\langle R \rangle$, but also because $\langle R \rangle$ undergoes an

objective (μ -independent) increase as a function of increasing n, l, s . (If one takes a “string view” of these compact loops, we see that these strings vibrate and are rather flexible to objectively enlarge and contract in relation to n, l, s .) Using this to eliminate $(l + s)$ and $\langle R \rangle$ from (5.13), we obtain:

$$\lambda = 4\pi n \frac{1}{Q} \sqrt{\frac{2}{\alpha}} L_p, \quad (5.15)$$

and using $E^5 = 2\pi\hbar c / \lambda$ with the Planck energy $E_p = \sqrt{\hbar c^5 / g}$ and $L_p = \sqrt{G\hbar / c^3}$, we also find:

$$E^5 = \frac{\sqrt{2\alpha}}{4} Q E_p \quad (5.16)$$

where we also employ $b^2 = 8$ (not derived here, but noted in an earlier footnote). At a given $\alpha(\mu)$, for a given Q , this energy is observed to be constant. For a given charge Q , the increase in wavelength as a function of n in (5.15) comes about not because of decreased energy component E^5 (especially because $E^5 = nhf$ increases with n for a given f), but because of the larger $\langle R \rangle$ of (5.13). It is also of interest, from (5.11), and defining the five-velocity vector $u^M \equiv dx^M / d\tau$ hence $u^5 \equiv dx^5 / d\tau$, to obtain via $p^5 = E^5 / c = mu^5$:

$$\frac{m}{M_p} \frac{u^5}{c} = \frac{\sqrt{2\alpha} Q}{4}. \quad (5.17)$$

If we employ $m = m_e = .511 \text{ MeV}$, or even the larger masses for the mu and tau leptons which proportionately reduce the linear velocity component u^5 thereby leaving the linear and angular momentum in x^5 unaffected (that is, x^5 velocity varies inversely with mass so all momenta are constant), the above *appears* to suggest a *superluminal* fifth-dimensional velocity in excess of $10^{20} c$ for the electron, and proportionately less for the remaining charge leptons. However, as we shall discuss at length in sections 9 and 10, these are *screened* masses, as observed from probe energies twenty orders of magnitude removed from the Planck scale. When probed directly at Planck scale, $\mu \sim E_p$, the unscreened masses are on the order of the Planck mass itself, and the *proper* velocity for each individual charge $|Q| = 1$ becomes subluminal.

Finally, returning to the $|n=1, l=0, m=0, s=\frac{1}{2}\rangle$ state, this means that:

$$J^{5^2} = \frac{1}{4}\hbar^2. \quad (5.18)$$

The Casimir operator $J^2 = J^{1^2} + J^{2^2} + J^{3^2} = j(j+1)\hbar^2$ of the rotation group $SU(2)$ commutes with the generators J^i , $i=1,2,3$ according to $[J^2, J^i]=0$. For $|n=1, l=0, m=0, s=\frac{1}{2}\rangle$, we have $J^{1^2} = J^{2^2} = J^{3^2} = \frac{1}{4}\hbar^2$ and $J^2 = J^{1^2} + J^{2^2} + J^{3^2} = \frac{3}{4}\hbar^2$. The x^5 hypercylinder $x^5 \equiv R\phi$, for which $g_{MN,5} = 0$, (3.4), and $g_{55,M} = 0$, (3.8), is its own $U(1)$ group. Denoting by $SU(2)_i$ the rotation group for the space coordinates x^i and by $U(1)_5$ the independent unitary group for x^5 , we form all four space coordinates into $SU(2)_i \times U(1)_5$. For $|n=1, l=0, m=0, s=\frac{1}{2}\rangle$, we find:

$$J^2 = J^{1^2} + J^{2^2} + J^{3^2} + J^{5^2} = \hbar^2, \quad (5.19)$$

where each of

$$J^{1^2} = J^{2^2} = J^{3^2} = J^{5^2} = \frac{1}{4}\hbar^2, \quad (5.20)$$

Therefore, the total magnitude of the intrinsic spin S , taken over all four space dimensions, is:

$$S = \hbar, \quad (5.21)$$

as opposed to $S = (\sqrt{3}/4)\hbar$ when only x^1, x^2, x^3 are considered.

