Decoding the Nuclear Genome: Is there and Unambiguous and Precise way to Define the Current Quark Masses and Relate them to Nuclear Binding Energies and Mass Defects, and what is the Underlying Theory?

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Abstract: To Be Added

Contents

1. Introduction: Is there a Valid Method for Defining Quark Masses with High Precision?

In two earlier peer-reviewed papers [1], [2] the author demonstrated within parts per 10^5 AMU and better precision how the binding and fusion energies of the ^{2}H, ^{3}H, ^{3}He and ^{4}He light nuclides as well as the binding energy of ^{56}Fe could be explained as a function of *only two parameters*, namely, the current masses of the up and down quarks, found with extremely high precision in AMU to be m_u = 0.002 387 339 327 u and m_d = 0.005 267 312 526 u, see [10.3] and [10.4] and section 4 of [2] as well as section 12 of [1]. Using the conversion 1 u = 931.494 061(21) MeV [3] this equates with some loss of precision [4] to m_u = 2.223 792 40 MeV and m_d = 4.906 470 34 MeV, respectively. In an International Patent Application published at [5], this analysis was extended to ^{6}Li, ^{7}Li, ^{7}Be, ^{8}Be, ^{10}B, ^{9}Be, ^{10}Be, ^{11}B, ^{11}C, ^{12}C and ^{14}N with equally-high precision. And in [6] this analysis was extended using the Fermi vev v_F=246.219651 GeV and the Cabibbo, Kobayashi and Maskawa (CKM) mass and mixing matrix as two additional parameters, to explain the proton and neutron masses M_N = 939.565379 MeV and M_P = 938.272046 MeV [7], *completely within all known experimental errors*.

Yet, there is one underlying point which has not been sufficiently explained in any of these prior papers: the Particle Data Group (PDG) lists these two current-quark masses to be to $m_u = 2.3^{+0.7}_{-0.5}\,\mathrm{MeV}$ and $m_d = 4.8^{+0.5}_{-0.3}\,\mathrm{MeV}$ with large error bars of almost 20% for the down quark and almost 50% for the up quarks, "in a mass-independent subtraction scheme such as $\overline{\mathrm{MS}}$ [modified minimal subtraction] at a scale $\mu \approx 2\mathrm{GeV}$." [8] (Note that $\overline{\mathrm{MS}}$ and similar renormalization schemes are used to absorb the divergences from perturbative calculations beyond leading order.) In other words, the PDG values are extracted for a given renormalization scale and are actually a function of this scale and of the renormalization scheme. So although these m_u = 2.223 792 40 MeV and m_d = 4.906 470 34 MeV found by the author are well-placed near the center of these PDG error bars, the claimed precision raises the question: can we really talk about and understand these quark masses with such high precision, in a fashion which is *independent* of renormalization scale and scheme? More plainly put: is there some sensible way to simply state that "the up and down quark masses are X and Y," with X and Y being some mass-energy numbers which have an extremely small error bar due to nothing other than the accuracy of our measuring equipment? Is there a sensible, definite, unambiguous, very precise scheme we can use to define the current quark masses, consistent with empirical data, which scheme is renormalization scale-independent?

Specifically, the author's prior findings that m_u = 2.223 792 40 MeV and m_d = 4.906 470 34 MeV (these same masses were shown above even more precisely in AMU) with a precision over a million times as tight as the PDG error bars, even if *mathematically* correct in relation to the nuclear masses with which these quark masses are interrelated, presuppose an understanding of how these quark masses are to be *physically* defined and measured and understood. Without such an understanding, the author's prior work is incomplete, and to date, the author has not directly and plainly articulated this understanding.

The intention of the present paper is to remedy this deficiency by making clear that the mass defects found in nuclear weights which are related in a known way to nuclear binding and fusion / fission energies, are in fact a sort of "nuclear DNA" or "nuclear genome" the proper decoding of which teaches about nuclear and nucleon structure and the masses of the quarks in a way that has not to date been fully appreciated. In contrast to the *nuclear scattering schemes* presently used to establish quark masses, which are all based on renormalization-dependent, energy scale-dependent experiments involving scattering of nuclides and nuclei, the scheme which has been implicitly used by the author which this paper will now make explicit, is a *nuclear mass defect scheme* in which quark masses are defined in relation to the objective, very precise, experiment-independent, scale-independent, long-known energy numbers that have been experimentally found and catalogued for the nuclear mass defects, weights, binding energies, and fusion / fission energies.

All scattering experiments essentially bombard a target and use forensic analysis of the known bombardment and the found debris to learn about the nature of the target prior to bombardment. In contrast, mass defects are no more and no less than an expression of nuclear weights requiring no bombardment of anything. In this context, the prevailing scheme for characterizing quark masses has wide error bars because it is based on "bombing" the nuclides and nuclei, while the scheme to be elaborated here has very high precision because it is a "weighing" scheme which uses only nuclide and nuclear weights and so inherits the precision with which these weights are known. Colloquially speaking, the scheme to be articulated here has very tight error bars because it is based on non-intrusive nuclear "weighing" rather than highly-intrusive nuclear "bombing," and because nuclear weights themselves are very precisely known while scattering experiments introduce renormalization and scale issues which ruin precision and the ability to establish an unambiguous approach for specifying quark masses.

2. Running Couplings, Vertical Confinement and Horizontal Freedom Asymptotes, Dimensional Transmutation, and the $Q \rightarrow 0$ Limit in QCD

The electromagnetic interaction, and the electron which is a most important fermion source of this interaction, furnish the best starting point for analyzing the questions about renormalization and ambiguity posed in the introduction. Maxwell's electrodynamics, when extended into non-abelian domains by Yang-Mills gauge theories, and when $SU(3)_C$ is the particular Yang-Mills group chosen for consideration, is the template that one customarily uses to initiate study of strong chromodynamic interactions. And the electron, which is an elementary spin ½ fermion subsisting in a $U(1)_{em}$ singlet following electroweak $SU(2)_W \times U(1)_Y$ symmetry breaking, is the template best used to contrast the quarks which also have spin ½, which are also regarded as "elementary" (at least to the same degree and in the same manner that electrons are elementary), but which form an $SU(3)_C$ color triplet.

It is also important to keep in mind that Quantum Chromodynamics (QCD) is a branch of *elementary particle physics* insofar as it is used to describe the strong interactions between *colored* (R, G, B) quarks such as up and down quark flavors, via *bi-colored* (e.g., $\overline{\mathrm{R}}\mathrm{G}$) gluons all confined within a baryon. Meanwhile, nuclear physics is used to describe the interactions between *color-neutral* baryons such as the proton and neutron baryon flavors with a wavefunction $\mathrm{R} \wedge \mathrm{G} \wedge \mathrm{B} \equiv \mathrm{RGB} + \mathrm{GBR} + \mathrm{BRG} - \mathrm{RBG} - \mathrm{BGR} - \mathrm{GRB}$ that is *antisymmetric* under color interchange. And these nuclear interactions transpire via *color-neutral* mesons such as the pion-flavored mesons originally predicted by Yukawa [9] with a wavefunction $\overline{\mathrm{R}}\mathrm{R}+\overline{\mathrm{G}}\mathrm{G}+\overline{\mathrm{B}}\mathrm{B}$ that is *symmetric* under color interchange and which have short range but are not confining. Although the elementary particle physics of colored quarks and bi-colored gluons and the nuclear physics of color-neutral baryons and mesons are often lumped together as one discipline in loose discourse, they are in fact distinct disciplines bridged via so-called hadronic physics in a fashion that to this date is still not fully understood. In many ways, this paper will seek to strengthen understanding of this hadronic bridge between elementary QCD particle physics and nuclear physics to advance unification among all of these physical disciplines, by showing how the masses of quarks which are elementary, are interrelated with the masses and binding energies of nucleons and nuclei which are not elementary.

It should also be kept in mind that the author's thesis first published in [1] that baryons are the chromo-magnetic monopoles of Yang-Mills gauge theory is a direct consequence of the fact baryons have a color symmetry $\mathrm{R} \wedge \mathrm{G} \wedge \mathrm{B} = \mathrm{R}[\mathrm{G},\mathrm{B}] + \mathrm{G}[\mathrm{B},\mathrm{R}] + \mathrm{B}[\mathrm{R},\mathrm{G}]$ which is *antisymmetric* under *color interchange* while magnetic monopoles which have the spacetime symmetries of $\partial_\sigma F_{\mu\nu} + \partial_\mu F_{\nu\sigma} + \partial_\nu F_{\sigma\mu}$ where $F_{\mu\nu} = -F_{\nu\mu}$ is an antisymmetric field strength tensor whether it is abelian or non-abelian, so that the monopoles are likewise *antisymmetric* under *spacetime index interchange*. In the former case there are three colors and in the latter three spacetime indexes, and in both cases the interchange symmetry is antisymmetric in identical fashion. The physically-meaningful link between these alike color and spacetime symmetries which demonstrates that baryons *are* the chromo-magnetic monopoles of non-Abelian gauge theory – i.e., the connection which advances us from like-symmetries to the *formal identification* of chromo-magnetic monopoles with baryons – is established in section 5 of [1] and deepened in section 10 of [10] which is presently under review at Physical Review D as manuscript DK11244, through the application of the Fermi-Dirac-Pauli Exclusion Principal.

Now, when we talk about the electromagnetic interaction, we can readily state that the dimensionless "running" coupling of this interaction is measured to be the rather precise $\alpha_{em} = e^2/4\pi\hbar c$ = 1/137.035 999 074 for low probe energies, where e is the electric charge strength, and specifically, that this "fine structure" number is the *horizontally-asymptotic* value of α_{em} as the renormalization scale $Q \to 0$, with Q plotted horizontally and the function $\alpha_{em}(Q)$ plotted vertically. We also know that as the renormalization scale Q is increased, so too is the strength of this interaction, which in quantum field theory is an important distinguishing feature between an abelian interaction and a non-abelian interaction. So, for example, when $Q \approx M_W$, we also have $\alpha_{em} \approx 1/128$. [3]

Likewise, when we talk about the mass of the electron, we can state that m_e = 0.510 998 928 ± 0.000000011 MeV, [11] which expresses an extremely high measurement precision limited only by the accuracy of our laboratory equipment. But just as the running coupling α_{em} is a function of renormalization scale Q, so too is the measured electron mass m_e. So when we make the foregoing statement as to the energy number associated with the electron mass, we are implicitly stating that this is the horizontally-asymptotic value of this mass for $Q \to 0$. At any deep probe scale, this mass is also expected to "run" just like the running coupling / charge strength. So whether stated explicitly or understood implicitly, we are *defining* the mass and electric charge strength of the electron based on what is asymptotically observed at $Q = 0$, and with this definition, we are able to express both α_{em} and m_e with a high precision limited only by our measuring instrumentation. *But we are only able to do this because the natural world obliges us by providing a running electromagnetic coupling and a running electron mass which <u>are in fact</u> horizontally-asymptotic in the $Q \to 0$ limit.*

So the question now arises, if we can define charge strength and mass in this way for electromagnetic interactions and electrons, can we not do the same for strong interactions and quarks? That is, why can't we just define the running strong coupling α_s and the up and down (and other) quark masses based on their horizontally-asymptotic values as the renormalization scale $Q \to 0$?

The answer is evident from the very asking of this question: we cannot establish a definition for the quark charges and masses similar to that for the electron charges and masses *precisely because quarks are confined and not free*. Quarks are not free particles in the same manner as electrons; they are only asymptotically free [12] deep inside a nucleon from which they can never be individually removed. Quantum Electrodynamics (QED) is abelian while QCD is non-abelian, so the running coupling curves are flipped in their qualitative features over the Q domain axis. In QCD, the running coupling α_s and quark masses m_q approach a *horizontal* asymptote, not as $Q \to 0$, but as $Q \to \infty$, or at least as Q reaches some very large energy associated with the horizontal asymptotic freedom observed deep inside a nucleon via deep inelastic scattering (DIS). So notwithstanding their similarities because they are both rooted in Maxwell's electrodynamics, the confining nature of $SU(3)_C$ as a non-abelian interaction is what makes strong interactions *qualitatively different* from $U(1)_{em}$ electromagnetic interactions which are abelian. And notwithstanding the similarities of quarks to electrons as spin ½ fermions which are equally-elementary, the confinement of quarks within nucleons is what makes them *qualitatively different* from electrons (and leptons generally).

The parameter Λ_{QCD} at which dimensional transmutation occurs in QCD provides a good quantitative vehicle to discuss these qualitative differences. Referring to Figure 9.4 of [13] reproduced as Figure 1 below for the reader's convenience, Λ_{QCD} specifies the energy-dimensioned domain value of a *vertical asymptote* approached by the dimensionless function $\alpha_s(Q)$ at $Q=\Lambda_{QCD}$ from right-to-left along the $Q>\Lambda_{QCD}$ domain. For example, for a six-flavor quark model in the $\overline{\text{MS}}$ scheme, as laid out in [9.24a] of [13] and the associated discussion, this vertical asymptote is determined to be situated at $\Lambda_{QCD}=90.6\pm 3.4\text{ MeV}$, which is one order of magnitude left of the leftmost domain of Figure 1. And, as *Q* grows larger beyond the rightmost domain of Figure 1, there is *also a horizontal asymptote* associated with asymptotic freedom. So in contrast to an abelian interaction like QED, the horizontal asymptote appears in the large-*Q* rather than the $Q\to 0$ domain, as discussed, and so is qualitatively flipped. Via the conversion constant $\hbar c=.197\,326\,971\,8\,\text{GeV fm}$ [3] which in natural units $\hbar=c=1$ may be rewritten as $1\,\text{GeV}=5.067\,730\,939\,\text{fm}^{-1}$, one is able to deduce using the median value $\Lambda_{QCD}=.0906\,\text{GeV}$, that $\Lambda_{QCD}=.0906\text{ GeV}=.0906\times 5.0677\text{ fm}^{-1}=.4591\,\text{fm}^{-1}=1/(2.1780\text{ fm})$. So in the six-flavor quark model, the deBroglie length associated with this vertical asymptote of confinement at Λ_{QCD} is $r_\Lambda \equiv \hbar / c\Lambda_{QCD}=2.1780\text{ fm}$, i.e., just over 2 Fermi in length dimension.

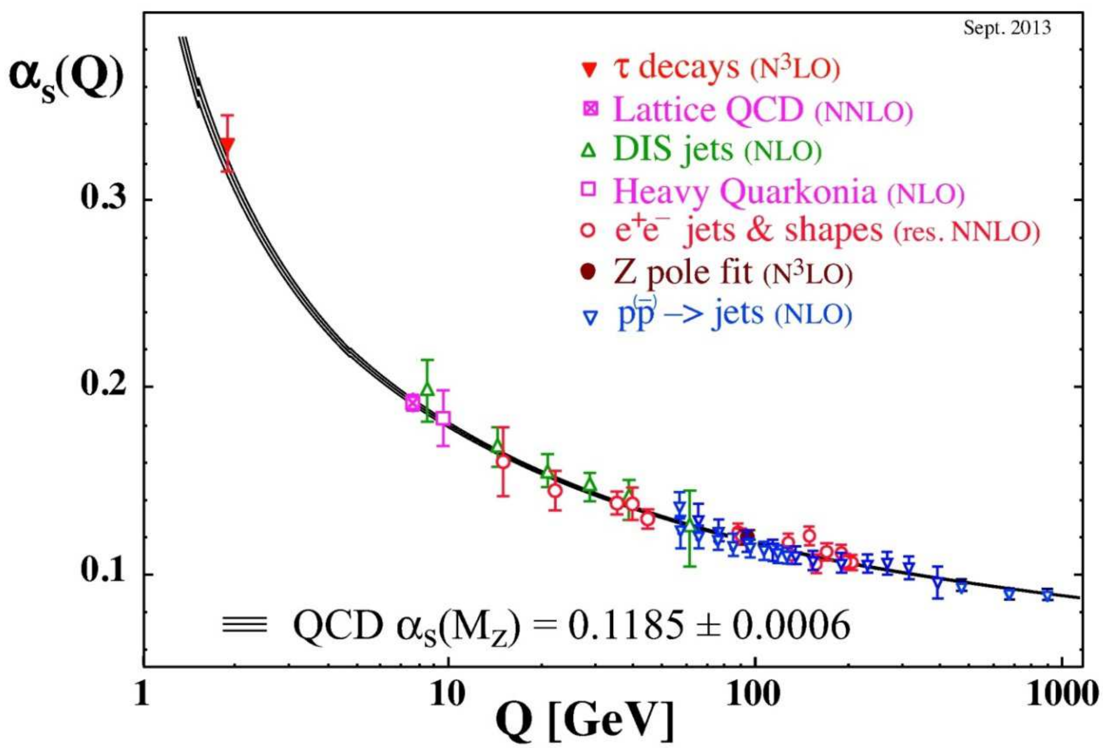

Figure 1: The Running Strong Coupling (reproduced from PDG's [13], Figure 9.4)

So while we are able in QCD to talk about the running of the strong coupling $\alpha_s = g_s{}^2 / 4\pi\hbar c$ and strong charge g_s acting between quarks for $Q > \Lambda_{\text{QCD}}$ as illustrated in Figure 1, it makes no sense to talk about the running of α_s for $Q < \Lambda_{\text{QCD}}$, or especially for $Q \to 0$, as we are able to do for α_{em} in QED. In fact, when we do experiments in the low-energy $Q < \Lambda_{\text{QCD}}$ domain, we are no longer observing *strong interactions between quarks* confined within a nucleon with a strength measured by α_s. Rather, we are observing *nuclear interactions between nucleons*. Further, these nuclear interactions are observed to have a very short range and exponentially diminish to zero beyond separations of a few Fermi in length. For example, because of this exponential strength diminution, nuclei heavier than about ^{56}Fe start to manifest inherent instability because nucleons (protons and neutrons) within the same nucleus become situated far enough apart that the nuclear force no longer holds them in the nucleus. So, in contrast to the *strong* interaction between quarks in the six-quark model which has a short range on the order of $r_\Lambda = 2.1780\,\text{fm}$ which grows vertically-asymptotically stronger and becomes infinite so as to enforce confinement as $Q \to \Lambda_{\text{QCD}}$ from right-to-left, the *nuclear* interaction is short range because it grows exponentially-smaller for $Q < \Lambda_{\text{QCD}}$ from right-to-left and exponentially attenuates to zero strength beyond a distance of several Fermi. Thus, as we move laterally across the vertical asymptote at the energy Λ_{QCD} and its length equivalent r_Λ, we are implicitly crossing the disciplinary boundary between the strong elementary particle physics of quarks, and the nuclear physics of nucleons and the assemblies thereof known as nuclei. That is the boundary sought to be bridged by hadronic physics.

