Dear Friends:

I am seeking some help with the following calculation. Start with a non-Gaussian wavefunction of the general form:

$$\psi(x) = e^{-\frac{1}{2}Ax^2 - Bx} \quad (1)$$

The goal is to calculate the variance $(\Delta x)^2 = \langle x^2 \rangle - \langle x \rangle^2$ of this non-Gaussian wavefunction. I am able to do this if I assume that the coefficients A and B are *independent*, that is $dA/dB = 0$.

But, suppose that $dA/dB \neq 0$, which is desirable, for example, with E denoting energy, if one were to consider $\psi(x) = e^{-\frac{1}{2}A_1 E^2 x^2 - B_1 E x}$ where these new coefficients can be independent, $dA_1/dB_1 = 0$, but where the original coefficients $dA/dB \propto 2E \neq 0$ are still interdependent.

I am able for $dA/dB \neq 0$ to write:

$$\langle x^2 \rangle = \frac{\int x^2 e^{-Ax^2 - 2Bx} dx}{\int e^{-Ax^2 - 2Bx} dx} = -\frac{d}{dA} \ln \int e^{-Ax^2 - 2Bx} dx - 2 \frac{\int \frac{dB}{dA} x e^{-Ax^2 - 2Bx} dx}{\int e^{-Ax^2 - 2Bx} dx} \quad (2)$$

and

$$\langle x \rangle = \frac{\int x e^{-\frac{1}{2}Ax^2 - Bx} dx}{\int e^{-\frac{1}{2}Ax^2 - Bx} dx} = -\frac{d}{dB} \ln \int e^{-\frac{1}{2}Ax^2 - Bx} dx - \frac{1}{2} \frac{\int \frac{dA}{dB} x^2 e^{-\frac{1}{2}Ax^2 - Bx} dx}{\int e^{-\frac{1}{2}Ax^2 - Bx} dx} . \quad (3)$$

This is easily checked by differentiating the ln term. These ln terms can be solved fully via the generalized Gaussian expression:

$$\int e^{-\frac{1}{2}Ax^2 - Bx} dx = \sqrt{\frac{2\pi}{A}} e^{\frac{B^2}{2A}} , \quad (4)$$

and one can thus obtain an exact answer for $(\Delta x)^2 = \langle x^2 \rangle - \langle x \rangle^2$ when $dA/dB = 0$. But, does anyone see a way to attack the terms $\frac{\int \frac{dB}{dA} x e^{-Ax^2 - 2Bx} dx}{\int e^{-Ax^2 - 2Bx} dx}$ and $\frac{\int \frac{dA}{dB} x^2 e^{-\frac{1}{2}Ax^2 - Bx} dx}{\int e^{-\frac{1}{2}Ax^2 - Bx} dx}$ in (2) and (3) to make those amenable to completing the calculation, even when $dA/dB \neq 0$ generally, or at least, when $dA/dB \propto 2E \neq 0$ specifically.

Thanks,

Jay.