We have thus established on a purely geometrodynamical basis, that the squared intrinsic angular momentum $J^{5^2} = \frac{1}{4}\hbar^2$ of the fifth dimension, is identical in magnitude to the squared intrinsic angular momentum of all three space dimensions. That is, this spin angular momentum, *squared*, is *isotropic* across all four space dimensions. Keeping in mind the group $SU(2)_i \times U(1)_5$, we may then write, for the intrinsic spin generators in all four space dimensions:

$$J^1 = \frac{1}{2}\hbar\sigma^1 = \frac{1}{2}\hbar \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; J^2 = \frac{1}{2}\hbar\sigma^2 = \frac{1}{2}\hbar \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}; J^3 = \frac{1}{2}\hbar\sigma^3 = \frac{1}{2}\hbar \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}; J^5 = \frac{1}{2}\hbar I = \frac{1}{2}\hbar \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}. \quad (5.22)$$

Note, for later reference when we consider Dirac's equations, that $J^5 = -\frac{1}{2}\hbar I$ can also be employed, consistently with the group product $SU(2)_i \times U(1)_5$. As an independent group factor,

$$[J^5, J^i] = 0 \quad (5.23)$$

commutes with all three of the remaining generators, and so the fifth-dimensional angular momentum is separately conserved from that in the ordinary three space dimensions.

6. Deducing Intrinsic Spin from Kaluza-Klein Geometry

The intrinsic spin $J^5 = s\hbar = \frac{1}{2}\hbar$ obtained in (5.10), is based solely upon the Riemannian geometry of five-dimensional Kaluza-Klein theory, combined with the requirement that oscillations in x^5 must “fit” around this compact dimension such that $n_1\lambda = 4\pi\langle R \rangle$, accounting for orientation and entanglement. Now, we shall show how intrinsic spin – and in the following section, the Heisenberg relationships – may be placed, fully intact, onto the firm underlying foundation of this five-dimensional Riemannian geometry.

Let us step back and suppose we knew nothing of intrinsic spin or the spin matrices (5.22), but desired to *deduce* these solely from the five dimensional geometry. So, we engage in all of the geometric analysis which led to $J^5 = s\hbar = \frac{1}{2}\hbar$ in (5.10). Or, with even less knowledge, suppose we had derived only $J^5 = \frac{1}{2}n_1n_2\hbar$ of (5.7). In fact, suppose all we knew was that $f\lambda = c$ and that $E^5 = hf$, i.e., that energy is a constant multiple of frequency, but that we did not know that $E^5 = nhf$, i.e., that energy comes in quantized packets, in which case (5.7) is merely $J^5 = \frac{1}{2}n_1\hbar$. Then, taking the $n_1 = 1$ ground state in which $\lambda = 4\pi\langle R \rangle$, and knowing to use 4π rather than 2π solely on the basis of orientation / entanglement considerations, we would be able to *derive* the “non-classical two-valuedness” of intrinsic spin, from pure five-dimensional geometry, as follows:

First, we would conclude from the ground state equation $J^5 = \frac{1}{2}\hbar$ that there exists an intrinsic angular momentum for the electron, at least in the x^5 dimension. This is a direct consequence of the electrostatic charge, see (4.2). We would also know that the squared intrinsic angular momentum in x^5 is given by $J^{5^2} = \frac{1}{4}\hbar^2$, equation (5.18). Now, one of the objections sometimes voiced with regards to a compactified fifth dimension – aside from the fact that it is too small to directly observe – is that the curled up x^5 dimension is “different” from the linear, uncurled x^1, x^2, x^3 . The question is sometimes posed: how does one “bias” the vacuum toward

one of four space dimensions, over the other three? While it may be perceived that there is a bias toward x^5 , there is one aspect in which there is no bias: in $J^{1^2} = J^{2^2} = J^{3^2} = J^{5^2} = \frac{1}{4}\hbar^2$ of (5.19). Suppose, therefore, that we knew nothing of intrinsic spin as represented in J^1, J^2, J^3 , and in fact, did not even know that J^1, J^2, J^3 must be quantum operators operating on wavefunctions and “states,” rather than being classical numbers. After all, $J^{5^2} = \frac{1}{4}\hbar^2$ is derived from a five dimensional, classical Riemannian geometry.

Now we introduce the postulate of *isotropic square intrinsic angular momentum*, which states that “the *square* magnitude of the intrinsic angular momentum of an electron must be isotropic in *all four* space dimension” That is, *squared* angular momentum shall not distinguish one spacetime dimension from the next! In this way, the vacuum shows no bias as regards intrinsic angular momentum, because the *squared* intrinsic spin exists equally in all four space dimensions. By this postulate, the intrinsic spin found in x^5 makes its way into x^1, x^2, x^3 .