Consequently, while in QED we can *define* 1/137.035 999 074 as the dimensionless strength of α_{em} for $Q = 0$ because electrodynamics is an abelian interaction which thereby has a *horizontal* asymptote as $Q \to 0$, we cannot employ a similar definition in QCD. Because of QCD's non-abelian character, the horizontal asymptote of QED as $Q \to 0$ is flipped to the horizontal asymptote of asymptotic freedom for $Q \gg \Lambda_{\text{QCD}}$, and the "low energy" domain is bounded on the left by a *vertical* asymptote at $Q = \Lambda_{\text{QCD}}$. The $Q \to 0$ limit for α_s is effectively meaningless in QCD, because as $Q \to 0$ the only pertinent interaction is the nuclear interaction and not the strong interaction between quarks. And that nuclear interaction, being short-range with exponential attenuation, has zero strength at $Q = 0$ rather than a finite number like the meaningful α_{em} = 1/137.035 999 074 found in electrodynamics. So instead of characterizing the strong interaction strength starting with a dimensionless value of $\alpha_s = 0$ at $Q = 0$ like we use α_{em} = 1/137.035 999 074 for QED, we define the strong interaction via the transmuted energy-dimensioned parameter Λ_{QCD} at which there is a vertical asymptote toward which $\alpha_s \to \infty$ from the right as in Figure 1. And then for $Q > \Lambda_{\text{QCD}}$, α_s depends very definitively on the energy scale *Q*, and in addition, it depends on the specific renormalization scheme used to absorb the higher-order perturbative divergences.

In sum: The dimensionally-transmuted energy number $\Lambda_{\text{QCD}} = .0906\,\text{GeV}$ in six-quark QCD serves the exact same role for QCD as does the dimensionless number α_{em} = 1/137.035 999

074 for QED in establishing the leftmost domain of the running couplings α_s and α_{em}. For QED, the "fine structure" number 1/137.035 999 074 tells us the dimensionless magnitude of α_{em} as $Q \to 0$ for which nature obliges us because the running coupling for an abelian interaction actually does approach a horizontal asymptote as $Q \to 0$. But nature does not similarly oblige us for a non-abelian interaction such as QCD. Now, at the low-energy boundary of the meaningful domain, for six quarks, there is a vertical asymptote for which $\alpha_s \to \infty$ at $\Lambda_{\mathrm{QCD}} = .0906\,\mathrm{GeV}$, and α_s has no meaning for $0 < Q < \Lambda_{\mathrm{QCD}}$ because that is the domain of nuclear interactions between baryons not strong interactions between quarks. So we are compelled to use the energy dimensioned number $Q = \Lambda_{\mathrm{QCD}} = .0906\,\mathrm{GeV}$ to tell us the Q at which the dimensionless number α_s approaches its low-energy vertical asymptote. Therefore, while the $Q \to 0$ limit is meaningful for QED because $\alpha_{em} \to 1/137.035\,999$ in this limit, the meaningful limit for QCD is $Q \to \Lambda_{\mathrm{QCD}} = .0906\,\mathrm{GeV}$ because $\alpha_s \to \infty$ in this limit. The $Q \to 0$ limit still does have meaning, but at least based on initial appearances, not for *strong interactions* between and among *quarks*. It has meaning for *nuclear interactions* between and among *baryons*, although at this limit, there is no interaction because of the exponential attenuation of the nuclear interaction strength.

Now, we have laid out sufficient background to return to the problem of how to define current quark masses.

3. Primary Relationships among the Up and Down Current Quark Masses, and the Electron, Proton and Neutron Masses, and the Three Questions they Raise

In QED we are able to use the $Q \to 0$ limit to define the electron rest mass m_e = 0.510 998 928 ± 0.000000011 MeV because there is a horizontal asymptote at α_{em} = 1/137.035 999 in this limit and because electrons are free particles which can have their attributes such as mass and charge and spin measured directly and with precision. But in QCD the $Q \to 0$ limit appears to be taken off the table, and the low-energy limit for meaningful discourse appears to be $Q = \Lambda_{\mathrm{QCD}} = .0906\,\mathrm{GeV}$ at which $\alpha_s \to \infty$ and quarks are confined. Plainly put: it is impossible to take a quark out of a baryon and measure its mass m_q in the $Q \to 0$ limit in the same way that we would measure an electron mass. Thus, to try to define current quark masses based on their measured values $m_q\left(Q=0\right)$ would appear to make no sense because this is a measurement which it is physically impossible to ever take for an individual quark! How can we *define* a quark mass m_q based on its value at $Q=0$ when it impossible to ever take such a measurement at $Q=0$? We would be using a definition that can never be experimentally validated!

But, as we do for free electrons, it *is* possible to take $Q=0$ mass measurements for baryons such as protons and neutrons, and indeed, we know very precise values for these measurements, namely M_P = 938.272046±0.000021 MeV and M_N = 939.565379±0.000021 MeV [7]. So while we certainly cannot *directly* measure quark masses $m_q\left(Q=0\right)$, we are able to directly measure baryon (*B*) masses $M_B(Q=0)$. And of course, baryons contain quarks, and protons and neutrons which are the most abundant and stable flavors of baryon contain the up and down flavors of quark. So the question arises whether it might be possible to measure $m_q(Q=0)$, not directly, but *indirectly, by inference,* from the direct measurements of $M_B(Q=0)$ which are well known with some substantial degree of precision, and whether this precision might then be inherited by the indirectly-defined $m_q(Q=0)$.

As we shall now start to explore, this is indeed possible, if, as stated in the introduction, we employ a scheme based on non-intrusive nuclear "weighing" rather than the highly-intrusive nuclear "bombing" of scattering experiments. Moreover, once we have defined the up and down current quark masses based on *indirect inference from nuclear weights* rather than direct inference from deep nuclear scattering, it becomes possible with high precision to use these quark masses to also explain the empirical binding energy and nuclear weight and mass defect and fusion energy data of multiple light nuclides which data has heretofore never been given a satisfactory explanation. This in turn serves to validate the initial indirect inference of quark masses from nuclear weights. Theoretically, all of this is rooted in and emerges from the author's thesis in [1] as further developed in [10] that baryons are the chromo-magnetic monopoles of Yang-Mills gauge theory.

The up and down quark masses are indirectly inferred from the $Q=0$ proton and neutron masses, as well as the $Q=0$ electron mass, using the following two relationships which for the moment will simply be stated, and which we shall later explain and support based on the thesis that baryons are the chromo-magnetic monopoles of Yang-Mills gauge theory. First, as initially found in [11.23] of [1], the *difference* between the up and down current quark masses is related to the electron rest mass according to:

$$m_d - m_u = \frac{(2\pi)^{\frac{3}{2}}}{3} m_e . \tag{3.1}$$

Second, as initially found in [A15] and [7.2] and section 10 of [2], the *difference* between the neutron and proton masses is related to the up and down current quark masses and the electron mass, and via (3.1) through which we can eliminate m_e, *exclusively* to the up and down current quark masses, according to:

$$M_N - M_P = m_u - m_e - \frac{2\sqrt{m_\mu m_d}}{(2\pi)^{\frac{3}{2}}} = m_u - \frac{3m_d - 3m_u + 2\sqrt{m_\mu m_d}}{(2\pi)^{\frac{3}{2}}} . \tag{3.2}$$

We shall regard (3.1) and (3.2) above to be *exact* relationships not only *Q* = 0, but for all *Q*, which is to say, we shall take these to be both exact and *Q*-invariant. And we shall use these relationships as the starting point to obtain many other relationships – most very close to empirical data albeit still approximate – intended to contradict or validate our treatment of (3.1) and (3.2) as exact, *Q*-invariant relationships. For these reasons, simply to provide a shorthand for discourse, we shall henceforth refer to (3.1) and (3.2) above as the “primary mass relationships” among the up and down current quark masses, and the electron, proton and neutron masses. It will be appreciated, because m_e in (3.1) is known with very high precision, and because $M_N - M_P$ in (3.2) is known with similarly high precision, that when we take (3.1) and (3.2) together, and if we do regard these as exact and Q-invariant as just discussed, that we can combine these to deduce m_u and m_d with commensurately-high precision.

This calculation is performed in section 10 of [2] using the AMU median empirical values $m_e = 0.000\,548\,579\,909\text{ u}$ [11], $M_N = 1.008\,664\,916\,0\text{ u}$ and $M_P = 1.007\,276\,466\,8\text{ u}$ [7] which all have been experimentally measured to ten or more digits of precision in AMU. So, using these values in (3.1) and (3.2) above leads us to deduce in [10.3] and [10.4] of [2], to the same ten-digit precision as the proton and neutron masses that:

$$m_u = 0.002\,387\,339\,3\text{ u} = 2.223\ 792\ 40\text{ MeV}\,, \tag{3.3}$$

$$m_d = 0.005\,267\,312\,5\text{ u} = 4.906\ 470\ 34\text{ MeV}\,. \tag{3.4}$$

As noted in the introduction, the median electron mass to the same precision level in MeV is m_e = 0.510 998 93 MeV. Certainly, (3.3) and (3.4) converted to MeV fit well within the PDG error bars which inform us that $m_u = 2.3^{+0.7}_{-0.5}\text{MeV}$ and $m_d = 4.8^{+0.5}_{-0.3}\text{MeV}$ [8], so we at least know that there is no direct empirical contradiction to these results from this particular data.

Starting from (3.3) and (3.4) as deduced from the primary mass relationships (3.1) and (3.2), there are three questions which now need to be explored:

1) <u>Legitimate, Unambiguous Measurement Scheme</u>: Can we make such a precise statement as to the masses of the up and down quarks, given: the wide PDG error bars $m_u = 2.3^{+0.7}_{-0.5}\text{MeV}$ and $m_d = 4.8^{+0.5}_{-0.3}\text{MeV}$; that these error bars reflect that quark masses are thought to be dependent upon the renormalization scheme and the renormalization scale Q; that quarks are confined and so can never have their $Q = 0$ masses *directly* measured in the same way we are able to measure the electron mass $Q = 0$; and that the only domain within which it even starts to make sense to talk about directly measuring a quark mass is the domain where $Q \ge \Lambda_{\text{QCD}}$? Indeed, these wide error bars emerge because it is widely perceived that $Q \ge \Lambda_{\text{QCD}}$ is the only domain in which it makes sense to talk about current quark masses, and because, as seen in Figure 1, measurement in this domain – invariably via scattering experiments at various depths – is so highly-dependent upon the scale Q and the renormalization scheme we use. In short, can we use (3.3) and (3.4) as precise statements about the $Q = 0$ up and down quark masses, in view of all the issues just reviewed in section 2?

2) <u>Clear Secondary Empirical Support</u>: If we can legitimately assert (3.3) and (3.4) to be the $Q = 0$ up and down quark masses by overcoming the “measurement” challenges of point 1 and section 2 above, are (3.3) and (3.4) supported by empirical particle data? This is a straightforward question as to whether nature supports (3.3) and (3.4) based on energies we

observe when we do experiments. As noted, the results m_u = 2.223 792 40 MeV and m_d = 4.906 470 34 MeV certainly are not contradicted by PDG's $m_u = 2.3^{+0.7}_{-0.5}\,\mathrm{MeV}$ and $m_d = 4.8^{+0.5}_{-0.3}\,\mathrm{MeV}$; indeed, they sit fairly near the mean of this data. But it would be desirable to see if (3.3) and (3.4) can be supported by *additional empirical data* beyond the electron, neutron and proton masses from which they were deduced via (3.1) and (3.2), via what we shall refer to as "secondary relationships." Specifically, *if* (3.3) and (3.4) are indeed correct valuations for the up and down current quark masses on a $Q = 0$ scale, and because the neutron, proton and electron masses are already related to these via (3.1) and (3.2), it seems plausible that other energies of interest, namely the binding, fusion, mass defect and nuclear weight energies of light nuclides such as hydrogen and helium and lithium and beryllium, etc., might also be related to and be secondary functions of these exact same $Q = 0$ quark masses. In other words, if (3.3) and (3.4) are legitimately-defined $Q = 0$ quark masses, then these masses will always be the $Q = 0$ quark masses, whether these quarks are in a free proton are neutron, or, for example, are in a proton or neutron inside of an alpha particle (^{4}He nucleus), or in a proton or neutron inside an ^{56}Fe nucleus, or are deep within the bowels of a lead or a uranium nucleus, etc. And that means that we *should* be able to specify the observed nuclear data for *any and all types of nuclei*, solely as a function of these two quark masses! This provides ample latitude for empirical contradiction. But at the same time, if a substantial number of nuclides can indeed have their nuclear data parameterized using secondary relationships based exclusively on the two masses (3.3) and (3.4), this would represent compelling empirical support for these results.

3) <u>Solid Theoretical Foundation</u>: If we can legitimately assert (3.3) and (3.4) to be the $Q = 0$ up and down quark masses and if we can find secondary support from a broad array of nuclear data, then we get to the third question: what is the overarching theory, and does that theory make sense within the overall framework of theoretical physics? As stated, that theory, first laid out in [1] and further developed and refined in [10], asserts that *baryons are the color-neutral chromo-magnetic monopoles of non-Abelian Yang-Mills gauge theory*. It is from this theory that the primary mass relationships (3.1) and (3.2) were initially discerned, and upon which the ^{2}H, ^{3}H, ^{3}He and ^{4}He [2] and ^{6}Li, ^{7}Li, ^{7}Be, ^{8}Be, ^{10}B, ^{9}Be, ^{10}Be, ^{11}B, ^{11}C, ^{12}C and ^{14}N [5] binding energies can be explained *exclusively* as a function of the two masses (3.3) and (3.4), via a series of secondary relationships, to at least parts per hundred thousand AMU in all cases. And it is from this theory that once the Fermi vev v_F=246.219651 GeV and the Cabibbo, Kobayashi and Maskawa (CKM) mass and mixing matrix are also admitted as parameters alongside of these two quark masses, the proton and neutron masses [6] can be fully explained within all known experimental errors.

So for the balance of this paper, we shall address each of these questions in turn.

4. Does Deduction of Very Precise *Q* = 0 Up and Down Current Quark Masses from the *Q* = 0 Electron, Proton and Neutron (EPN) Masses Establish a Legitimate Measurement Scheme?

As discussed at the start of section 3, because quarks are confined it is impossible to ever measure their $Q = 0$ masses *directly*, because to access a quark in the six quark model (which clearly looks to be what nature chooses and which we shall henceforth regards as nature's choice) one must provide an impact energy at least on the order of $Q = \Lambda_{\text{QCD}} = .0906\ \text{GeV}$. In other words, to directly detect of *any attributes* of an individual quark, and indeed its very existence, one must supply an impact energy north of about 90 million electron volts. So whatever attributes we observe *by definition* will not be the $Q = 0$ attributes of the observed quark. This is the measurement problem which leads to the large error bars $m_u = 2.3^{+0.7}_{-0.5}\ \text{MeV}$ and $m_d = 4.8^{+0.5}_{-0.3}\ \text{MeV}$ wherein the quark masses are dependent upon the chosen measurement scheme, and once a scheme is chosen, on the choice of Q given that $Q = 0$ quark attributes appear to not be measurable because quarks are confined, not free, particles.

But in (3.1) and (3.2) we have chosen a measurement scheme by which the up and down quark masses are *inferred indirectly* from the $Q = 0$ electron, proton and neutron masses. Just like minimal subtraction MS and modified minimal subtraction $\overline{\text{MS}}$, (3.1) and (3.2) do represent a measurement scheme for quark masses, albeit a different scheme from the usual. The question here is whether this is different scheme is a *legitimate* measurement scheme.

Now, any time that we do an experiment for which $Q > 0$ we are necessarily doing a scattering experiment, which is to say, we are bombarding a target in some fashion and discerning information about the target via forensic analysis of the post-bombardment debris coupled with knowledge of the bombardment we employed. No matter how it is couched in its specifics, any experiment with $Q > 0$ is *by definition* causing an impact with the target we seek to study, and in the course of obtaining information about the target we are necessarily altering the target. Thus, when we use several different Q at several different times, we have to prepare for the possibility that what we are measuring about the target will take on several different values, with no one particular value being any more correct or unique than any other value. Thus, we will have error bars stemming from more than just the limitations of our measuring equipment. To use the colloquialism of section 1, such an experiment entails bombing the target, not weighing the target.

