Dear Friends:

I'd like below to try to generalize the "baby problem" on pp 42-43 of Zee[1] to understand, as I inquired about earlier in this thread, the "precise origins of the term d/dJ which appears as the argument in the potential $V(d/dJ)$ in the so-called "Central Identity of Quantum Field Theory," given on page 460 of Zee's QFT in an Nutshell, and especially how one gets from V(x) → $V(d/dJ)$." I think this is how we do this:

We start with the well-known Gaussian integral:

$$\int e^{-\frac{1}{2}Ax^2+Bx}dx=\sqrt{\frac{2\pi}{A}}e^{\frac{B^2}{2A}}. \quad (1)$$

Now, let us seek a closed expression for the integral:

$$\int e^{-\frac{1}{2}Ax^2-V(x)+Bx}dx=\int e^{-V(x)}e^{-\frac{1}{2}Ax^2+Bx}dx=\int\left(1-V(x)+\frac{1}{2!}V(x)^2+...\right)e^{-\frac{1}{2}Ax^2+Bx}dx, \quad (2)$$

where in the final expression we show the first two terms in the series expansion for $e^{-V(x)}$.

The so-called "potential" $V(x)$, we now take be given by the perfectly-general polynomial:

$$V(x)\equiv\sum_{n=0}^{\infty}C^{(n)}x^n, \quad (3)$$

where the $C^{(n)}$ represent an infinite set of coefficients. While we refer to $V(x)$ as a "potential," it is perhaps better to think about this simply as some unspecified, perfectly-general polynomial in *x*. Substituting (3) into (2) then allows us to write:

$$\begin{aligned}\int e^{-\frac{1}{2}Ax^2-V(x)+Bx}dx&=\int\left(1-\sum_{n=0}^{\infty}C^{(n)}x^n+\frac{1}{2!}\left(\sum_{n=0}^{\infty}C^{(n)}x^n\right)^2+...\right)e^{-\frac{1}{2}Ax^2+Bx}dx\\&=\int\left(1-\sum_{n=0}^{\infty}C^{(n)}\left(\frac{d}{dB}\right)^n+\frac{1}{2!}\left(\sum_{n=0}^{\infty}C^{(n)}\left(\frac{d}{dB}\right)^n\right)^2+...\right)e^{-\frac{1}{2}Ax^2+Bx}dx \quad . \quad (4)\\&=\int e^{-\sum_{n=0}^{\infty}C^{(n)}\left(\frac{d}{dB}\right)^n}e^{-\frac{1}{2}Ax^2+Bx}dx=e^{-\sum_{n=0}^{\infty}C^{(n)}\left(\frac{d}{dB}\right)^n}\int e^{-\frac{1}{2}Ax^2+Bx}dx\end{aligned}$$

Between the first two lines, the polynomial in *x* becomes a polynomial in the operator d/dB. In the final step, we are able to move $e^{-\sum_{n=0}^{\infty}C^{(n)}\left(\frac{d}{dB}\right)^n}$ outside the integral, because it is no longer a direct function of *x*.

---

[1] Zee, A., *Quantum Field Theory in a Nutshell*, Princeton (2003)

Now, we define the "potential" polynomial as a function of d/dB:

$$V\left(\frac{d}{dB}\right) \equiv \sum_{n=0}^{\infty} C^{(n)} \left(\frac{d}{dB}\right)^n , \quad (5)$$

so that (4), using the Gaussian integral (1), may finally be rewritten as:

$$\boxed{\int e^{-\frac{1}{2}Ax^2 - V(x) + Bx} dx = e^{-V\left(\frac{d}{dB}\right)} \int e^{-\frac{1}{2}Ax^2 + Bx} dx = e^{-V\left(\frac{d}{dB}\right)} \sqrt{\frac{2\pi}{A}} e^{\frac{B^2}{2A}} = \sqrt{\frac{2\pi}{A}} e^{-V\left(\frac{d}{dB}\right) + \frac{B^2}{2A}}}. \quad (6)$$

This is the integral expression which underlies what Zee at 460 refers to as the "Central Identity of Quantum Field Theory."

Does this explain the origin of $V(d/dJ)$ in a sufficiently general way?

Thanks,

Jay R. Yablon