In that event, having deduced $J^{5^2} = \frac{1}{4}\hbar^2$ from geometry, one could immediately require that $J^{1^2} = J^{2^2} = J^{3^2} = J^{5^2} = \frac{1}{4}\hbar^2$ which is (5.19). In fact, *this is the mathematical statement of the foregoing postulate*. This would also mean that the total magnitude of the squared angular momentum in x^1, x^2, x^3 is $J^2 = J^{1^2} + J^{2^2} + J^{3^2} = \frac{3}{4}\hbar^2 = s(s+1)\hbar^2 = \frac{1}{2} \cdot \frac{3}{2}\hbar^2$. Then, one would make note of the fact that $g_{MN,5} = 0$, (3.4), and $g_{55,M} = 0$, (3.8), which effectively isolates x^5 from the other x^1, x^2, x^3 . Therefore, whatever the individual J^i might be in the other three dimensions, we can deduce that J^5 must be conserved separately from J^1, J^2, J^3 . Therefore, we would know that $[J^5, J^i] = 0$. Then the question arises, what about the J^i themselves?

Although $J^5 = \frac{1}{2}\hbar$ is conserved separately from the J^1, J^2, J^3 which sum all tolled to $J^2 = J^{1^2} + J^{2^2} + J^{3^2} = \frac{3}{4}\hbar^2$, we know that J^1, J^2, J^3 can be redistributed on an individual basis among x^1, x^2, x^3 via the rotation group. Put into mathematical language, $[J^i, J^j] \neq 0$ for $i \neq j$. On the other hand, the fact that J^2 is conserved over the three space dimensions, is stated as $[J^2, J^i] = 0$. So, we know right away that J^1, J^2, J^3 cannot be ordinary scalar numbers, but rather must be mutually-non-commuting operators. The smallest such operators are the 2x2 spin

matrices J^i of (5.22), and so we are led to define the $[J^i, J^j] = \epsilon^{ijk} J^k$ to express the non-commutation of the J^i .

Thus, from $J^5 = s\hbar = \frac{1}{2}\hbar$, from the isolation of x^5 from x^1, x^2, x^3 by $g_{MN,5} = 0$ and $g_{55,M} = 0$, by postulating the isotropy of squared intrinsic angular momentum across all four space dimensions, and given that $[J^i, J^j] \neq 0$ among the three space dimensions, even classically no matter the magnitude of the J^i , we are led inexorably from a classical five-dimensional Riemannian geometry, to the intrinsic spin matrices (5.22).

By virtue of the foregoing, we are thereby able to establish a *classical geometric foundation* for the ostensibly *quantum-mechanical*, “*non-classical two-valuedness*” of the intrinsic spin of the charged leptons. [11]

7. Derivation of the Heisenberg Commutators for Position and Momentum

Having deduced the intrinsic spin matrices (5.22) from the Kaluza-Klein geometry based on intrinsic spin isotropy, we are but a step away from the Heisenberg principle, because of the fact that spin angular momentum can only be projected with certainty onto a single axis of quantization, customarily taken to be the z-axis of the diagonalized operator σ^3 (and more generally expressed via the helicity operator $\frac{1}{2}\Sigma \cdot \hat{\mathbf{p}} \equiv \frac{1}{2} \begin{pmatrix} \sigma \cdot \hat{\mathbf{p}} & 0 \\ 0 & \sigma \cdot \hat{\mathbf{p}} \end{pmatrix}$).

Specifically, the spin matrices (5.22) obey the well-known commutation relationship:

$$[J^i, J^j] = \epsilon^{ijk} J^k, \quad (7.1)$$

Then, we assume that “components of . . . angular momentum in quantum mechanics can be defined in an analogous manner to the corresponding components of classical angular momentum.” That is, we assume that the classical equations

$$L^i = \epsilon^{ijk} x^{[j} p^{k]}, \quad (7.2)$$

which specify the angular momentum L^i in terms of the position x^i and linear momentum p^i , also specify the quantum mechanical “angular momentum *operators* in terms of the position and linear momentum *operators*.” [12]

Therefore, taking (7.2) to an equation among quantum mechanical operators as well as among classical numbers, and treating (7.1) as an angular momentum, we may set:

$$J^i = L^i. \quad (7.3)$$

This leads directly to the commutation relationship of the Heisenberg principle in the following manner.

Using (7.1), (7.2) and (7.3), we may write:

$$[L^i, L^j] = [J^i, J^j] = i\hbar \epsilon^{ijk} L^k = i\hbar \epsilon^{ijk} J^k. \quad (7.4)$$

Equation (7.4) will be true, if and only if, the quantum mechanical operators for position x^i and linear momentum p^i are given by:

$$[x^i, x^j] = 0. \quad (7.5)$$

$$[p^i, p^j] = 0. \quad (7.6)$$

$$[x^i, p^j] = i\hbar \delta^{ij}. \quad (7.7)$$

Equations (7.5) and (7.6) specify that position commutes with position and momentum with momentum, while (7.7) is the *Heisenberg canonical commutation relationship* between position with momentum, from which the so-called ‘‘uncertainty principle’’ can be derived in a well-known manner.