Conversely, merely taking the weight of a body is *the quintessential* $Q = 0$ *experiment*, whether that body is a person or a baseball or an electron, proton or neutron. *Subject to the caveat in the next paragraph*, we do not have to impact a body in order to weigh that body; we merely place it on a scale and then rely upon the equivalence of gravitational and inertial mass. So we are able to say that at $Q = 0$ the mass of the electron is $m_e = 0.000\,548\,579\,909\ \text{u}$, period. And we are similarly able to say that at $Q = 0$ the masses of the proton and the neutron are $M_N = 1.008\,664\,916\,0\ \text{u}$ and $M_P = 1.007\,276\,466\,8\ \text{u}$, period. We do not need to talk about the measurement scheme, and we do not need to talk about the renormalization scale Q other than to understand that by definition we are using $Q = 0$. Of course we have the option if we wish to study how these masses may vary from their $Q = 0$ values for various $Q \neq 0$. But $Q = 0$ does provide a uniqueness which is not provided by any other Q, with the possible exception of $Q = \Lambda_{\text{QCD}} = .0906\ \text{GeV}$ which happens to coincide with the confining $\alpha_s = \infty$ and so presents other measurement challenges.

Now, of course, someone who is familiar with experiments used to obtain the above-recited electron, proton and neutron masses will understand the caveat that nobody can really put one of these particles on a scale and "weigh" that particle in the same manner that we can weigh ourselves or weigh a macroscopic object. The experiments used to establish these masses themselves do have some $Q \neq 0$ scattering aspect. However, the electron, proton and neutron are all free particles unlike quarks, and their masses approach asymptotic values as $Q \to 0$. So by doing enough experiments on these free particles – even with some impact – it is possible to deduce the asymptote that is approached by the masses of each of these particles. Therefore, the precision with which the experimental community has succeeded in doing this is effectively expressed by the mass values and associated experimental errors for m_e, M_P and M_N given in [11] and [7]. The same can also be said for measurements of the masses of composite nuclides, such as ^{2}H, ^{3}H, ^{3}He, ^{4}He, etc.

So when we take the expressions (3.1) and (3.2), plug in the $Q = 0$ "weights" of the electron, proton and neutron, and thereby deduce (3.3) and (3.4) for the up and down quark masses, what we have discerned – albeit indirectly – must also be regarded as the $Q = 0$ "weights" of these two quarks. Again, this is certainly a different scheme from the minimal subtraction schemes which are usually employed to specify quark masses and other running attributes.

But irrespective of the specifics of relations (3.1) and (3.2), if we were to establish *some pair of valid relations* which express the up and down quark masses in relation to the electron, proton and neutron masses such that these two quark masses are *uniquely fixed* once these other three masses are fixed, then by employing the $Q = 0$ values of the electron, proton and neutron masses, we would necessarily be deducing the $Q = 0$ values of the up and down quark masses, and we would have a legitimate measurement scheme. The point here is that this "weighing, not bombing" scheme is not wedded to the specifics of (3.1) and (3.2), but rather, to the question of whether *any valid relationships* which might *uniquely* output the up and down quark masses once the $Q = 0$ electron, proton and neutron masses are given can be said to yield legitimate values for the $Q = 0$ quark masses.

Understood in this manner, it should be clear that it is perfectly legitimate *as a matter of defining a measurement scheme* to specify $Q = 0$ confined quark masses in relation to the known masses of other particles which are free and which can be observed asymptotically in the low-*Q* energy domain, *if* such relationships exist and can be found. So the real question becomes *whether there do in fact exist some of valid relations* in nature by which the up and down quark masses can be uniquely deduced from the electron, proton and neutron masses (or any other free particle $Q = 0$ masses), and if so, what those relationships are and whether (3.1) and (3.2) are in fact those relationships.

If it should turn out that (3.1) and (3.2) are valid relationships, then (3.3) and (3.4) are indeed the $Q = 0$ masses of the up and down quarks, and the measurement scheme for defining these quark masses in this way is perfectly legitimate. Further, by having these two mass values (3.3) and (3.4), we now know the quark masses to a precision that is *close to a billion times more* *precise* than what we learn from $m_u = 2.3^{+0.7}_{-0.5}\,\text{MeV}$ and $m_d = 4.8^{+0.5}_{-0.3}\,\text{MeV}$ based the $\overline{\text{MS}}$ scheme. It is the foregoing elaboration of how the quark masses $m_u = 0.002\,387\,339\,3\ \text{u}$ and $m_d = 0.005\,267\,312\,5\ \text{u}$ can be *legitimately defined* from the proton, neutron and electron masses with a precision vastly exceeding the PDG data based on $\overline{\text{MS}}$, which was absent from the authors prior work, and which should remedy this deficiency. And it should also be very clear that a second scheme which allows the quark masses to be defined close to a billion times more accurately than a first scheme is manifestly preferable to the first scheme. So that is the scheme that the author is proposing for defining the up and down quark masses. Because this scheme *defines* $Q=0$ up and down current quark masses in (3.3) and (3.4) from the relationships (3.1) and (3.2) using the $Q=0$ electron (E), proton (P) and neutron (N) masses, we shall refer to this as the EPN measurement scheme with an EPN-0 definition for the up and down quark masses. Of course, relationships (3.1) and (3.2) should apply at all *Q*. So if one were to know how each of $m_e(Q)$, $M_P(Q)$ and $M_N(Q)$ run as a function of *Q*, one would then use (3.1) and (3.2) to further derive $m_u(Q)$ and $m_d(Q)$. In this way the EPN scheme provides a consistent and unambiguous basis for first defining the up and down quark masses at $Q=0$, and for then ascertaining how they run as a function of increasing scale *Q*, all based on three masses m_e, M_P and M_N which are each known at $Q=0$ with very high precision. And it avoids the pitfalls and ambiguities of having to define quark masses based on probing inside the nucleons in a fashion that will necessarily make these masses a function of our experiment.

So with the measurement question of how best to define the current quark masses now addressed, we next turn to question whether (3.3) and (3.4) are indeed the correct physical, $Q=0$ quark masses. If they are, then this in turn would validate the relationships (3.1) and (3.2) and the theory from which these are obtained. Certainly, the fact that masses (3.3) and (3.4) fit well within $m_u = 2.3^{+0.7}_{-0.5}\,\text{MeV}$ and $m_d = 4.8^{+0.5}_{-0.3}\,\text{MeV}$ provides preliminary validation for these masses by failing to invalidate these masses. But this is a starting point, not an endpoint. Now we arrive at the second question posed in section 3, whether the quark masses (3.3) and (3.4) have clear secondary empirical support.

5. Origins of the Primary Mass Relationships used in the EPN Measurement Scheme

In section 3, we simply stated the primary mass relationships (3.1) and (3.2). Now it is appropriate to begin discussing their physical origins based on the thesis that baryons are the chromo-magnetic monopoles of Yan-Mills gauge theory. First, let us just lay out some general background.

It is well-known that $T^{\mu\nu} = \partial^\mu \phi \left(\partial \mathfrak{L} / \partial \left(\partial_\nu \phi \right) \right) - g^{\mu\nu} \mathfrak{L}$ is the canonical energy-momentum tensor for a given field φ with associated Lagrangian density $\mathfrak{L}$. If we require the spatially-integrated Lagrangian $L = \iiint \mathfrak{L} d^3x$ to be stationary under small field variations, then the $\partial^\mu \phi \left(\partial \mathfrak{L} / \partial \left(\partial_\nu \phi \right) \right)$ term can be neglected and this becomes $T^{\mu\nu} = -g^{\mu\nu} \mathfrak{L}$. So in flat spacetime with $g^{00}=1$ we have $T^{00}=-\mathfrak{L}$. Therefore the total energy E of the system associated with $\mathfrak{L}$ will be $E=\iiint T^{00}d^3x=-\iiint \mathfrak{L}d^3x=-L$. And more simply, $E=-L$.

Now, in abelian electrodynamics, the Lagrangian density associated with a pure gauge field $F^{\mu\nu}$ is given by $\mathfrak{L}=-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$, and so $E=-L=-\iiint \mathfrak{L}d^3x=\iiint \frac{1}{4}F_{\mu\nu}F^{\mu\nu}d^3x$ will specify the energy arising from the pure gauge field terms. In Yang-Mills gauge theory the field strength may still be written with $F_{\mu\nu}$ as shorthand, but it contains additional internal symmetry structure which must be understood. Particularly, for any simple unitary gauge group SU(N) there are a set of generators λ^i with $i=1...N^2-1$ forming a closed group and commuting according to $\left[\lambda^i,\lambda^j\right]=if^{ijk}\lambda^k$, conventionally normalized to $\mathrm{Tr}\lambda^{i2}=\frac{1}{2}$. Each of these generator matrices has rank 2 with an N×N dimensionality, so to be fully explicit we must represent these matrices by $\lambda^i{}_{AB}$ with $A,B=1...N$. So in reality, the field strength $F_{\mu\nu}$ is a shorthand for $F_{\mu\nu AB}=\lambda^i{}_{AB}F^i{}_{\mu\nu}$, where the "adjoint form" $F^i{}_{\mu\nu}$ consists of N^2-1 individual 4×4 field strength tensors, and the "matrix form" $F_{\mu\nu AB}$ is an N×N internal symmetry matrix of 4×4 field strength tensors. The pure-gauge field Lagrangian density represented in the matrix form is now $\mathfrak{L}_{\text{gauge}}=-\frac{1}{2}\mathrm{Tr}\left(F_{\mu\nu}F^{\mu\nu}\right)$, with the doubling of the coefficient owing to the generator normalization, and so the energy is $E=\iiint \frac{1}{2}\mathrm{Tr}F_{\mu\nu}F^{\mu\nu}d^3x$.

Now, if we want to be as explicit as possible, then rather than using the trace (Tr) notation, we can use the matrix form $F_{\mu\nu AB}$ and explicitly show the index contractions which yield this trace, namely, $\mathfrak{L}=-\frac{1}{2}\mathrm{Tr}\left(F_{\mu\nu}F^{\mu\nu}\right)=-\frac{1}{2}F_{\mu\nu AB}F^{\mu\nu}{}_{BA}$. That is, the trace is formed first by taking an *inner product* $F_{\mu\nu AB}F^{\mu\nu}{}_{BC}$ which yields a new N×N internal symmetry matrix. Then we contact the A and C indexes to obtain $F_{\mu\nu AB}F^{\mu\nu}{}_{BA}$. It is by this latter contraction that we obtain the trace, and more specifically, the *inner product trace*. But mathematically, there is a second trace available from $F_{\mu\nu}F^{\mu\nu}$, and that is the *outer product trace* which for any two matrices A and B is given by $\mathrm{Tr}(A\otimes B)=\mathrm{Tr}(A)\mathrm{Tr}(B)$. So using explicit indexes, the outer product trace is $F_{\mu\nu AA}F^{\mu\nu}{}_{BB}$. Thus, if we wish to be as general as possible, we should entertain the possibility of constructing the pure Yang-Mills gauge field Lagrangian density using some linear combination of both the inner product trace $F_{\mu\nu AB}F^{\mu\nu}{}_{BA}$ and the outer product trace $F_{\mu\nu AA}F^{\mu\nu}{}_{BB}$.

With this general background in mind, we start with an $F_{\mu\nu AB}$ which is carefully developed for the chromo-magnetic monopoles of Yang-Mills gauge theory, see [10.1] of [1] which is more deeply developed into [10.4] of [10]. This employs the gauge group SU(3)$_C$ of strong chromodynamic interactions with colors R, G, B, which means that the internal symmetry matrices have a 3x3 dimensionality, see, e.g., the matrix [9.20] of [10] which explicitly shows this. We then represent a (duu) proton by assigning the R quark color to the down quark flavor and the G and B quark colors to the up quark flavors via $\mathrm{R}\to d;\mathrm{G}\to u;\mathrm{B}\to u$, and a (udd) neutron by an analogous assignment $\mathrm{R}\to u;\mathrm{G}\to d;\mathrm{B}\to d$, all as laid out in sections 7 and 8 of [1] and the second half of section 10 in [10]. Finally, as laid out in sections 9, 11 and 12 of [1] we calculate an energy $E=\iiint \frac{1}{2}\mathrm{Tr}F_{\mu\nu}F^{\mu\nu}d^3x$ using the *outer product trace* $E=\iiint \frac{1}{2}F_{\mu\nu AA}F^{\mu\nu}{}_{BB}d^3x$ for each of the so-represented proton and neutron. It turns out that these respective energies, showing both the matrix form and the scalar expression after the outer product trace is taken, see (12.4) and (12.5) of [1], are:

$$E_P=\frac{1}{(2\pi)^{\frac{3}{2}}}\mathrm{Tr}\begin{pmatrix}\sqrt{m_d}&0&0\\0&\sqrt{m_u}&0\\0&0&\sqrt{m_u}\end{pmatrix}\otimes\begin{pmatrix}\sqrt{m_d}&0&0\\0&\sqrt{m_u}&0\\0&0&\sqrt{m_u}\end{pmatrix}=\frac{m_d+4\sqrt{m_u m_d}+4m_u}{(2\pi)^{\frac{3}{2}}}\,,\quad(5.1)$$
$$\equiv(2\pi)^{-\frac{3}{2}}\mathrm{Tr}K_P\otimes K_P=(2\pi)^{-\frac{3}{2}}K_{P\,AA}K_{P\,BB}$$
$$E_N=\frac{1}{(2\pi)^{\frac{3}{2}}}\mathrm{Tr}\begin{pmatrix}\sqrt{m_u}&0&0\\0&\sqrt{m_d}&0\\0&0&\sqrt{m_d}\end{pmatrix}\otimes\begin{pmatrix}\sqrt{m_u}&0&0\\0&\sqrt{m_d}&0\\0&0&\sqrt{m_d}\end{pmatrix}=\frac{m_u+4\sqrt{m_u m_d}+4m_d}{(2\pi)^{\frac{3}{2}}}\,.\quad(5.2)$$
$$\equiv(2\pi)^{-\frac{3}{2}}\mathrm{Tr}K_N\otimes K_N=(2\pi)^{-\frac{3}{2}}K_{N\,AA}K_{N\,BB}$$

In the final lines of each of the above, we denote the matrix appearing twice in (5.1) as $K_{P\,AB}$ and in (5.2) as $K_{N\,AB}$. We also point out that as elaborated in sections 2 through 4 of [6] that these matrices K can be used to restate the Koide mass relationships [14], which is why we choose the symbol "K" for these. We further point out as elaborated in the rest of [6] that by supplementing the energy square roots $\sqrt{m_u}$ and $\sqrt{m_d}$ with $\sqrt{v_F}$ where v_F=246.219651 GeV is the Fermi vev, one can make extended use of these "Koide matrices" to explain the proton and neutron masses themselves.

If we then take the *difference* E_N-E_P between (5.2) and (5.1), the expression we get is

$$E_N-E_P=\frac{3}{(2\pi)^{\frac{3}{2}}}(m_d-m_u)\equiv m_e\,,\quad(5.3)$$

where we *define* (really, hypothesize) this to be equal to the electron rest mass. It will be seen that this is just another way of writing (3.1). So this is how (3.1) comes about. Why do we make this hypothesis?

Originally in [1], the author found (5.1) and (5.2), then calculated E_N-E_P using the PDG data $m_u=2.3^{+0.7}_{-0.5}\,\mathrm{MeV}$ and $m_d=4.8^{+0.5}_{-0.3}\,\mathrm{MeV}$, and found that $E_N-E_P=.476^{+.228}_{-.190}\,\mathrm{MeV}$, which nicely encompasses the electron rest mass $m_e=.511\,\mathrm{MeV}$ pretty much near the center of the error bar. This was the first plausible point of contact that was made from the theory that baryons are the chromo-magnetic monopoles of Yang-Mills gauge theory to empirical data, particularly because a neutron decaying into a proton via $n \to p^+ + e^- + \overline{\nu} + \text{Energy}$ which can be recast as $n - p^+ = e^- + \ldots$, and a down quark decaying into an up quark via $d \to u + e^- + \overline{\nu} + \text{Energy}$ which recasts as $d - u = e^- + \ldots$ would, at least at a "linear" or "lowest order" level, support a relationship of the form $E_N - E_P \propto m_d - m_u \propto m_e$ in (5.3). So given both this empirical concurrence and the $n - p^+ = e^- + \ldots$ and $d - u = e^- + \ldots$ decay sensibilities, (5.3) was elevated into a *hypothesized* relationship with the electron rest mass, to be confirmed or contradicted based on additional empirical data. Subsequent theoretical development in section 9 of [10] demonstrated that (5.1) through (5.3) are in fact all relationships taken in the zero-order limit of Yang-Mills gauge theory. And subsequent empirical development which will be summarized momentarily appears to validate rather than refute (5.3), and to show that this zero-order limit appears to govern what is observed in nuclear binding and fusion events and the nuclear mass defects.

Now, we turn to the origins of (3.2), and for this, we must begin to discuss nuclear binding energies. While (5.3) was the first plausible point of contact between theory and experiment uncovered by the author, it was (5.1) and (5.2) themselves which opened up fertile new vistas via some extremely compelling connections to nuclear binding energies. Let us explain how this is developed.

If (5.1) and (5.2) represent some to-be-determined form of energy associated with the proton and neutron, then it is certainly a good idea to calculate these energies. We may do so using $m_u = 2.3^{+0.7}_{-0.5}\,\text{MeV}$ and $m_d = 4.8^{+0.5}_{-0.3}\,\text{MeV}$ from PDG which is what the author first did in [12.4] and [12.5] of [1]. But rather than retread this same ground, let us use the much-more-precise masses (3.3) and (3.4) which are be the correct quark masses *if (3.1) and (3.2) are valid relationships*, which is what we are testing out at present. So, if we use (3.3) and (3.4) in each of (5.1) and (5.2), and then also apply 1 u = 931.494 061(21) MeV, we calculate to ten significant digits in AMU and seven significant digits in less-precise MeV [4] that:

$$E_P = \frac{m_d + 4\sqrt{m_u m_d} + 4m_u}{(2\pi)^{\frac{3}{2}}} = 0.001\,837\,399\,7\ \text{u} = 1.711\,526\,9\ \text{MeV}\,, \tag{5.4}$$

$$E_N = \frac{m_u + 4\sqrt{m_u m_d} + 4m_d}{(2\pi)^{\frac{3}{2}}} = 0.002\,387\,693\,9\ \text{u} = 2.224\,122\,7\ \text{MeV}\,. \tag{5.5}$$

Now at first sight, these energies are a bit mysterious. After all, M_N = 939.565379 MeV and M_P = 938.272046 MeV, so these energies are certainly not the proton and neutron masses themselves. But we know that the proton and neutron contain three quarks each, that the current masses of the quarks contribute only slightly to the overall proton and neutron masses, and that the rest of the mass is generated through extensive interactions involving quarks and gluons. So let us strip out all of these interactions and focus solely on the current quark masses, which when properly summed together, should represent something of a "zero order" value for the proton and neutron masses. Continuing to use the masses (3.3) and (3.4), the sums Σ of these current quark masses, for the duu proton and udd neutron respectively, are:

$$\Sigma_P = 2m_u + m_d = \mathrm{Tr}\begin{pmatrix} \sqrt{m_d} & 0 & 0 \\ 0 & \sqrt{m_u} & 0 \\ 0 & 0 & \sqrt{m_u} \end{pmatrix} \cdot \begin{pmatrix} \sqrt{m_d} & 0 & 0 \\ 0 & \sqrt{m_u} & 0 \\ 0 & 0 & \sqrt{m_u} \end{pmatrix} = \mathrm{Tr}K_P \cdot K_P = K_{P\,AB}K_{P\,BA} \quad , \tag{5.6}$$
$$= 0.010\,023\,9911\,\mathrm{u} = 9.337\,288\,2\ \mathrm{MeV}$$

$$\Sigma_N = 2m_d + m_u = \mathrm{Tr}\begin{pmatrix} \sqrt{m_u} & 0 & 0 \\ 0 & \sqrt{m_d} & 0 \\ 0 & 0 & \sqrt{m_d} \end{pmatrix} \cdot \begin{pmatrix} \sqrt{m_u} & 0 & 0 \\ 0 & \sqrt{m_d} & 0 \\ 0 & 0 & \sqrt{m_d} \end{pmatrix} = \mathrm{Tr}K_N \cdot K_N = K_{N\,AB}K_{N\,BA} \quad . \tag{5.7}$$
$$= 0.0129129643\,\mathrm{u} = 12.0283496\ \mathrm{MeV}$$

We note that these sums $\Sigma_P = 2m_u + m_d = \mathrm{Tr}K_P \cdot K_P$ and $\Sigma_N = 2m_d + m_u = \mathrm{Tr}K_N \cdot K_N$ employ the *inner product trace* of the same Koide matrices for which the outer product trace was taken in (5.1) and (5.2).

These energy numbers deepen the mystery further, because one would expect the predicted energies (5.4) and (5.5) to at least be as much as the masses (5.6) and (5.7), and yet, they are substantially less. That is, some of the mass we expect to see in (5.6) and (5.7) is "missing" from (5.4) and (5.5), in very much the same way that some of the mass one might expect to see by combining two nuclides if we naively add their separate masses together, goes missing in the mass defect and is released as fission energy. So now the question becomes, how much mass has gone missing in (5.5)? We can easily calculate this missing mass difference $\Delta = \Sigma - E$ for each of the proton and neutron by subtracting (5.4) from (5.6) and (5.5) from (5.7) as was first done using the PDG data in [12.6] and [12.7] of [1], to obtain:

$$\Delta_P = \Sigma_P - E_P = 2m_u + m_d - \frac{m_d + 4\sqrt{m_u m_d} + 4m_u}{(2\pi)^{\frac{3}{2}}} = 0.008\,186\,591\,4\,\mathrm{u} = 7.625\,761\,3\ \mathrm{MeV} \quad , \tag{5.8}$$
$$= \mathrm{Tr}K_P \cdot K_P - (2\pi)^{-\frac{3}{2}}\,\mathrm{Tr}K_P \otimes K_P$$

$$\Delta_N = \Sigma_N - E_N = 2m_d + m_u - \frac{m_u + 4\sqrt{m_u m_d} + 4m_d}{(2\pi)^{\frac{3}{2}}} = 0.010\,525\,270\,4\,\mathrm{u} = 9.804\,226\,8\ \mathrm{MeV} \quad . \tag{5.9}$$
$$= \mathrm{Tr}K_N \cdot K_N - (2\pi)^{-\frac{3}{2}}\,\mathrm{Tr}K_N \otimes K_N$$

We see that these missing masses Δ combine *both inner and outer product traces* of the 3x3 Koide matrices in (5.1), (5.2), (5.6) and (5.7).

We may then easily calculate that the average of these two missing masses $\frac{1}{2}(\Delta_P + \Delta_N) = 8.714\,994\,1\ \mathrm{MeV}$, and this number starts to reveal some very deep meaning. For, if we refer to the well-known empirical curve for the binding energy per nucleon which is reproduced below as Figure 2, and we keep in mind that most nuclides have roughly the same number of protons as neutrons but with larger proportion of neutrons over protons as the nuclides get heavier, we see that this number also is very close to the peak per-nucleon energy at about 8.75 MeV per nucleon. In particular, we know that the heaviest nuclides do give up approximately 8.75 MeV per nucleon in order to bind together, which very closely tracks the missing mass $\frac{1}{2}\left(\Delta_P + \Delta_N\right) = 8.714\,994\,1\,\mathrm{MeV}$.

It is this observation, first reported in section 12 of [1], which caused the author to initially suspect that these missing masses are very closely related to nuclear binding. And to be clear, the author had no *a priori* suspicion that these missing masses might be related to nuclear binding. Had the result of the foregoing calculation been $\frac{1}{2}\left(\Delta_P + \Delta_N\right) = 20\,\mathrm{MeV}$, or $\frac{1}{2}\left(\Delta_P + \Delta_N\right) = 3\,\mathrm{MeV}$, or some other number, then this would not have implicated nuclear binding and mass defect as the source of this missing mass. It is *only because* the missing mass was theoretically predicted to be $\frac{1}{2}\left(\Delta_P + \Delta_N\right) = 8.714\,994\,1\,\mathrm{MeV}$ and this is so close to the peak of the nuclear binding curve, that these missing masses were first suspected to be related to the mass defect. So here, the matching of a theoretical prediction to empirical data gave birth to a new theoretical understanding that was unanticipated at the outset.

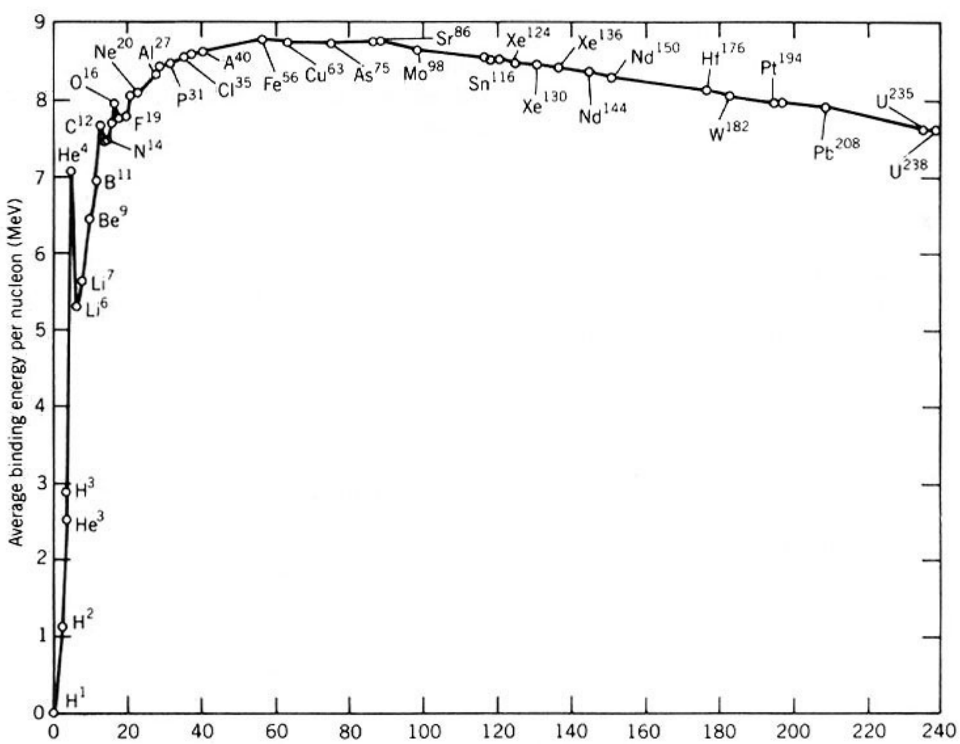

Figure 2: Empirical Binding Energy per Nucleon

Once this connection is discerned, it becomes interesting to actually use (5.8) and (5.9) to examine the binding energies of nuclides right near the peak of Figure 1. The two best examples are ^{56}Fe and ^{62}Ni which have two of the highest per-nucleon binding energies of all the nuclides in nature. The former has 26 protons plus 30 neutrons, with a median empirical binding energy of 492.253892 MeV [15], and the latter has 28 protons and 34 neutrons with an empirical binding energy of 545.2590 MeV (calculated from [16]). So if we use (5.8) and (5.9) to ascertain how much mass is "missing" from each of these nuclides, we find that:

$$\Delta\left({}^{56}\mathrm{Fe}\right) = 26\Delta_P + 30\Delta_N = 492.396\,598\,5\ \mathrm{MeV} \quad \text{versus} \quad 492.253\,892\ \mathrm{MeV\ observed}\,, \tag{5.10}$$

$$\Delta\left({}^{62}\mathrm{Ni}\right) = 28\Delta_P + 34\Delta_N = 546.865\,028\,4\ \mathrm{MeV} \quad \text{versus} \quad 545.259\,0\ \mathrm{MeV\ observed}\,. \tag{5.11}$$

So for ^{56}Fe the observed binding energy is 99.9710% of the theoretical missing mass $\Delta\left({}^{56}\mathrm{Fe}\right)$, and for ^{62}Ni this same percentage is 99.7063%. And if one does a similar calculation for all of the other nuclides near ^{56}Fe and ^{62}Ni it turns out – importantly – that *no nuclide* reaches or exceeds 100%, and that the very highest percentage is the one just shown for ^{56}Fe. This means that (5.8) and (5.9) – in some manner that needs to be understood – are establishing the upper limit that we see on the nuclear binding curve in Figure 1. And clearly, the results in (5.10) and (5.11) validate that (5.8) and (5.9) are revealing something very real and very important about nuclear binding, which gives further credence to the validity of the relationships (5.1) and (5.2) and thus the primary mass relationship (3.1) a.k.a. (5.3) with which they are integrally interconnected..

From here, we shall avoid repetition and instead refer the reader to the primary reference [2] in which the author first deciphers and explores the meaning of these results in detail. But the most important highlights which do need to be conveyed in the context of the present paper, specifically to explain the origins of the primary mass relationship (3.2) presently under consideration, are the following:

1) Nuclear Binding and Quark Confinement: The energies (5.8) and (5.9), in physical reality, are "latent binding energies" of the proton and neutron respectively. When a proton or a neutron (nucleon) is *free*, i.e., not bound to any other nucleon, then the entirety of this latent binding energy is used to confine quarks within the nucleon. But when a proton or neutron is *fused and bound* into a nucleus with at least one other nuclide, some, but never all (which is why the numbers above are always less than 100%) of the latent binding energy in (5.8) / (5.9) is released as fusion energy, the mass of the fused nucleus as a whole becomes less than the sum of the masses of all its separate nucleons which underlies the mass defect, and this lost mass / energy goes into the binding energy fusing together the nucleus, all in a sort of energetic nuclear "see saw" between confinement and binding. For ^{56}Fe which at 99.9710% channels a higher percentage of its latent binding energies than any other nuclide into *actual nuclear binding*, there is still a small 0.0290% share of its latent binding energy amounting to 0.142706 MeV (less than 1/3 the mass of a single electron) which does not get released and thereby going into nuclear binding, but remains behind to continue confining all of the quarks within the ^{56}Fe nuclides. Because *no nuclide* ever uses up more than 100% of its latent binding energies for actual binding, but always reserves at least some energy for confinement, quarks are always confined. Quarks inside the nucleons of ^{56}Fe are less-tightly confined than the quarks inside any other nuclide (which is a basis for understanding the "first EMC effect" [17]), but they do assuredly remain confined. The peak in Figure 2 at ^{56}Fe at which sits at 99.9710% of what it would take to

de-confine quarks, is one very direct way in which nature displays confinement. Indeed, the fact that the observed binding energies in (5.10) and (5.11) and any other nuclides are *always* less than the total latent binding energies reveals the energetic explanation for why *quarks always remain confined*.

2) Observed and Latent Nuclear Binding Energies: In general, for a nuclide with Z protons and N neutrons hence $A = Z + N$ nucleons, the latent binding energy which we denote by ${}^{A}_{Z}B$ is calculated from (5.8) and (5.9) using:

$$ {}^{A}_{Z}B = Z \cdot \Delta_P + N \cdot \Delta_N \,. \tag{5.12} $$

So for example, (5.10) and (5.11) may be represented as specific application of this formula for ${}^{56}_{26}B = \Delta\left({}^{56}\mathrm{Fe}\right)$ and ${}^{62}_{28}B = \Delta\left({}^{62}\mathrm{Ni}\right)$. And the percentage ratios discussed earlier are ${}^{56}_{26}B_0 \,/\, {}^{56}_{26}B = 99.9710\%$ and ${}^{62}_{28}B_0 \,/\, {}^{62}_{28}B = 99.7063\%$. These latent binding energies ${}^{A}_{Z}B$ thereby establish *upper limits* for actual, observed binding energies which we denote generally as ${}^{A}_{Z}B_0$ with the 0 subscript. But as ^{56}Fe demonstrates, these limits are never reached or exceeded, that is, ${}^{A}_{Z}B_0 < {}^{A}_{Z}B$, or alternatively, ${}^{A}_{Z}B_0 \,/\, {}^{A}_{Z}B < 100\%$, *always*. So this now leads us to ask how it is that we can explain the specific *observed* binding energies ${}^{A}_{Z}B_0$ for *all* the nuclides. This is especially of interest for the lightest nuclides which have the lowest ${}^{A}_{Z}B_0 \,/\, {}^{A}_{Z}B$ ratios, and for which the observed binding energies to date have not yet been satisfactorily explained. So, what do we now know to help us figure this out?

3) The Binding and Fusion Energy "Toolkit": We know that the latent binding energies ${}^{A}_{Z}B = Z \cdot \Delta_P + N \cdot \Delta_N$ employ linear combinations of (5.8) and (5.9), and these in turn involve inner and outer product traces of the matrices (5.1), (5.2), (5.6) and (5.7). The elements of these matrix products in turn are very limited to only the energy numbers m_u, m_d, $\sqrt{m_u m_d}$, the foregoing divided by $(2\pi)^{\frac{3}{2}}$, and *integer multiples* of all these. We make the conservative and very stringent assumption that *every single observed nuclear binding energy* ${}^{A}_{Z}B_0$ must be constructed out of some combination of the foregoing energy number "toolkit" and "structurally sensible" integer multiples thereof, which means that the observed ${}^{A}_{Z}B_0$ must *all* be functions of the $Q = 0$ up and down quark masses (3.3) and (3.4). This is stringent because it gives us no room to adjust anything. If we cannot construct the observed binding energies from these energy numbers with some fairly high degree of precision, which means as functions of the up and down quark masses and nothing more, then this approach is contradicted. But if we can construct a fair number of observed binding energies in this way, then that would lend solid empirical support to this approach. We know that the latent binding energies ${}^{A}_{Z}B = Z \cdot \Delta_P + N \cdot \Delta_N$ comes readily packaged, so for any given nuclide, we should consider both adding to and subtracting from a pertinent ${}^{A}_{Z}B$, i.e., we should ask how much its binding energy either exceeds or falls below some ${}^{A}_{Z}B$. We should also sensibly include in our "toolkit" scalar traces of the Koide matrices,

namely, $\mathrm{Tr}K_P = \sqrt{m_d} + 2\sqrt{m_u}$ and $\mathrm{Tr}K_N = \sqrt{m_u} + 2\sqrt{m_d}$ multiplied by $\sqrt{m_u}$ or $\sqrt{m_d}$. Finally, to extend this approach, we should consider matching the energy numbers not only to binding energies, but also to the energies released during various fusion or fission and other decay reactions. From here, with toolkit assembled, the task of characterizing individual observed binding energies ${}^A_Z B_0$ involves elbow grease, a good spreadsheet or computer program, and educated trial and error. In this venture, one is using empirical data in combination with the foregoing toolkit to try to discern systematic but hidden theoretical patterns in the nuclear binding energies – in broad scope, seeking to "decode" the nuclear "genome."