To see that (7.5)-(7.7) must be a consequence of (7.4), explicitly write out one of the three equations in (7.4), say, $i = 1, j = 2$. Employing (7.1) and (7.2), this expands to:

$$[L^1, L^2] = [x^2 p^3 - x^3 p^2, x^3 p^1 - x^1 p^3] = [J^1, J^2] = i\hbar J^3 = i\hbar L^3 = i\hbar(x^1 p^2 - x^2 p^1). \quad (7.8)$$

This relationship may be solved if we *define* the bridge term $x^2[p^3, x^3]p^1 + x^1 p^2[x^3, p^3]$ by:

$$[x^2 p^3 - x^3 p^2, x^3 p^1 - x^1 p^3] \equiv x^2[p^3, x^3]p^1 + x^1 p^2[x^3, p^3] \equiv i\hbar(x^1 p^2 - x^2 p^1). \quad (7.9)$$

Repeating this for $i = 2, j = 3$ and $i = 3, j = 1$, the full expansion and re-consolidation of these terms demonstrates that (7.9) is true if and only if (7.5)-(7.7) are true.

Finally, by general covariance, we extend (7.5)-(7.7) to include the time components, so:

$$[x^\mu, x^\nu] = 0. \quad (7.10)$$

$$[p^\mu, p^\nu] = 0. \quad (7.11)$$

$$[x^\mu, p^\nu] = i\hbar \delta^{i\mu\nu}. \quad (7.12)$$

This final relationship, now includes the time / energy commutation $[x^0, p^0] = i\hbar$.

Thus, we have demonstrated how two foundations of quantum theory – the Pauli spin matrices and the Heisenberg canonical commutation relationship – can both be canonically generated from an even deeper level, out of the compactified fifth space dimension of a Kaluza-Klein theory, by requiring an isotropic *squared* intrinsic angular momentum over all four space dimensions. Quantum theory thereby acquires a foundation in Riemannian geometry, and the compactified fifth-dimension, often regarded as the Achilles heel of Kaluza-Klein theory because it is not directly observed, now becomes the geometrodynamics foundation standing at the root of quantum theory.

8. Positrons and the Dirac Equation.

Let us now take a next step, and ask about the positron. If all classical gravitational and electrodynamic motion is to be along geodesics in the five-dimensional Kaluza-Klein geometry, then equation (2.5) must describe the motion of all charges, positive and negative, in an invariant manner. That is, the form of (2.5) should not change, based on whether a charge in an electromagnetic field is positive or negative. On the other hand, the Lorentz force, as expressed in (2.6), explicitly contains the charge strength $-q$, and this will change sign to $+q$ when the charge goes from negative to positive. Therefore, when applied to a positron, the relation (2.7) changes to:

$$2\Gamma^\mu_{5\sigma} \frac{dx^5}{d\tau} \frac{dx^\sigma}{d\tau} = + \frac{q}{m} F^\mu{}_\sigma \frac{dx^\sigma}{d\tau}. \quad (8.1)$$

Now, the sign of equation (3.1), $\Gamma^M_{5\Sigma} \equiv -\frac{1}{4} b \bar{\kappa} F^M{}_\Sigma$, which relates the electromagnetic field strength to the Christoffel connections should not be affected, because that describes the external field through which the positron is moving. Therefore, if we follow the same steps as before, (8.1) leads us to find that for a positron, the equation corresponding to (4.2) is:

$$-\frac{dx^5}{d\tau} = \frac{2}{b\kappa} \frac{q}{m} = \frac{2c^2}{b\sqrt{16\pi G}} \frac{q}{m}, \quad (8.2)$$

According to Feynman-Stückelberg interpretation of negative energy solutions to Dirac's equation, a negative energy electron moving backwards through time is a positive energy positron moving forwards through time. Therefore, in the term $-dx^5/d\tau$ above, it is not the proper time $d\tau$ which is responsible for the flipped sign in relation to (4.2). It is the dx^5 motion. That is, by (8.2) versus (4.2), a positron travels oppositely from an electron through x^5 , $dx^5(e^+) = -dx^5(e^-)$. Because $dx^5 \equiv Rd\phi + \phi dR$, and requiring the compactification radius R to remain positive, this means that for a positron, $d\phi$ (and dR) is what flips sign in relation to the electron, that is, $d\phi(e^+) = -d\phi(e^-)$. Following the previous development all the way through, this means that the equation corresponding to the earlier (5.4), for the positron, is given by:

$$-J^5 = -p^5 \langle R \rangle = -m \frac{dx^5}{d\tau} \langle R \rangle = -m \langle R \rangle \frac{Rd\phi + \phi dR}{d\tau} = \frac{1}{2} n_1 n_2 \hbar = \frac{2c}{b\sqrt{16\pi G}} Qq \langle R \rangle = \frac{Q \langle R \rangle}{b} \frac{\sqrt{\hbar c^3 \alpha}}{\sqrt{G}}. \quad (8.3)$$