4) <u>Hydrogen-2</u>: The easiest place to start is with the ^{2}H deuteron, consisting of one proton and one neutron. In AMU, the observed binding energy is ${}^2_1 B_0 = 0.002\,388\,170\,100\ \mathrm{u}$. We then refer to our energy number "toolkit" m_u, m_d, $\sqrt{m_u m_d}$, the foregoing divided by $(2\pi)^{\frac{3}{2}}$, and integer multiples of these. But we need not search very far. From (3.3) the mass of the up quark is $m_u = 0.002\ 387\ 339\ 3\ \mathrm{u}$. The difference is ${}^2_1 B_0 - m_u = 8.308 \times 10^{-7}\ \mathrm{u}$, which is to say, the accuracy is to better *eight parts per ten million AMU*. It should be pointed out that in [1] the author originally *hypothesized* that the deuteron binding energy is *exactly the same* as the up quark masse due to how close they in fact appeared to be. That is, the author originally employed ${}^2_1 B_0 = m_u$ rather than (3.2) as a primary mass relationship in combination with (3.1). Then, on this basis, over the course of the development in sections 1 through 9 of [2] the author was able for the first time to derive the primary mass relationship (3.2) with eight parts per ten million AMU accuracy. Once this (3.2) had been derived, for the reasons elaborated at length in section 10 of [2], the author shifted hypotheses and advanced (3.2) to a primary, exact mass relationship while withdrawing ${}^2_1 B_0 = m_u$, so that the sub-parts-per-million accuracy error was shifted from (3.2) to ${}^2_1 B_0$. It must also be pointed out that this error is *outside* of experimental error margins because ${}^2_1 B_0$ is known with greater than ten-digit accuracy, and so it still warrants understanding as will be discussed later in this paper.

5) <u>Helium-3 and Helium-4</u>: From there we attempt to explain some other light nuclide binding energies in like fashion based on the foregoing toolkit, particularly hydrogen and helium isotopes. For the highly stable alpha particle – the ^{4}He nucleus – it was found through trial and error that the observed binding energy ${}^4_2 B_0 = 0.030\,376\,586\,5\ \mathrm{u}$ is less than the latent binding energy ${}^4_2 B = 2 \cdot \Delta_P + 2 \cdot \Delta_N = 0.037\ 465\ 212\ 2\ \mathrm{u}$ by approximately $2\sqrt{m_u m_d}$. So we then calculate $2 \cdot \Delta_P + 2 \cdot \Delta_N - 2\sqrt{m_u m_d} = 0.030\,373\,002\,0\ \mathrm{u} \approx {}^4_2 B_0$, to find that this differs from the observed alpha binding energy by under *four parts per million AMU*. The integer factor 2 used with $\sqrt{m_u m_d}$ is "structurally sensible" because the alpha particle has 2 protons and 2 neutrons, i.e., 2 neutron / proton pairs. And this overall expression for ${}^4_2 B$ is structurally sensible because just like the alpha particle itself, it is completely symmetric under both $P \leftrightarrow N$ and $u \leftrightarrow d$ interchange. This is first developed in detail in section 5 of [2] and the numerical results are recalibrated in section 10 of [2] after (3.2) is used to replace ${}^2_1 B_0 = m_u$ as a primary mass relationship.

For the ^{3}He nucleus (helion) with observed binding energy ${}_{2}^{3}B_0 = 0.008\,285\,602\,8\text{ u}$ we calculate $\sqrt{m_u}\,\text{Tr}K_P = 2m_u + \sqrt{m_u m_d} = 0.008\,320\,783\,9 \approx {}_{2}^{3}B_0$ by employing the trace of the Koide proton matrix $\text{Tr}K_P = \sqrt{m_d} + 2\sqrt{m_u}$ from our toolkit. Having $\sqrt{m_d} + 2\sqrt{m_u}$ involved here is "structurally sensible" because ^{3}He has one neutron (one extra down quark) and two protons (two extra up quarks). This differs from the empirical data by *under four parts per hundred thousand AMU* after recalibration in section 10 of [2], and was first developed in detail in section 6 of [2].

6) <u>Hydrogen-3 and the Neutron minus Proton Mass Difference</u>: It was in the course of attempting to obtain a binding energy for the ^{3}H triton that the author finally discovered the mass relationship (3.2) which was then advanced to a *primary* exact relationship in section 10 of [2]. While ${}_{1}^{2}B_0 \approx m_u$, ${}_{2}^{4}B_0 \approx 2\cdot\Delta_P + 2\cdot\Delta_N - 2\sqrt{m_u m_d}$ and ${}_{2}^{3}B_0 \approx 2m_u + \sqrt{m_u m_d}$ for ^{2}H, ^{4}He and ^{3}He respectively could be ferretted out relatively straightforwardly using binding energies, latent binding energies (5.12), and the toolkit from point 3, finding ${}_{1}^{3}B_0$ for ^{3}H proved to me impossible working with binding energies alone. So at that point in time, as detailed in the appendix of [2], we begin to consider certain nuclear fusion reactions to see if the energies released in these reactions might provide a close empirical connection to the point 3 toolkit. And we also begin to make use of the general mass defect relationship

$$ {}_{Z}^{A}B_0 = Z\cdot M_P + N\cdot M_N - {}_{Z}^{A}M_0 \tag{5.13} $$

through which one can related the observed binding energy ${}_{Z}^{A}B_0$ to the observed nuclear mass (weight) ${}_{Z}^{A}M_0$ for any nuclide with Z protons, N neutrons and $A = Z + M$ nucleons. (Note: $M_P = {}_{1}^{1}M$ and $M_N = {}_{0}^{1}M$.)

First, we consider the fusion ${}_{1}^{1}H + {}_{1}^{2}H \to {}_{1}^{3}H + e^+ + \nu + \text{Energy}$ of a proton and a deuteron into a triton and ask: how much energy is released? Empirically, this energy is observed to $\text{Energy} = {}_{1}^{1}M + {}_{1}^{2}M - {}_{1}^{3}M - m_e = 0.004\,780\,386\,2\text{ u}$. Dipping into the toolkit, we find a close connection using $2m_u = 0.004\,774\,678\,6\text{ u}$ which differs from the observed fusion energy by 5.7076×10^{-6} u, i.e., just under *six parts per million AMU*. And the factor of 2 makes some structural sense because we are fusing two nuclides. So we make the close association $\text{Energy}\left({}_{1}^{1}H + {}_{1}^{2}H \to {}_{1}^{3}H + ...\right) \approx 2m_u$. After some calculations using (5.13) and leading to [A9] in [2] we obtain the expression ${}_{1}^{3}B_0 \approx M_N - M_P + 3m_u + m_e$ for the ^{3}H binding energy, which requires us to find the neutron minus proton mass difference $M_N - M_P$ which is the primary relationship (3.2).

To do this, we do a second fusion study, this time of fusion ${}_{1}^{1}H + {}_{1}^{1}H \to {}_{1}^{2}H + e^+ + \nu + \text{Energy}$ of two protons into a deuteron, and again ask: how much energy is released? The observed empirical energy is $\text{Energy} = 2M_P - {}_{1}^{2}M - m_e = 0.000\,451\,141\,0\text{ u}$.

We again return to trial and error with the toolkit, this time dipping into the $(2\pi)^{\frac{3}{2}}$ divisor to find that $2\sqrt{m_\mu m_d} / (2\pi)^{\frac{3}{2}} = 0.000\,450\,424\,1\,\mathrm{u}$. This differs from the empirical fusion energy by 7.169×10^{-7} u, and so has an accuracy of *better than one part per million AMU!* So we make the close association $\text{Energy}\left({}^1_1H + {}^1_1H \rightarrow {}^2_1H + ...\right) \approx 2\sqrt{m_\mu m_d} / (2\pi)^{\frac{3}{2}}$. Thereafter, we arrive in [A15] of [2] at $M_N - M_P = m_u - m_e - 2\sqrt{m_\mu m_d} / (2\pi)^{\frac{3}{2}} = m_u - \left(3m_d + 2\sqrt{m_\mu m_d} - 3m_u\right) / (2\pi)^{\frac{3}{2}}$, which is the primary mass relationship (3.2). With this, we have completed the explanation of how this relationship (3.2) is obtained.

Of course, when (3.2) was first obtained in [A15] of [2], this was as an intermediate step that was necessitated to reduce ${}^3_1B_0 \approx M_N - M_P + 3m_u + m_e$ to obtain the binding energy for the ^{3}H triton, which has the empirical value ${}^3_1B_0 = 0.009\,105\,585\,4\,\mathrm{u}$. So we then completed the calculations in the appendix of [2] using all of these results to arrive in [A17] at the approximate expression $4m_u - 2\sqrt{m_\mu m_d} / (2\pi)^{\frac{3}{2}} = 0.009\,099\,047\,1\,\mathrm{u} \approx {}^3_1B_0$ for the triton bending energy, which differs from the observed value by 6.5383×10^{-6} u, *just under seven parts per million AMU.*

7) Recalibration of Mass Relationships: As just discussed, the primary mass relationship (3.2) was first uncovered as a byproduct in the course of pursuing the triton binding energy. But based on the initial hypothesis in place at the time that ${}^2_1B_0 = m_u$, this relationship (3.2) itself predicted a neutron minus proton mass difference which was off by a few parts per ten million AMU. Then, for the reasons detailed in section 10 of [2] the author withdrew ${}^2_1B_0 = m_u$ as a primary relationship and instead hypothesized (3.2) to be a primary, exact relationship among the electron, proton and neutron masses. It is with this hypothesis that (3.2) joined (3.1) as a "primary mass relationship" then then was used in accordance with the EPN-0 quark mass definition to deduce very precise quark masses (3.3) and (3.4) which have been used in the development here ever since, and all mass relationships previously developed were recalibrated to reflect this revised hypothesis.

6. Is there Clear Secondary Empirical Support for the Deduced *Q* = 0 Up and Down Current Quark Masses?

Having shown how the primary mass relationships (3.1) and (3.2) are obtained, we now return to the second of the three questions posed in section 3, namely, whether these primary mass relationships (3.1) and (3.2) and the very precise $Q = 0$ up and down current quark masses (3.3) and (3.4) deduced therefrom can be supported by other "secondary relationships" rooted in nuclear data, or whether there are contradictions to be found.

When discussing in general whether a theory is "valid" or has "support," one must keep in mind that for scientific work, one can never truly "validate" a theory. One can simply show that at multiple places where the theory might be open to contradiction, no contradiction is found. This takes place at two levels: the empirical level, and the theoretical level.

At the empirical level, the question is whether efforts to make contact with empirical data are contradicted or not contradicted: do the experiments rule out the theory, or do they fail to rule out the theory. If a sufficient number of efforts are made to contradict and no contradictions are in fact found with the experimental data, then the weight of those "failures to contradict" start to translate into "empirical support" for the theory. But there is no objective, scientific measurement as to when there are enough failures to contradict so as to constitute theoretical validation. That is a subjective judgment which must first be made by individual scientists and then, eventually, by the scientific community as a whole.

At the theoretical level, the question is whether a proposed theory is consistent with, i.e., not contradictory to, other settled theories and theoretical elements which have advanced to the point of having gained wide acceptance in the scientific community based on multiple failures to contradict those settled theories. There are other corollary questions related to this: is the theory economical, which in a conservative view of science might be reframed as whether the theory requires brand new notions to be injected into the theoretical discourse of the community, or whether the theory can be rested solely on novel combinations of well-established and well-settled theories and theoretical elements to uniquely and unambiguously deduce new results and new explanations for previously-unexplained observational data. From a conservative scientific stance, the latter (combination of settled science) is preferable, and the former (brand new notions) is not ruled out but should be used as a last resort when there is no apparent way to succeed by restricting oneself to combining known elements in unknown ways.

In this section, we shall discuss empirical support, which is the second of the three questions posed in section 3. In the next section we shall discuss theoretical support, which is the third and final of the three questions posed in section 3

To a very large degree, section 5 has already developed very substantial empirical support that the validity (3.3) and (3.4) are correct quark masses, and therefore (3.1) and (3.2) are correct relationships. Now, we shall review this empirical support, and introduce additional empirical support.

Thus far, we started out by hypothesizing (3.1) and (3.2) to be valid, exact, Q-invariant relationships, and thereby hypothesizing (3.3) and (3.4) to be valid, very precise up and down $Q=0$ quark masses. Based on this, we have thus far been able to deduce the following non-contradictory, supporting empirical results:

1) <u>Hyrdrogen-2 and -3, Helium-3 and -4 Binding Energies</u>: Secondary relationships for the ^{2}H, ^{3}H, ^{3}He and ^{4}He (1s shell) nuclide binding energies strictly terms of m_u and m_d with very close matches to parts per 10^5, 10^6 or even 10^7 AMU. Respectively, these secondary relationships are: ${}^2_1B_0 \approx m_u$ (section 5, point 4); ${}^3_1B_0 \approx 4m_u - 2\sqrt{m_\mu m_d} / \left(2\pi\right)^{\frac{3}{2}}$ (section 5, point 6); ${}^3_2B_0 \approx 2m_u + \sqrt{m_u m_d}$); (section 5, point 6); and in view of (5.8) and (5.9), ${}^4_2B_0 \approx 2\cdot\Delta_P + 2\cdot\Delta_N - 2\sqrt{m_u m_d}$ (section 5, point 6).

2) <u>Deuteron and Triton Fusion Energies</u>: Interrelated to the point 1 secondary relationships and the primary relationship (3.2), the secondary relationships $\text{Energy}\left({}^1_1H+{}^2_1H\to{}^3_1H+...\right)\approx 2m_u$ for the fusion energy released when a fusing proton and a deuteron into a triton, and $\text{Energy}\left({}^1_1H+{}^1_1H\to{}^2_1H+...\right)\approx 2\sqrt{m_\mu m_d}\,/\left(2\pi\right)^{\frac{3}{2}}$ for the fusion energy released when fusing two protons into a deuteron (section 5, point 6).

3) <u>The Nuclear Binding Peak near 8.75 MeV</u>: The relationships (5.8) and (5.9) Δ_P and Δ_N which represent "missing mass," and which have a value of $\frac{1}{2}\left(\Delta_P+\Delta_N\right)=8.714\,994\,1\,\text{MeV}$ which is right at the peak of the empirical nuclear binding curve in Figure 2.

4) <u>Iron-56 and other Tightly-Bound Nuclides</u>: Based on (5.8) and (5.9), the relationship $\Delta\left({}^{56}\text{Fe}\right)=26\Delta_P+30\Delta_N=492.396\,598\,5\,\text{MeV}$ in (5.10) which is *extremely close* to the empirical ${}^{56}_{26}B_0=492.253\,892\text{ MeV}$, such that ${}^{56}_{26}B_0\,/\,{}^{56}_{26}B=99.9710\%$. This, and other relationships such as (5.11) which are deduced via (5.12), provide the basis for recognizing that Δ_P and Δ_N are latent energies available to be used for binding, which confine quarks in free nucleons, but which are partially released as fusion energies for nuclear binding, in a percentage that varies for each type of nuclide but never exceeds 100% and is greatest for ^{56}Fe than for any other nuclide. And this enables us to understand quark confinement on an energetic basis and explain the first EMC effect [17] whereby quarks inside bound nuclei are observed to be less-combined than those in free nucleons.

All of the foregoing provide secondary empirical validation to the view that (3.1) and (3.2) are empirically-valid relationships, and that (3.3) and (3.4) are therefore empirically-valid quarks masses because they can be used to closely and correctly characterize a broad range of other empirical data. But there are further supporting empirical results as well:

5) <u>Solar Fusion</u>: By combining the ^{2}H, ^{3}H, ^{3}He and ^{4}He binding results in point 1 above with $\text{Energy}\left({}^1_1H+{}^2_1H\to{}^3_1H+...\right)\approx 2m_u$ and $\text{Energy}\left({}^1_1H+{}^1_1H\to{}^2_1H+...\right)\approx 2\sqrt{m_\mu m_d}\,/\left(2\pi\right)^{\frac{3}{2}}$ for the fusion events in point 2 above, it is possible as detailed in section 9 of [2] to accurately express the 26.73 MeV energy observed to be released during a single solar fusion event by the relationship [9.8] of [2]:

$$\begin{aligned}&\text{Energy}\left(4\cdot{}^1_1H+2e^-\to{}^4_2He+\gamma(12.79MeV)+2\gamma(5.52MeV)+2\gamma(.42MeV)+4\gamma(e)+2\nu\right)\\&=4m_u+6m_d-2\sqrt{m_u m_d}+\frac{2m_d-22m_u-12\sqrt{m_u m_d}}{\left(2\pi\right)^{\frac{3}{2}}}=26.73\,MeV\end{aligned} \quad . \qquad (6.1)$$

Like the other binding and fusion results, this is also expressed wholly and exclusively in terms of the same two parameters: the up quark mass (3.3) and the down quark mass (3.4).

6) Stable Neutron-Rich Nuclides: The fact that the latent binding energy of the neutron in (5.9) is greater than that of the proton in (5.8) by a factor of $\Delta_N / \Delta_P = 1.284\,295\,230\,4$ teaches that neutrons inherently carry 28.42% more latent binding energy than does a proton. This immediately explains the clear empirical evidence that for all nuclei heavier than helium, the stable isotopes *always* have either equal numbers of protons and neutrons, or are neutron-rich. If one has a given nucleus, and seeks to fuse on an extra proton or neutron, it is clear that the nucleon which brings in more energy available for nuclear binding will have an easier time becoming and staying bound.