This is the same as in (5.4), except now, because the direction of motion through x^5 is reversed, so too will be the direction of the associated angular momentum J^5 . To be clear: $J^5 = -\frac{1}{2} n_1 n_2 \hbar$ does not indicate a negative *magnitude* for the angular momentum, but rather, a positive magnitude, but with opposite *orientation* because of the reversed-direction of x^5 movement. The compactification radius based on (8.3), is the same as which was derived in (5.6).

From (8.3), following the same reasoning as before, we find that equations (5.19), $J^2 = J^{1^2} + J^{2^2} + J^{3^2} + J^{5^2} = \hbar^2$, and (5.20), $J^{1^2} = J^{2^2} = J^{3^2} = J^{5^2} = \frac{1}{4} \hbar^2$, apply equally to the electron and the positron. However, for the ground state $n_1 = n_2 = 1$, we now have $J^5 = -\frac{1}{2} \hbar$.

To create a set of operators which simultaneously satisfy (5.19), (5.20), and now, also, $J^5 = -\frac{1}{2} \hbar$ when operating on positrons, we must now move beyond the 2×2 , Pauli matrices, and consider 4×4 matrices. If we consider a four-component wavefunction ψ in which $\psi_{1,2}$ are electron states and $\psi_{3,4}$ are positron states (as is the case for solutions to Dirac's equation in the Dirac-Pauli representation), then it is easy to see that these requirements for the J^i for both *electron and positrons* can be satisfied by the Dirac gamma matrices $\frac{1}{2} (\gamma^\mu \gamma^\nu + \gamma^\nu \gamma^\mu) = \eta^{\mu\nu}$,

which will transform covariantly when used to form a vector current $\bar{\psi}\gamma^\mu\psi$, given a Minkowski spacetime metric tensor $\text{diag}(\eta^{\mu\nu}) = (1, -1, -1, -1)$. That is, the Dirac algebra will ensure that $J^{1^2} = J^{2^2} = J^{3^2} = J^{5^2} = \frac{1}{4}\hbar^2$ for the electron and positron both, that $J^5 = \pm\frac{1}{2}\hbar$ for electron and positron, and that all of the other commutation relationships of the $SU(2)_i \times U(1)_5$ rotation group are fully satisfied. Backtracking, some of the relationships in section 5 also need to be updated, to recognize that $J^5 = s\hbar = \pm\frac{1}{2}\hbar$, rather than merely $J^5 = s\hbar = \frac{1}{2}\hbar$ as earlier.

9. Fifth-Dimensional Super- or Sub-Luminosity?

Equation (5.17) derived earlier, $\frac{m}{M_p} \frac{u^5}{c} = \frac{\sqrt{2\alpha Q}}{4}$, prompts consideration of the question

whether an electron, when moving through the compactified fifth dimension to generate its intrinsic spin and its q/m ratio, is traveling at super- or sub-luminal velocity. This, in turn, compels us to a direct examination of physics at the Planck scale, and to the development of a rudimentary Planck-scale “model” of the electron.

The problem now to be considered is seen most easily by returning to (5.17) and observing that at low probe energy, with $\alpha \rightarrow 1/137.036$ and $b^2 = 8$ (a result that requires a detailed analysis beyond this paper), and with $m_e = .511 \text{ MeV}$ and $M_p = 1.22 \times 10^{22} \text{ MeV}$, one can derive that for the electron or oppositely-moving positron with $|Q| = 1$, the fifth-dimensional linear velocity $u^5/c = 7.211 \times 10^{20}$. This is extremely superluminal, for which there are but three explanations: First, this result, and all that leads to it, is just plain wrong. Second, superluminosity is permitted in the fifth-dimension, even for a massive particle like the electron.

Third – and the path we shall now explore – is the prospect that the only reason (5.17) leads to $v/c = 7.211 \times 10^{20}$, is because the value we are using for the electron rest mass, $m_e = .511 \text{ MeV}$, is the *screened* electron mass, as observed from probe energies twenty orders of magnitude removed from the Planck scale. We know that probe energy is a key parameter in particle physics, and that how deeply we probe does affect what we observe. In particular, we shall consider the prospect that when observed unscreened from a Planck-scale probe energy, the electron rest mass is very much larger than $m_e = .511 \text{ MeV}$, and is, indeed, on the order of the

Planck mass itself. Note, for example, from (5.1), that $p^5 = (Q\sqrt{\alpha}/b)M_p c$, which is already on the order of the Planck scale. This puts the ratio m/M_p and the running coupling α in (5.17) on the order of unity (1), renders the true velocity of the electron into a sub-luminal range, and explains that $v/c = 7.211 \times 10^{20}$ results as an illusory outgrowth of the twenty orders of magnitude worth of charge and mass screening which sits between the perch of our experimental observations, and the actual physics which occurs tucked right in, riding an electron at the Planck scale. Recognizing that at Planck scale, the gravitational curvatures are so great as to generate a sea of virtual black holes, * this is not unlike observing a black hole from an intergalactic distance rather than from right at its boundary. In effect, these twenty orders of magnitude of screening act as a powerful “lens,” distorting greatly, our view of what actually happens at the Planck scale. The principle of “relativity” extends in this way to probe energy, such that one must always specify the probe energy “relative to” which the phenomena in question are being observed.

10. Wheeler’s Quantum Geometrodynamics: A 5th-Dimensional Perspective

The touchstone for this approach is Wheeler’s article [10], which lays the foundation for the geometrodynamics approach to physics, and makes quite clear in the very last paragraph that “the question of the origin of spin is decisive for the assessment of quantum geometrodynamics.” In this section, we engage in a complete review of this article, in light of the foregoing results.

In this seminal article [10], Wheeler admits a multiply-connected spacetime topology in lieu of one that is simply-connected, and, at 604, regards “classical charge . . . as the flux of lines of force trapped in a multiply connected metric.” Here, we make a course correction, and by the relationship (6.1), regard the electron charge to be emanating from the motion of the electron through the fifth compactified spacetime dimension, which simultaneously accounts for the electron’s intrinsic spin and via isotropic projection of the x^5 intrinsic spin into the three

* By way of review, the Planck mass, defined from the term atop Newton’s law as a mass for which $GM_p^2 \equiv \hbar c$, is thus $M_p = \sqrt{\hbar c/G}$. In the geometrodynamics vacuum, the negative gravitational energy between Planck masses separated by the Planck length $L_p = \sqrt{G\hbar/c^3}$ precisely counterbalances and cancels the positive energy of the Planck masses themselves. The Schwarzschild radius of a Planck mass $R_S = 2GM_p/c^2 = 2\sqrt{G\hbar/c^3} = 2L_p$.

ordinary dimensions of space, for the non-classical two-valuedness of this spin when observed in ordinary spacetime, and for the Heisenberg commutation relationships, as earlier demonstrated.

Continuing, Wheeler shows how positive Planck mass fluctuations which are typical of the Planck vacuum are typically separated by the Planck length and so render a negative gravitational energy which precisely counterbalances the positive Planck masses. And so, at 610, Wheeler concludes, “the possibility is open to have a vacuum state with zero net energy density.” We adopt the same view here, but add to it the understanding gleaned in recent decades that Higgs-type fluctuations in the vacuum are spinless (scalar) fluctuations. We apply this to model Wheeler’s vacuum as one filled with *scalar* fluctuations, with an expected mass equal to the Planck mass, with an expected separation equal to the Planck length, and with an expected negative gravitational energy equal to the negative of the Planck energy. Thus, the net vacuum expectation energy over any region substantially larger than the Planck length, is precisely equal to zero. And, there is an expected Schwarzschild black hole radius equal to twice the Planck-length enveloping this vacuum, so by Hawking’s celebrated analysis from 1974, [13] it is not implausible to suppose that the statistical distribution which leads to the foregoing expectation values is a blackbody spectrum, and perhaps, that when observed in a screened state, this is related in some manner to the cosmic blackbody radiation background. So, how do we now get real quantum electrons with intrinsic spin and non-zero masses?

By the forgoing results derived here, the electron is moving through a compactified fifth spacetime dimension with a $dx^5 = Rd\phi + \phi dR \neq 0$ which, starting with (5.1), is made to account with precision for the intrinsic spin not only of the electron, but of the muon and tauon (recall from after (5.17) that the x^5 velocity varies inversely with mass so all momenta are constant). On the other hand, the spinless scalar Planck-scale masses which inhabit Wheeler’s vacuum foam must, by definition as spinless, be stationary along the fifth dimension, with $dx^5 = 0$. Thus, we posit here, a rudimentary model of the electron as a Planck mass which distinguishes itself from all the other Planck scale fluctuations in the vacuum, by having a non-zero dx^5 specified by (5.1). Now, by virtue of this motion through dx^5 , there is an additional amount of kinetic energy that arises, and this provides the energy needed to perturb the vacuum ever-so-slightly above its natural energy density of zero. The electron, from this view, is a Planck mass which has gotten “trapped” into fifth dimensional motion (as opposed to a wormhole which has “trapped” electric flux lines), moves through the fifth dimension according to (5.1), and thereby

acquires its electric charge and its intrinsic spin and its “undressed” rest mass all at once, and all because of this $dx^5 = Rd\phi + \phi dR \neq 0$ motion.