7) Lithium-6 and -7 and Beryllium-7 and -8: Thus far we have only examined the ^{2}H, ^{3}H, ^{3}He and ^{4}He binding energies. But there is further support available from some heavier nuclides as well. To date, the author has characterized eleven additional nuclides ^{6}Li, ^{7}Li, ^{7}Be, ^{8}Be, ^{10}B, ^{9}Be, ^{10}Be, ^{11}B, ^{11}C, ^{12}C and ^{14}N with equally-high precision, exclusively as a function of the up and down quark masses. All of these derivations are detailed at length in [5], so we shall simply summarize them here.

The detailed derivations for ^{6}Li, ^{7}Li, ^{7}Be, ^{8}Be, which are 2s shell nuclides, are contained in section 13 of [5] and are exceptionally revealing in terms of the requirement that the integer multiples of the m_u, m_d, $\sqrt{m_u m_d}$ and these divided by $(2\pi)^{\frac{3}{2}}$ must be "structurally sensible." We have already applied this in points 5 and 6 of section 5 for the hydrogen and helium derivations, but when applied to Li and Be, this requirement provides deep support for the approach being laid out here.

The respective binding energies for ^{6}Li, ^{7}Li, ^{7}Be, ^{8}Be are found in [13.21] and [13.12] of [5] to be:

$$ {}^{6}_{3}B_0 \approx 7m_u + 6m_d - 2\sqrt{m_u m_d} + \left(-10m_u - 10m_d - 9\sqrt{m_u m_d}\right) \Big/ (2\pi)^{\frac{3}{2}} = 0.034\,336\,427\,2 \text{ u} . \quad (6.2) $$

$$ {}^{7}_{3}B_0 \approx 8m_u + 6m_d - 2\sqrt{m_u m_d} + \left(2m_u + 2m_d - 11\sqrt{m_u m_d}\right) \Big/ (2\pi)^{\frac{3}{2}} = 0.042\,105\,716\,0 \text{ u} . \quad (6.3) $$

$$ {}^{7}_{4}B_0 \approx 7m_u + 6m_d - 2\sqrt{m_u m_d} + \left(-10m_u + 8m_d - 9\sqrt{m_u m_d}\right) \Big/ (2\pi)^{\frac{3}{2}} = 0.040\,356\,362\,0 \text{ u} . \quad (6.4) $$

$$ {}^{8}_{4}B_0 \approx 4 \cdot \Delta E_P + 4 \cdot \Delta E_N - 2\sqrt{m_u m_d} - 32\sqrt{m_u m_d} \,/\, (2\pi)^{1.5} = 0.060\,633\,250\,9 \text{ u} . \quad (6.5) $$

The respective *empirical* values out to seven digits are ${}^{6}_{3}B_0 = 0.034\,347\,1\text{ u}$ (difference of -1.07×10^{-5} u); ${}^{7}_{3}B_0 = 0.042\,130\,3\text{ u}$ (difference of -2.45×10^{-5} u); ${}^{7}_{4}B_0 = 0.040\,365\,1\text{ u}$ (difference of -8.74×10^{-6} u), and ${}^{8}_{4}B_0 = 0.060\,654\,8\text{ u}$ (difference of -2.16×10^{-5} u). So as with H and He, these all have accuracy to parts in 10^5 or 10^6 u.

Now, while the existence of the coefficients 6, 7 and 8 multiplying the quark masses provides some "structural sensibility" for nuclides with 6, 7 or 8 nucleons, the deep and striking structural sense emerges from the fusion relationships which were used in section 13 of [5] to establish (6.2) through (6.5) above. Specifically, to arrive at (6.2) for ^{6}Li we considered the fusion reaction ${}^4_2He+2p\rightarrow {}^6_3Li+e^+ +\nu+\text{Energy}$ for which the empirical energy to seven digits is $0.002\,033\,5\text{ u}$, and after using the toolkit and "structurally-sensible" integer multiples, we found in [13.3] of [5] that:

$$\text{Energy}\left({}^4_2He+2p\rightarrow {}^6_3Li+e^+ +\nu+\text{Energy}\right)\approx 9\sqrt{m_u m_d}\,/\left(2\pi\right)^{1.5}=0.002\,026\,4\text{ u}, \tag{6.5}$$

which is a difference of $-7.1\times10^{-6}\text{ u}$, with the coefficient 9. And to arrive at (6.3) for ^{7}Li we developed the β^+ decay reaction ${}^7_4Be+e\rightarrow {}^7_3Li+\nu+\text{Energy}$ for which the empirical energy is $0.000\,925\,3\text{ u}$. Using the toolkit and "structurally-sensible" integer multiples, we found in [13.9] of [5] that:

$$\text{Energy}\left({}^7_4Be+e\rightarrow {}^7_3Li+\nu+\text{Energy}\right)\approx 6m_u\,/\left(2\pi\right)^{1.5}=0.000\,909\,5\text{ u}, \tag{6.6}$$

which differs by $-1.58\times10^{-5}\text{ u}$, with a coefficient of 6. And to arrive at (6.4) for ^{7}Be we worked with the reaction ${}^6_3Li+p\rightarrow {}^7_4Be+\text{Energy}$ which has an empirical energy of $0.006\,018\,0\text{ u}$. Here, we found in [13.6] of [5] that:

$$\text{Energy}\left({}^6_3Li+p\rightarrow {}^7_4Be+\text{Energy}\right)\approx 18m_d\,/\left(2\pi\right)^{1.5}=0.006\,019\,9\text{ u}, \tag{6.7}$$

differing by $1.9\times10^{-6}\text{ u}$, with a coefficient of 18. It is these three coefficients, 9, 6 and 18 which not only yield very close results to parts per 10^5 or 10^6, but provide structural sensibility as well.

When we build the ^{6}Li nucleus onto an alpha particle in (6.5), we are creating a nucleus with 9 up quarks and 9 down quarks, i.e., with 9 up / down quark pairs. And what is the toolkit number that gets us a to ^{6}Li? $9\sqrt{m_u m_d}\,/\left(2\pi\right)^{1.5}$! How better to formally state that there are 9 up / down quark pairs than with $9\sqrt{m_u m_d}\,/\left(2\pi\right)^{1.5}$, and to state that both the beginning and end-products ^{4}He and ^{6}Li are absolutely symmetric under $P\leftrightarrow N$ and $u\leftrightarrow d$ interchange. In (6.6) we have the isotopic β^+ decay from unstable proton-rich ^{7}Be to stable neutron-rich ^{7}Li for which the toolkit gives us $6m_u\,/\left(2\pi\right)^{1.5}$. (Keep in mind point 6 where we explained based on latent binding energies why nature favors extra neutrons over extra protons for anything heavier than He.) In this reaction a proton is being traded for a neutron, but the unchanging nucleus during thus reaction is the underlying stable ^{6}Li nucleus with is an isotope of ^{7}Li and an isotone of ^{7}Be. The invariant structural piece of the nucleus which does not change, is the underlying ^{6}Li with 6 nucleons. So what is the coefficient here? Why, it is 6! In (6.7) we are adding a proton to ^{6}Li, and the toolkit yields $18m_d\,/\left(2\pi\right)^{1.5}$. Why 18? The nucleus at the root of this fusion event is ^{6}Li, which contains 18 quarks! It is also interesting to observe that thee three of the main toolbox elements $\sqrt{m_u m_d}$, m_u and m_d are used in these decays $9\sqrt{m_u m_d}\,/\left(2\pi\right)^{1.5}$,

$6m_u/(2\pi)^{1.5}$ and $18m_d/(2\pi)^{1.5}$, that the ^{6}Li nucleus common to all three reactions appears to drive these coefficients.

All of this suggests that when any nuclear transition occurs and some energy is being released, there is definitive set of energy "dosages" which are released or otherwise used in the process, and which are allocated discretely to each of the quarks or quark pairs or nucleons, etc. So for ${}^4_2He+2p\to{}^6_3Li+...$ with $9\sqrt{m_u m_d}/(2\pi)^{1.5}$, each of the nine quark pairs gives up an single energy dose $\sqrt{m_u m_d}/(2\pi)^{1.5}$ to be able to establish the ^{6}Li with the start of a new shell overlaid on the alpha nucleus, that is, to "entice" an extra proton and neutron to join the alpha core. For ${}^7_4Be+e\to{}^7_3Li+...$ with $6m_u/(2\pi)^{1.5}$ each of the six nucleons – three protons and three neutrons – in the ^{6}Li core gives up a single energy dose $m_u/(2\pi)^{1.5}$ to the β^+ decay. And for ${}^6_3Li+p\to{}^7_4Be+...$ with $18m_d/(2\pi)^{1.5}$, every single quark in the ^{6}Li core needs to give up a single $m_d/(2\pi)^{1.5}$ energy dose to "entice" the proton into the core. This then tells us retrospectively to point 2, that to create a deuteron which is symmetric under $P\leftrightarrow N$ and $u\leftrightarrow d$ interchange, via $\text{Energy}\left(p+p\to{}^2_1H+...\right)\approx 2\sqrt{m_\mu m_d}/(2\pi)^{\frac{3}{2}}$ each proton has to contribute a $\sqrt{m_\mu m_d}/(2\pi)^{\frac{3}{2}}$ dose of energy which dose is similarly symmetric. And to create a triton via $\text{Energy}\left({}^1_1H+{}^2_1H\to{}^3_1H+...\right)\approx 2m_u$ each of the proton and the deuteron must contribute an energy does valued at m_u. This provides a deeper picture of what it means to say that the "toolbox" elements need to be used with coefficients which are "structurally sensible," and we come to understand that when we observe some fusion or fission energy released during some reaction, that this energy originates from a collection of "doses" of the toolbox energies in relations to the structural elements of the involved nuclei.

We also see that the method of fitting the toolkit to observed fusion or β decay energies is extremely important in building up larger nuclides. In section 13 of [5], we started with the ^{4}He nucleus and built that into ^{6}Li which is diagonally-adjacent upper left to lower right in the nuclide table, per (6.5). Then we added a proton as in (6.7) and built this into its isotone ^{7}Be. Then we diagonally beta-decayed this upper right to lower left into ^{7}Li as in (6.6). Once lighter nuclides are so-characterized, we have the ability to "weave" over from one nuclide to horizontally or vertically-adjacent nuclides by examining their decay energies, and then convert over to binding energies via (5.13).

Further, we see from the ^{4}He binding energy ${}^4_2B_0\approx 2\cdot\Delta_P+2\cdot\Delta_N-2\sqrt{m_u m_d}$ and from the ^{8}Be binding energy ${}^8_4B_0\approx 4\cdot\Delta E_P+4\cdot\Delta E_N-2\sqrt{m_u m_d}-32\sqrt{m_u m_d}/(2\pi)^{1.5}$ that the $Z=N=\text{even}$ nuclides appear to form something of a nuclear "backbone" which are $N\leftrightarrow P$ and $u\leftrightarrow d$ invariant, and that their binding energies are perhaps best uncovered by first using (5.12) ascertain their latent binding energies, then using the toolkit to see how much of this latent energy is retained for confinement, and throughout being guided by the $N\leftrightarrow P$ and $u\leftrightarrow d$ symmetry of these nuclides.

So the basic approach to "decoding the nuclear genome" is to establish the diagonal $Z = N = \text{even}$ backbones via the latent binding energy formula and determination of how much latent energy goes unused (5.12), then "weave" our way over to nearby nuclides while making use of the various emergent coefficients to provide clues as the nuclear substructure and which elements within the nucleus are emitting what energy dosages.

8) <u>Stability of Helium-4 over Beryllium-8</u>: By now having close fits for both ${}^{8}_{4}B_0$ and ${}^{4}_{2}B_0$ with the ratio ${}^{8}_{4}B_0 / {}^{4}_{2}B_0 = 1.996\,052\,2$ based on (6.5) and point 5 of section 5, we implicitly explain that why ^{8}Be is unstable and always decays rapidly into two ^{4}He nuclei. This is another important empirical feature of nuclear physics which now supports the approach here.

9) <u>Boron-10</u>: Further empirical validation is obtained through characterizing the ^{10}B, ^{9}Be, ^{10}Be, ^{11}B, ^{11}C, ^{12}C and ^{14}N nuclides as the authors has previously done in section 14 of [5]. We shall not repeat those derivations here because they are available at the original source [5]. But the patterns which stated to emerge for ^{6}Li, ^{7}Li, ^{7}Be, ^{8}Be do appear in some places for these even-heavier nuclides. An excellent example of this is the ${}^{8}_{4}Be + 2p \to {}^{10}_{5}B + e^{+} + \nu + \text{Energy}$ reaction, which is analogous to ${}^{4}_{2}He + 2p \to {}^{6}_{3}Li + e^{+} + \nu + \text{Energy}$ as summarized in (6.5). The empirically-released energy in this reaction is $0.006\,921\,0\ \text{u}$. And as found in [14.3] of [5], symmetric under $u \leftrightarrow d$ interchange as expected for any $Z = N$ nuclides, we obtain:

$$\text{Energy}\left({}^{8}_{4}Be + 2p \to {}^{10}_{5}B + e^{+} + \nu + \text{Energy}\right) = \sqrt{m_u m_d} + 15\sqrt{m_u m_d} / \left(2\pi\right)^{1.5} = 0.006\,923\,4\ \text{u}, \quad (6.8)$$

which differs from the empirical energy by $2.4\times10^{-6}\ \text{u}$. What is extremely striking is that the creation of ${}^{6}_{3}Li$ with 9 up / down quark pairs from ${}^{4}_{2}He$ contained a $9\sqrt{m_u m_d} / \left(2\pi\right)^{1.5}$ term shown in (6.5), and the creation of ${}^{10}_{5}B$ with 15 up / down quark pairs from ${}^{8}_{4}Be$ contains a exactly the same term, but now $15\sqrt{m_u m_d} / \left(2\pi\right)^{1.5}$. *This cannot be mere coincidence. This reveals a very definite and meaningful data pattern.* As with ${}^{4}_{2}He + 2p \to {}^{6}_{3}Li + ...$, each quark pair in the ${}^{8}_{4}Be + 2p \to {}^{10}_{5}B + ...$ contributes a single $\sqrt{m_u m_d} / \left(2\pi\right)^{1.5}$ energy dose, except now there are more quark pairs – 15 rather than 9 – to make such a contribution. But the new feature in (14.3) is that there is also a single overall $\sqrt{m_u m_d}$ dose. Because structural sensibility is important in discerning which possible relationships are true signal and which are mere noise, we need to closely look at the structure of the nuclides involved. Earlier, ${}^{6}_{3}Li$ opened up a new 2s shell for a protons and a neutrons alike, but in 2s, the orbital angular momentum is *l*=0 as it is for 1s. Now, however, ${}^{10}_{5}B$ is opening up a new 2p shell for a proton and a neutron, and these shells have *l*=1. So to create this shell, so as to sustain both a proton (extra up quark) and a neutron (extra down quark) in an *l*=1 state, we need some additional energy. The $\sqrt{m_u m_d}$ term appears to tell us that the *l*=1 proton contributes the m_u and the *l*=1 neutron contributes the m_d to this

$\sqrt{m_u m_d}$ energy does an the price for entry and maintenance in an orbital state. Again: decoding the nuclear genome!

10) Carbon-12: The ^{12}C nuclide is seat of biological life and the chosen standard of nuclear weight measurement with an isotopic mass exactly equal to 12 u by definition, also is of keen interest in terms of confirming certain patterns already seen for the ^{4}He and ^{8}Be which are the first three nuclides with $Z = N = \text{even}$. This sits on the nuclear backbone, and so we go straight to (5.12) with $Z = N = 6$ to obtain the latent binding energy and then see how much is subtracted away, i.e., held in reserve to confine quarks rather than bind the nucleus. The empirical binding energy ${}^{12}_{6}B_0 = 0.098\,939\,8\ u$. What we discern in [14.30] of [5] is that:

$$ {}^{12}_{6}B_0 \approx 6 \cdot \Delta E_P + 6 \cdot \Delta E_N - (m_u + m_d) - 12(m_u + m_d)/(2\pi)^{1.5} = 0.098\,908\,7\ \text{u}\,. \tag{6.9} $$

The empirical difference is -3.10508×10^{-5} u. Thus far the $u \leftrightarrow d$-symmetric energy number we have used is $\sqrt{m_u m_d}$, yet the above makes clear that $m_u + m_d$ is a good tool to add to the toolkit (by corollary it is already there because m_u and m_d are already there, but it helps to be cognizant of the equally-weighted sum $m_u + m_d$ especially for $u \leftrightarrow d$-symmetric nuclides). The coefficient 12 clearly makes structural sense: there are after all, 12 nucleons in ^{12}C, so each nucleon is responsible for one of the $(m_u + m_d)/(2\pi)^{1.5}$ energy doses. But like ^{10}B, ^{12}C has nucleons in the 2p shell must sustain yet another proton and neutron in an *l*=1 orbital state. So in the same way that $\sqrt{m_u m_d}$ sustained the first proton / neutron pair in an orbital state in (6.8), $m_u + m_d$ sustains the second proton / neutron pair in the *l*=1 orbital. *This also establishes a very definite and meaningful data pattern.* For the remaining ^{9}Be, ^{10}Be, ^{11}B, ^{11}C and ^{14}N nuclides which the authors has also characterized, we will take no further space here, but refer the reader to section 14 of [5].

11) The Proton and Neutron and Constituent Quark Masses: If the foregoing are not yet overwhelmingly convincing evidence that the primary relationships (3.1) and (3.2) are correct, that (3.3) and (3.4) are indeed the $Q = 0$ masses of the up and down quarks, and that the quark masses can systematically be used to decode the nuclear genome in a way that has never been done before, then the crowning empirical validation comes through using an extension of the foregoing approaches to explain the observed proton and neutron masses M_N = 939.565379 MeV and M_P = 938.272046 MeV themselves, in relation to these very same quark masses, *within all experimental errors!* This was the central result in [6], which will be summarized here to establish overwhelming empirical support beyond any reasonable doubt. The next section will then turn to the underlying theory, that baryons are the chromo-magnetic monopoles of Yang-Mills gauge theory.

It will be understood from basic algebra that if we know the difference *A*-*B* between any two numbers *A* and *B* and also know their sum *A*+*B*, then we can then deduce these two separate numbers. Because we already know the neutron minus proton mass difference $M_N - M_P$ in relation to the up and down quark masses from the primary relationship (3.2), we are one step away from knowing the proton and neutron masses themselves if we can also determine $M_N + M_P$. So the objective is to deduce the sum of these two masses. Once that is the objective, there is an important symmetry benefit we have already seen with the $Z = N$ nuclides: we expect that $M_N + M_P$ which represents baryons with a combined total of 3 up and three down quarks, must be symmetric under $u \leftrightarrow d$ interchange. This greatly restricts the toolkit elements we may use to either $m_u m_d$ products or $m_u + m_d$ sums.

The problem we have, however, is that the proton and neutron masses are at least two orders of magnitude larger than m_u = 2.223 792 40 MeV and m_d = 4.906 470 34 MeV, so the "sensible integer multiples" approach does not help us here. But we know from electroweak theory that the Fermi vev v_F=246.219651 GeV is used to set the scale of certain observed masses, notably the masses for the *W* and *Z* bosons, and we might expect on general principles that this vev will also turn up in the proton and neutron masses. So knowing that we are going to need $u \leftrightarrow d$ symmetric constructs such as $\sqrt{m_u m_d}$ to obtain $M_N + M_P$, and entertaining the possibility of employing $\sqrt{v_F}$ as an additional energy square root to supplement $\sqrt{m_u}$ and $\sqrt{m_d}$ which we are already using, we perform an exploratory calculation in [3.8] of [18] to find that the construct $\sqrt{v \cdot \sqrt{m_u m_d}} = 901.835259\text{ MeV}$ lands within about 3% of the actual proton and neutron masses. To use a golf analogy, this places the ball on the green; now we need to figure out how to hit it into the cup.

The next step was to employ $\text{diag}(\Phi_F) = v_F \text{diag}Q = v_F\left(0, \tfrac{2}{3}, -\tfrac{1}{3}, -\tfrac{1}{3}, -1, -\tfrac{1}{3}, \tfrac{2}{3}, \tfrac{2}{3}\right)$ which is a Fermi vacuum in the adjoint presentation for elementary fermions which were grouped into an $\left(\nu, (u_R, d_G, d_B), e, (d_R, u_G, u_B)\right)$ octet in the fundamental representation of an SU(8) Grand Unified Theory (GUT) that the author had used to break the electroweak symmetry and which naturally explained the existence of three fermion generations and CKM mixing and so answered Rabi's long ago quip about the muon, "who ordered that?" Plainly put: the electric charges *Q* of the up and down quarks needed to enter $\sqrt{v \cdot \sqrt{m_u m_d}} = 901.835259\text{ MeV}$ in the form of $v_F Q$.

So supplementing the Koide matrices *K* which were first discussed at (5.1) and (5.2) above with the quark electric charge magnitudes via Φ_F, the author in [5.8] of [6] constructed and then calculated the following inner product trace between a first Koide-type matrix with the duu (proton) charges and mass, and a second matrix the udd (neutron) charges and masses:

$$\text{Tr}\begin{pmatrix} \sqrt[4]{\tfrac{1}{3} v_F m_d} & 0 & 0 \\ 0 & \sqrt[4]{\tfrac{2}{3} v_F m_u} & 0 \\ 0 & 0 & \sqrt[4]{\tfrac{2}{3} v_F m_u} \end{pmatrix} \cdot \begin{pmatrix} \sqrt[4]{\tfrac{2}{3} v_F m_u} & 0 & 0 \\ 0 & \sqrt[4]{\tfrac{1}{3} v_F m_d} & 0 \\ 0 & 0 & \sqrt[4]{\tfrac{1}{3} v_F m_d} \end{pmatrix} = 3 \cdot \sqrt[4]{\tfrac{2}{9} {v_F}^2 m_u m_d} \tag{6.10}$$
$$= 1857.570635\text{ MeV}$$

which was understood to apply to all but the current quark mass sum $3m_u + 3m_d$ associated with $M_N + M_P$. Upon adding this sum to (6.10), it was found in [5.10] of [6] that:

$$M_N + M_P \approx 3\left(\sqrt[4]{\tfrac{2}{9} v_F{}^2 m_u m_d} + m_u + m_d\right) = 1878.961\,415\,\text{MeV} . \tag{6.11}$$

which differs from the observed $M_N + M_P = 1877.837\,425\,\text{MeV}$ by a scant 0.0599%!

The balance section 6 of [6] was devoted to closing this gap. In sum, it was found in [6.6] of [6] (see also [5.14]) that the *exact* $M_N + M_P$ includes a mixing angle θ_1 and a phase δ parameters which also need to be in (6.11) growing out of the fact that the up and down quarks have oppositely signed electric charges, and that the complete expression is:

$$M_N + M_P = 3\left(\sqrt[4]{\tfrac{2}{9} v_F{}^2 m_u m_d} \exp(i\delta) + (m_u + m_d)\cos\theta_1\right). \tag{6.12}$$

It was then deduced in [6.28] from the *empirical* $M_N + M_P$ that $\cos\theta_1 = 0.9474541242$ and in [6.30] that $\delta = 0$ *by mathematical identity*. The latter result tells us that there are no CP-violating effects associated with neutron and proton, which is validated by empirical data that the mass of the antiproton is equal to that of the proton, and similarly for the neutron, see, e.g., [19], [20], while the former result boils down and bundles up the problem of explaining the proton and neutron masses within all experimental errors, to the problem of explaining the value of the "nucleon fitting angle" $\cos\theta_1 = 0.9474541242$ within all experimental errors.

Because this θ_1 and the phase δ emerged from matrices with were mathematically the same as the CKM mixing matrices, it made sense to see if $\cos\theta_1 = 0.9474541242$ could be related in some way to the observed CKM mixing angles themselves. Equations [11.2], [11.3] and [11.27] (for empirical magnitude-only data) of PDG's [21] coupled with [22] tell us that:

$$\begin{aligned} V &= \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \\ &= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} , \\ &= \begin{pmatrix} 0.97427 \pm 0.00015 & 0.22534 \pm 0.00065 & 0.00351^{+0.00015}_{-0.00014} \\ -0.22520 \pm 0.00065 & 0.97344 \pm 0.00016 & 0.0412^{+0.0011}_{-0.0005} \\ -0.00867^{+0.00029}_{-0.00031} & -0.0404^{+0.0011}_{-0.0005} & 0.999146^{+0.000021}_{-0.000046} \end{pmatrix} \end{aligned} \tag{6.13}$$

and the Jarlskog determinant which is a phase-convention-independent measure of CP violation is $J = 2.96^{+0.20}_{-0.16} \times 10^{-5}$. A comparison of the empirical data with $\cos\theta_1 = 0.9474541242$ suggests that the *determinant* $|V|$ might be of help. We see from the product of three separate matrices in the first line above that $|V| = V_{ud}V_{cs}V_{tb} + V_{us}V_{cb}V_{td} + V_{ub}V_{cd}V_{ts} - V_{ub}V_{cs}V_{td} - V_{us}V_{cd}V_{tb} - V_{ud}V_{cb}V_{ts} = 1$ by construction, but this has two parts which we call the "major" and "minor" determinants $|V|_+ = V_{ud}V_{cs}V_{tb} + V_{us}V_{cb}V_{td} + V_{ub}V_{cd}V_{ts}$ and $|V|_- = V_{ub}V_{cs}V_{td} + V_{us}V_{cd}V_{tb} + V_{ud}V_{cb}V_{ts}$ such that $|V| = |V|_+ - |V|_- = 1$. From the median empirical magnitude-only data, we calculate $|V|_+ = 0.947535$ and $|V|_- = -0.052355$ thus $|V| = |V|_+ - |V|_- = 0.999889$, while the CP violating aspects of V are captured by $J = 2.96^{+0.20}_{-0.16} \times 10^{-5}$. Then, comparing the data number $\cos\theta_1 = 0.9474541242$ with $|V|_+ = 0.947535$, it begins to appears as if $\cos\theta_1$ may in fact be synonymous with $|V|_+$. In fact, when considering the experimental errors in (6.13), then we find in [7.4] of [6] that $|V|_+ = 0.947454^{+0.000400}_{-0.000262}$, i.e., that $0.947273 < |V|_+ < 0.947935$. This places the nucleon fitting angle $\cos\theta_1 = 0.9474541242$ predicted from the actual proton and neutron masses, well within the experimental errors for $|V|_+$.

So, once again driven by empirical data, we identify $\cos\theta_1 \equiv |V|_+$ which connects the CKM matrix with the nucleon fitting angle, and also using $\delta = 0$, we then rewrite (6.12) as:

$$M_N + M_P = 3\left(\sqrt[4]{\tfrac{2}{9} v_F{}^2 m_u m_d} + (m_u + m_d)|V|_+\right). \tag{6.14}$$

Now, this sum becomes specified within *all experimental errors*, when (6.14) is then solved together with the primary relationship (3.2) for $M_N - M_P$, we obtain theoretical values for the proton and neutron masses which are a function of only four parameters: m_u and m_d from (3.3) and (3.4), the Fermi vev, and $|V|_+$ obtained from the CKM mixing matrix. Solving in combination with the mass difference of the primary relationship (3.2) then yields the separate masses in [6.31] and [7.6] of [6], namely ($\sqrt[4]{\tfrac{2}{9} v_F{}^2 m_u m_d} \equiv \sqrt{M_u M_d}$, see [5.14] of [6]):

$$M_N = \tfrac{1}{2}\left(3\left(\sqrt[4]{\tfrac{2}{9} v_F{}^2 m_u m_d} + |V|_+ (m_u + m_d)\right) + m_u - \left(3m_d + 2\sqrt{m_\mu m_d} - 3m_u\right)/(2\pi)^{\frac{3}{2}}\right), \tag{6.15}$$

$$M_P = \tfrac{1}{2}\left(3\left(\sqrt[4]{\tfrac{2}{9} v_F{}^2 m_u m_d} + |V|_+ (m_u + m_d)\right) - m_u + \left(3m_d + 2\sqrt{m_\mu m_d} - 3m_u\right)/(2\pi)^{\frac{3}{2}}\right), \tag{6.16}$$

This then provides the basis in [8.3] through [8.6] of [6] for obtaining the so-called "constituent" quark masses (which we shall refer to as "contributive" quark masses) in which the current quark masses are bundled together with all of their associated non-linear behaviors to specify their separate contributions in the order of 310 to 320 MeV to the overall observed free nucleon masses.

12) <u>Charm, Strange, Top and Bottom-Flavored Baryon Masses</u>: If the proton and neutron can be expressed in terms of the up and down current quark masses as we see in (6.14), then this suggests that other flavors of baryon containing c, s, t and b quarks can similarly be expressed on these second and third generation quark flavors are included. In this regard, the culmination of the development leads us in [6.17] of [6] to a "mass and mixing matrix" Θ given by:

$$\Theta = 27\begin{pmatrix} \begin{matrix} -m_u\sqrt{m_s m_c}\sqrt{m_b m_t}\,\mathrm{c}_1\mathrm{s}_2\mathrm{s}_3 \\ +\sqrt{M_u M_d}\,m_s m_b\,\mathrm{c}_2\mathrm{c}_3\mathrm{e}^{i\delta} \end{matrix} & \begin{matrix} m_u\sqrt{m_s m_c}\,m_t\,\mathrm{c}_1\mathrm{s}_2\mathrm{c}_3 \\ +\sqrt{M_u M_d}\,m_s\sqrt{m_b m_t}\,\mathrm{c}_2\mathrm{s}_3\mathrm{e}^{i\delta} \end{matrix} & \sqrt{m_u m_d}\sqrt{m_s m_c}\sqrt{M_t M_b}\,\mathrm{s}_1\mathrm{s}_2 \\ \begin{matrix} -m_u m_c\sqrt{m_b m_t}\,\mathrm{c}_1\mathrm{c}_2\mathrm{s}_3 \\ -\sqrt{M_u M_d}\sqrt{m_s m_c}\,m_b\,\mathrm{s}_2\mathrm{c}_3\mathrm{e}^{i\delta} \end{matrix} & \begin{matrix} m_u m_c m_t\,\mathrm{c}_1\mathrm{c}_2\mathrm{c}_3 \\ -\sqrt{M_u M_d}\sqrt{m_s m_c}\sqrt{m_b m_t}\,\mathrm{s}_2\mathrm{s}_3\mathrm{e}^{i\delta} \end{matrix} & \sqrt{m_u m_d}\,m_c\sqrt{M_t M_b}\,\mathrm{s}_1\mathrm{c}_2 \\ \sqrt{m_u m_d}\sqrt{M_c M_s}\sqrt{m_b m_t}\,\mathrm{s}_1\mathrm{s}_3 & -\sqrt{m_u m_d}\sqrt{M_c M_s}\,m_t\,\mathrm{s}_1\mathrm{c}_3 & m_d\sqrt{M_c M_s}\sqrt{M_t M_b}\,\mathrm{c}_1 \end{pmatrix} \tag{6.17}$$

which includes the shorthand definitions $M_{u,c,t} \equiv \sqrt{\tfrac{2}{3}v_F m_{u,c,t}}$ and $M_{d,s,b} \equiv \sqrt{\tfrac{1}{3}v_F m_{d,s,b}}$ for "vacuum-amplified" quark masses containing the current quark masses amplified by the Fermi vev and attenuated by their electric charge magnitudes. The mathematics in the above was developed in the original parameterization of the Kobayashi and Maskawa matrices, but can be developed if desired in the standard parameterization appearing in (6.13). If we set the c, s, t, b masses equal to 1, set $s_2 = s_3 = 0$ and take the trace, then we obtain $\tfrac{1}{9}\mathrm{Tr}\Theta = 3\left(\sqrt[4]{\tfrac{2}{9}v_F{}^2 m_u m_d}\,\exp(i\delta) + (m_u + m_d)\cos\theta_1\right) = M_N + M_P$ in view of the above shorthands for $M_{u,c,t}$ and $M_{d,s,b}$. This is identical to the $M_N + M_P$ sum in (6.12), and it means that the proton and neutron masses are embedded in Θ as a special case. Thus, it must be considered that upon further study, this matrix will help provide an explanation of the various c, s, t and b flavored baryons. It should be kept in mind for any study in this direction, that in (3.2) we *defined* the up and down current quark masses from the proton and neutron masses which are known with much better precisions because they can be studies as free $Q \to 0$ particles whereas quarks are confined. It is to be expected that a similar approach will be warranted when it comes to these second and third generation quarks and the baryons within which they are confined.

13) <u>Who Ordered That?: Why are there Three Fermion Generations?</u>: Having just discussed the second and third generation quarks and baryons, it is worth now going back to Rabi's original quip "who ordered that?" about the muon. While the second and third generation quarks and leptons and their mixing properties have been well-characterized since then, Rabi's question remains unanswered to this day. Nobody has yet shown the theoretical *imperative* for having three generations, or for the mixing of these generations. These have been *described*, but why nature manifests itself in this way remains unexplained. The author in [18] shows how three stages of symmetry breaking of the SU(8) octuplet $(\nu,(u_R,d_G,d_B),e,(d_R,u_G,u_B))$ already mentioned in point 11 above and integrally used in deriving the proton and neutron masses, *leads inexorably to the appearance of three generations of quark and lepton and CKM-type mixing*. In retrospect, it was the author's unfortunate omission not to reference this finding as to the three generations

in the title of [18]. Unlike what has been discussed in points 1 through 12, this is a *qualitative*, not quantitative concurrence with empirical data. But it is equally important because the existence of multiple generations has, until now, remained one the great unexplained empirical mysteries of nature.

14) <u>Resonant Nuclear Fusion</u>: All of fundamental science has technological implications which may be developed over time, and the foregoing is no exception. Protons and neutrons bind together to form nuclei. When they do so they release fusion energies and the fused nuclei harbor mass defects which are very precise energy numbers which never vary from one experiment to the next. There must be an explanation why, for example, the deuteron *always* has a binding energy of 2.224 52 ± 0.00020 MeV, each and every time, and indeed, why all the binding energies shown in Figure 1 and all the energies of the fusion and fission events related to these are as they are. As we have now seen, the explanation rests in the current masses of the up and down quarks which these nucleons contain. Stepping back and applying hindsight, there is little else that *could* account for these energies, because protons and neutrons are no more and no less than systems containing quarks and their highly-non-linear interactions. But if that is the case, as pointed out in section 9 of [2] and more completely elaborated in [5], then the binding and fusion energy "toolkit" discussed in point 3 of section 5 which specifies the most elemental energy dosages released during a fusion event may be not only a theoretical toolkit, but also a *technological* one.