Then, we come to the question of how the electron acquires its experimental mass. Continuing at 610, Wheeler observed that “in electron theory one distinguishes between the mass and charge of the ‘undressed electron’ and the mass and charge of the experimental electron.” As regards the charge strength, as is well-known in particle theory, this of course scales with the running coupling $\alpha \rightarrow 1/137/036$ at low energy, which is hypothesized, at the Planck scale, to merge in magnitude with all other interaction couplings (and thus exhibit one aspect of “unification”). Regarding the electron mass, Wheeler at 611 considers the view that “the electron is nothing but a collective state of excitation of the foam-like medium [which] is suggested in Fig. 2 by the slightly closer spacing of the wormholes within the dashed circle. The fractional increase in the concentration of electromagnetic mass-energy within the electron . . . is fantastically small compared to the concentration of . . . energy already present in the vacuum.”

Here, we take a slightly-modified view, that the electron mass arises from the differential energy provided by its $dx^5 = Rd\phi + \phi dR \neq 0$ motion through the fifth dimension which also distinguishes the electron in a clear and stable way from the unstable scalar masses of the foam which characterize the vacuum. This additional dx^5 -derived kinetic energy is still “fantastically small compared to the concentration of . . . energy already present in the vacuum,” such that when it is viewed from the same perch at which $\alpha \rightarrow 1/137/036$, also leads one to observe that $m_e = .511 \text{ MeV}$. We leave unanswered for now, whether the muon and tauon are distinguished from the electron because their fifth-dimensional velocity, at Planck scale, is slower than that of the electron, or whether this distinction arises because, at low energies, these three charged leptons are differently-screened. That is, one must consider whether the difference in mass between the electron and the other two charge leptons arises from differences already rooted at the Planck scale, or by virtue of screening differences down to low energies.

Wheeler’s ultimate goal, stated at 612, is “to find out, not whether pure quantum geometrodynamics can account for elementary particle physics, but whether there is some way to prove that it *cannot*” (original emphasis). Here, we have demonstrated that with slight changes, the results developed within, whereby the electron spin and charge and mass all derive from fifth

dimensional motion, remain fully consistent with this analysis by Wheeler, with one bonus of inestimable value:

The final question posited by Wheeler is this: “How can a classical theory endowed with fields of integral spin possibly give on quantization a spin $\frac{1}{2}$ such as is required to account for the neutrino, the electron, and other particles? From the beginning Pauli referred to spin as ‘nonclassical two valuedness.’ Is there anything . . . that forces the introduction of any such non-classical two-valuedness?*

Unless there is, *pure quantum geometrodynamics must be judged deficient as a basis for elementary particle physics*. Therefore, the question of the origin of spin is decisive for the assessment of quantum geometrodynamics.”

Understanding intrinsic spin on the basis of motion through the compactified fifth dimension, and the isotropic projection of fifth-dimensional *squared* intrinsic spin into the remaining three space dimensions, with its concomitant production of the electron’s charge and mass, and of the Pauli spin operators and the Heisenberg canonical commutation relationship, suggests that quantum geometrodynamics remains alive and well as a foundation for the description of nature, rooted in real physical phenomena occurring at the Planck scale.

11. Limitations, Discussion, and Conclusion

While the above was derived by considering the charged leptons, there are questions which arise when other particles / field quanta are considered. For example, from (5.1), it appears at first sight that for a neutral body, $Q=0$, such as the neutrino, we have $dx^5 = Rd\phi + \phi dR = 0$, and so there is no fifth-dimensional rotation. One might take this to suggest that the neutrino has no fifth-dimensional motion and thus no intrinsic spin, the latter of which, of course, is contradicted by empirical knowledge. Strictly speaking, however, in the context of a Kaluza-Klein theory based on $U(1)_{em}$, one really ought not try to discuss any particles other than charged leptons and photons. But to provide at least some guidance, we know that in electroweak theory, $Q = Y/2 + I^3$, so there are in fact other charge generators implicit in (6.1). Therefore, it is plausible that this question about the neutrino may be resolved if one considers Kaluza-Klein in a non-Abelian (Yang-Mills) $SU(2)_w \times U(1)_Y$ rather than the present abelian $U(1)_{em}$ context. Here, the neutrino will have a non-zero weak isospin $I^3 = +\frac{1}{2}$

* We defer the question of the neutrino for the following section, sticking for the moment with just the electrons.

coupled via a weak running coupling α_w related to the electromagnetic running coupling by $\alpha = \alpha_w \sin^2 \theta_w$ via the weak mixing angle θ_w . Perhaps, this can lay a foundation for the intrinsic spin of the neutrinos similar to that of (5.1) for the charged leptons. It should be observed also, that (5.1) suggests that any elementary scalar particle which has no intrinsic spin, must be electrically neutral. This is, in fact, true of the hypothesized Higgs boson. One must finally point out, that (5.1) does not shed any direct light on the intrinsic spin of *massless* particles, such as the photon. However, we also note that in electroweak theory, the absence of mass for the photon emerges from a spontaneous breaking of symmetry which does leave three other bosons, the $W^{\pm\mu}$ and the Z^μ , with a non-zero mass. We also note that for the photon, the fundamental geometric starting point is (2.1), with $d\tau^2 = 0$.

So, with the clear caution that particles other than the charged leptons do raise further questions, *might it nevertheless be possible that the compactified fifth dimension of Kaluza-Klein, which has long begged for a physical foundation, is in fact the Riemannian-geometric foundation of intrinsic spin?* If one is scrupulous about the use of language, and if one is so quixotic as to still be looking for a geometric foundation even for an ostensibly-quantum phenomenon such as intrinsic spin, one may discern that using the term “intrinsic” to describe an “inherent” quantized angular momentum of elementary particles, linguistically papers over a deep ignorance of what “intrinsic spin” really means, *geometrically*. [11] Why?

For a material body to have an angular momentum, there must implicitly be a radius R with which that body circles about a rotational origin. Even the smallest bodies, if they have an angular momentum, must be rotating or spinning – at some *finite spatial radius* – about an origin. At the same time, nobody believes that intrinsic spin represents an angular momentum about a radius R in the three usual spatial dimensions. By associating intrinsic spin with motion through a fourth, compactified, hyper-cylindrical spatial dimension, and requiring the isotropic projection of this (squared) spin into all four space dimensions, one simultaneously makes sense of intrinsic spin and of a compact fourth spatial dimension. The material body now has a spatial radius R of rotation through a fourth spatial dimension other than the usual three spatial dimensions to give geometric plausibility to its “intrinsic” spin. At the same time, the compactified fourth spatial dimension now takes on real, physical meaning as something which is physically observed, via the phenomenon of intrinsic spin, and not merely a fictional

mathematical device which by its perceived contrivance, gives people pause about Kaluza-Klein theories specifically, and dimensional compactification in general. Especially, considering that such (squared) intrinsic angular momentum must project isotropically the fifth dimension, and therefore will reside non-classically along all three ordinary space dimensions, we are led directly to the Pauli spin operators (6.5), and from there, to the Heisenberg canonical commutation relationship (7.7).

In sum, this provisional, tentative, cautious, hypothesized understanding of intrinsic spin as emanating from cyclical motion through a fourth dimension of space which is curled up into a radius on the order of the Planck length, if it can be developed further and sustained for particles other than the charged leptons, may be useful to overcome one of the most nagging objections about Kaluza-Klein theories. It would do so by underscoring a clearly-observed, physical manifestation of the fourth space dimension, namely, the intrinsic spins which pervade particle physics and which have, up to the present time, defied explanation on any sort of classical geometric foundation. This in turn, would remove a primary cause which has led many to abandon entirely, not only the attractive possibility of explaining nature on a solely geometrodynamical footing, but also the compelling gravitational and electrodynamic union otherwise provided by Kaluza-Klein theories. Finally, this would also remove the need for string theorists generally, and Kaluza-Klein theorists specifically, to reply with some disingenuity to skeptics, that the extra space dimension(s) is/are “so small that nobody will ever see it/them anyway.”

Thus, with all of the above-noted caveats, and recognizing that the foregoing exposition is based strictly and solely on particles with non-zero mass which carry a single unit of electrostatic charge, i.e., an electron, muon, or tauon or their antiparticles, we conclude by cautiously affirming the provisional, tentative hypothesis, to be studied in other contexts and for other elementary particles, that the fourth spatial dimension in Kaluza-Klein theories, may best be thought of as the “intrinsic spin dimension” of a real, physical, five-dimensional spacetime in which classical gravitation and classical electrodynamics stand united under one roof.

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