Nikola Tesla, who possessed one of the greatest historical aptitudes for extracting technology from science, once stated "if you want to find the secrets of the universe, think in terms of energy, frequency and vibration." So if the secret we wish to extract from nature is how to extract energy via nuclear fusion in the best way possible, and if think about vibrating nuclei and nucleons in resonance with certain energies and frequencies that might facilitate fusion better than can be done absent this vibration, then the foregoing toolkit energies which explain the nuclear binding and fusion data provide a compelling approach. It is on this basis that the author has proposed and filed the international patent application [5] for catalyzing nuclear fusion by bathing a nuclear fuel in gamma radiation at energies established by the discrete energies in the dosage toolkit. This needs to be tested and if viable, developed, but the testing is very simple: In experiment 1, Carry out a given fusion reaction in the "usual" and "ordinary" way and carefully assemble and monitor all of the variables, e.g., temperature, power, density, etc. which are involved, as an experimental "control." Then in experiment 2 apply gamma radiation proximate the toolkit frequencies which are pertinent to that fusion reaction, and change nothing else. Make certain that the only difference is that in experiment 2 the gamma radiation is applied and in experiment 1 it is not. See if the fusion moves any of the key variables in a "fusion-favorable" direction. If it does, then the further development of those results may provide the path for more practical and widespread applications of nuclear fusion to produce commercial energy. And, any favorable change based on using the toolkit energies would be a further empirical validation of these scientific results.

15) <u>Decoding the Nuclear Genome</u>: The many ways, the fundamental purpose of this paper is to present overwhelmingly-convincing evidence empirical evidence for the viewpoint that there is in fact a nuclear genome which needs to be decoded if humankind is to advance its understanding nuclear and elementary particle physics beyond where it stands at present, that this

nuclear genome is physically manifest through multiple relationships in which the nuclear masses and mass defects and binding and fusion / fission energies are expressed in terms of current quark masses (and in certain instances the Fermi vev and the CKM quark generation mixing matrices) which quarks masses can be established with the same level of precision as these other mass / energy parameters, and that all of this can be achieved using an unambiguous electron-proton-neutron (EPN) measurement system for defining the $Q \to 0$ quark masses notwithstanding the fact that quarks are confined and so can never be directly observed in the quiescent $Q = 0$ states of being.

This exposition began with the postulated "primary mass relationships" (3.1) and (3.2) from which we then deduced $Q = 0$ up and down quarks masses with a high precision inherited from the EPN masses and then posed the three questions whether 1) it is legitimate and unambiguous as a measurement system, to establish $Q = 0$ quark masses in this way, 2) whether such an approach relating the quark masses to nuclear masses and energies could be validated by empirical data and 3) whether and how the thesis that baryons are the chromo-magnetic monopoles of Yang-Mills gauge theory provides a firm theoretical foundation upon which all of this may be supported.

The evidence presented in this section of parts-per 10^5, 10^6 and even 10^7 AMU empirical fits between the up and down quark masses and multiple light nuclide binding energies ^{2}H, ^{3}H, ^{3}He, ^{4}He, ^{6}Li, ^{7}Li, ^{7}Be, ^{8}Be, ^{10}B, ^{9}Be, ^{10}Be, ^{11}B, ^{11}C, ^{12}C and ^{14}N, very tightly-bound nuclides like ^{56}Fe, and even the proton and neutron masses themselves within all experimental errors, demonstrate beyond any reasonable doubt that there really do exist definitive relationships in nature between the up and down current quark masses and a plethora of energies observed in the nuclear world, and that the up and down quark masses are indeed the masses deduced in (3.3) and (3.4) with a precision close to a billion times better than anything that has been achieved to date by defining quark masses from the results of nuclear scattering experiments. If our purpose was to validate the primary relationships (3.1) and (3.2) and thus the up and down quark masses (3.3) and (3.4) by showing that *if* these relationships and masses are regarded as true many other nuclear energies could also be similarly-related to these masses, then every single one of points 1 through 11 of this section contain further examples of secondary nuclear energy relationships which can be expressed in terms of the up and down current quark masses, just like the primary relationships (3.1) and (3.2), thus providing overwhelming empirical validation. Point 12 suggests possible additional validation (or contradiction) through the study of other baryon masses, and it is also very important as we are reminded of in point 13, that this approach allows us to finally answer Rabi's questions about the higher fermion generations, "who ordered that?"

So at this point, the primary relationships (3.1) and (3.2) have been amply validated by empirical data, and this validation also demonstrates that the EPN measurement system laid out here yields sensible and unambiguous results. Now the time has arrived to summarize the theoretical considerations from which the author originally deduced the mass / energy relationships (3.1), (5.1) and (5.2) from which all of the other empirical connections elaborated here were developed via comparison with empirical data. The underlying theory, of course, is that baryons are the chromo-magnetic monopoles of Yang-Mills gauge theory as originally presented by the author in [1], and thereafter, more-deeply developed in [10] which for the first time fully lays out the quantum field theory for this via an exact, non-linear path integration of classical Yang-Mills gauge theory. In short, we now turn to the third question from section 3: is there a firm theoretical foundation upon which all of this may be supported?

7. The Theoretical / Empirical Interface

The author's thesis that the observed baryons are the chromo-magnetic monopoles of Yang-Mills gauge theory is what initially led following development in [1] and later deeper elaboration in [10] to equations (5.1) and (5.2) and then by subtraction of (5.1) from (5.2), to equation (3.1). These three equations, in turn, became the foundation for all of the empirical connections elaborated in the last section which cumulatively provide overwhelming evidence for the validity of the underlying theory, as has been reviewed here. So because it is equations (5.1) and (5.2) which are the "interface equations" between the underlying theory and the ability to prove that theory by reference to the wealth of empirical data enumerated in the last section, we shall briefly review the underlying theory as to the connection between Yang-Mills chromo-magnetic monopoles and baryons, but leave the details of this theory to the original source materials [1] and [10], and place particular emphasis on how it is that (5.1) and (5.2) ultimately derive from that theory.

We start by returning to the question posed in point 3 of section 3: "If we can legitimately assert (3.3) and (3.4) to be the $Q=0$ up and down quark masses and if we can find secondary support from a broad array of nuclear data [which has now been done], then we get to the third question: what is the overarching theory, and does that theory make sense within the overall framework of theoretical physics?"

And as to theoretical sensibility, the thesis that the observed baryons are the chromo-magnetic monopoles of Yang-Mills gauge theory is in fact exceptionally conservative, and is grounded solely in widely-accepted, highly-settled, thoroughly-tested science. Its novelty rests in its deductive combination of known, accepted and well-validated scientific theories and theoretical elements to uniquely and unambiguously deduce new results and new explanations for previously-unexplained observational data, such as what was reviewed in the last section. As suggested near the start of section 6, while brand new ideas ought not to be ruled out out-of-hand, a combination of settled science and scientific elements is preferable, and brand new notions should only be used as a last resort when there is no apparent way to succeed by restricting oneself to combining known elements in unknown ways.

Specifically, setting aside the empirical validations already reviewed, in order to accept this theory from a *theoretical standpoint*, one is required simply to believe and accept no more and no less than: a) that Maxwell's electrodynamics which includes (vanishing) magnetic monopoles is a correct theory of nature; b) that Yang-Mills gauge theory which extends Maxwell's electrodynamics to non-abelian domains is a correct theory of nature; c) that Dirac's theory is a correct theory of nature particularly insofar as it relates fermion wavefunctions to current densities via $J^\sigma = \overline{\psi}\gamma^\sigma\psi$; d) believing that Dirac-Fermi-Pauli were correct when they asserted that multiple fermions within a single system must occupy exclusive states distinguished from one another by one or more quantum numbers (the "Exclusion Principal"); and e) for the quantum theory of chromodynamics QCD, believing that Feynman's method of path integration is the correct way to start with a classical field equation in spacetime (configuration space) for a field φ with source J and its related Lagrangian density $\mathfrak{L}(\varphi, J)$ and action $S(\varphi, J) = \int d^4x\mathfrak{L}(\varphi, J)$, and convert this over to a quantum field theory by performing the integration $Z = \exp iW(J) = \mathfrak{C}\int D\varphi \exp iS(\varphi, J)$ and then extracting the quantum field $W(J)$ in (Fourier-transformed) momentum space. And to cross the threshold from theory to empirical confirmation by obtaining the interface equations (5.1) and (5.2), one also needs to believe and accept f) that the quarks inside a baryon, although confined, are asymptotically free and can thus be treated at least in an approximate manner as free fermions.

If one accepts and believes a) through d), then the inexorable result of *merely combining all of these together* leads one to conclude that the classical magnetic monopoles of Yang-Mills gauge theory – specifically the sources of a non-vanishing magnetic field flux $\oiint F \neq 0$ across closed spatial surfaces – do indeed have the earlier noted antisymmetric $\mathrm{R} \wedge \mathrm{G} \wedge \mathrm{B}$ color symmetry of a baryon and confine everything but entities with the symmetric $\overline{\mathrm{R}}\mathrm{R}+\overline{\mathrm{G}}\mathrm{G}+\overline{\mathrm{B}}\mathrm{B}$ color symmetry of a meson with $\oiint F \neq 0$ being the classical representation of this meson flux, as established in detail in Part I of [10]. This combination also teaches that employing $SU(3)_C$ as the color group of chromodynamics is not a *choice*, but is *required* (the only choice is how to name the three mandated eigenstates). *So chromodynamics is not a theory of first principle, but is a corollary theory* emerging inexorably from the combination of a) through d). And if one further accepts and believes e), then the quantum theory which emerges via theoretical deduction following path integration leads to a running QCD coupling which matches up to Figure 1 above within experimental errors, as established generally in section 18 and specifically in [18.22] and Figure 14 of [10]. Finally, if one accepts f), then it becomes possible to use this theory to obtain (5.1) and (5.2) which is the bridge to empirical testing. But the fact that (5.1) and (5.2) and their offspring (3.1) lead to all of the empirical confirmations already enumerated here provides comfort that this treatment of quarks inside a baryon as approximately-free particles is empirically-valid. So let us now turn as directly as possible to how the interface equations (5.1) and (5.2) are obtained and then work backwards to place that in the overall theoretical context.

The starting point for deriving the interface equations (5.1) and (5.2) in the original formulation of the baryon / monopole thesis was equation [11.2] of [1]. In the later formulation presented in [10], the equivalent starting point is equation [10.4], which is reproduced below:

$$\begin{aligned} &i\mathrm{Tr}\Sigma F_{\mathrm{eff}\,\mu\nu}\left((0)\right)_0 = \mathrm{Tr}\Sigma\left[G_\mu, G_\nu\right]\left((0)\right)_0 \\ &= \overline{\psi_R}\gamma_{[\mu}\left(\not{p}_R - m_R\right)^{-1}\gamma_{\nu]}\psi_R + \overline{\psi_G}\gamma_{[\mu}\left(\not{p}_G - m_G\right)^{-1}\gamma_{\nu]}\psi_G + \overline{\psi_B}\gamma_{[\mu}\left(\not{p}_B - m_B\right)^{-1}\gamma_{\nu]}\psi_B \end{aligned} . \tag{7.1}$$

The notation in $\Sigma F_{\mathrm{eff}\,\mu\nu}\left((0)\right)_0$ is a bit cumbersome so let us simplify this a bit, and also remind the reader what this means. The Σ in (7.1) simply reminds is of the use of the spin sum $\Sigma_{\mathrm{spins}}u\overline{u} = N^2/(E+m)(\not{p}+m)$ during the course of the derivation starting with [9.12] of [10]. If we simply keep in mind that a spin sum was used to get to that point then we can drop the Σ from the notation. The $\left((0)\right)_0$ notation developed in section 8 of [10] tells us that that (7.1) is taken in the abelian limit of non-abelian gauge theory for which $G_\mu\left(\left(0\right)\right)_0 = \left(k_\tau k^\tau - m^2 + i\varepsilon\right)^{-1} J_\mu$ and in which we have not recursed G_μ into itself at all. As shown in section 7 of [10], a natural consequence of the non-linearity of Yang-Mills gauge theory is that when we invert the classical Maxwell chromo-electric charge equation between G_μ and J_μ, we find that $G_\mu\left(G_\mu, J_\mu\right)$ is a function of itself along with J_μ, and if we recurse *n* time before cutting off then we denote this as $G_\mu\left(\left(0\right)\right)_n$. To simplify, we shall simply keep the subscript "0" as a reminder that $F_{\mu\nu}$ above is taken at the zero recursive order which is the abelian limit and drop the nested parenthesis.

Finally, the "eff" subscript for "effective" in (7.1) is used to denote that this is the portion of the field strength tensor $F_{\mu\nu}$ which actually net-flows $\oiint F = \oiint F_{\text{eff}} = -i\oiint \left[G,G\right] \neq 0$ across the closed surfaces surrounding the "faux" magnetic sources $P' = -id\left[G,G\right] = -idGG$ of Yang-Mills gauge theory. This is because the term dG in the complete field strength $F = dG - i\left[G,G\right]$ identically drops out of any expression for $\oiint F$ because $ddG = 0$ because the exterior derivative of an exterior derivative is zero in differential geometry which is why $\oiint F = 0$ in electrodynamics, which combines Gauss' law for magnetism and Faraday's law for induction. This is the heart of how baryons are theoretically developed from the monopoles of Yang-Mills gauge theory by deductively combining points a) and b) above (Maxwell and Yang-Mills are both correct theories of nature). Thus we shall retain the "eff" subscript as a reminder of this. Therefore, $\Sigma F_{\text{eff}\,\mu\nu}\left(\left(0\right)\right)_0$ above shall now be denoted simply $F_{\text{eff}\,0\mu\nu}$ to mean the net-flowing $\oiint F \neq 0$ portion of *F* in the zero-recursive order of Yang-Mills gauge theory.

The final aspect of (11.1) which we have not yet discussed, that this is a trace equation. If we backtrack to an earlier equation such as [9.20] of [1] from which this is descended to write this in matrix form prior to taking the trace, then (7.1) can be put in its matrix form:

$$F_{\text{eff}\,0\mu\nu} = -i\begin{pmatrix} \overline{\psi_R}\gamma_{[\mu}\left(\not{p}_R - m_R\right)^{-1}\gamma_{\nu]}\psi_R & 0 & 0 \\ 0 & \overline{\psi_G}\gamma_{[\mu}\left(\not{p}_G - m_G\right)^{-1}\gamma_{\nu]}\psi_G & 0 \\ 0 & 0 & \overline{\psi_B}\gamma_{[\mu}\left(\not{p}_B - m_B\right)^{-1}\gamma_{\nu]}\psi_B \end{pmatrix}. \quad (7.2)$$

This is the formal starting point via $E = \iiint \frac{1}{2}\text{Tr}F_{\mu\nu}F^{\mu\nu}d^3x$ using both inner and outer product traces as reviewed in this paper near the start of section 5, for deriving (5.1) and (5.2) which are the interface equations leading to all the empirical connections reviewed in section 6. So let us proceed to show how this connection is made. This will essentially review section 11 of [1], but with additional clarity. We begin by looking at the generic expression $\overline{\psi}\gamma_{[\mu}\left(\not{p} - m\right)^{-1}\gamma_{\nu]}\psi$ in (7.2) for each of the three colors of quark.

First, we separate the propagators as $(\not p - m)^{-1} = (\not p + m)/(p^2 - m^2)$ into two parts:

$$\overline{\psi}\gamma_{[\mu}(\not p - m)^{-1}\gamma_{\nu]}\psi = \frac{\overline{\psi}\gamma_{[\mu}(\not p + m)\gamma_{\nu]}\psi}{p^2 - m^2} = \frac{m\overline{\psi}[\gamma_\mu, \gamma_\nu]\psi}{p^2 - m^2} + \frac{\overline{\psi}\gamma_{[\mu}\not p\gamma_{\nu]}\psi}{p^2 - m^2}. \tag{7.3}$$

Now we expand out the numerator in the latter term using $\not p = p^\sigma \gamma_\sigma$, as such:

$$\overline{\psi}\gamma_{[\mu}\not p\gamma_{\nu]}\psi = p^\sigma\overline{\psi}\gamma_{[\mu}\gamma_\sigma\gamma_{\nu]}\psi = p^0\overline{\psi}\gamma_{[\mu}\gamma_0\gamma_{\nu]}\psi + p^1\overline{\psi}\gamma_{[\mu}\gamma_1\gamma_{\nu]}\psi + p^2\overline{\psi}\gamma_{[\mu}\gamma_2\gamma_{\nu]}\psi + p^3\overline{\psi}\gamma_{[\mu}\gamma_3\gamma_{\nu]}\psi. \tag{7.4}$$

We evaluate each of the independent components $\mu\nu = 01,02,03,12,23,31$ and apply the Dirac relation $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$ in various combinations to terms which do not drop out via the $[\mu,\nu]$ commutator. Using $g_{\mu\nu} = \eta_{\mu\nu}$ for flat spacetime, one may summarize the result by:

$$\overline{\psi}\gamma_{[\mu}\not\rho\,\gamma_{\nu]}\psi = 2i\varepsilon_{\mu\nu\alpha\beta}\rho^{[\alpha}\overline{\psi}\gamma^{\beta]}\gamma^5\psi \tag{7.5}$$

So we use this as well as the Dirac covariant $[\gamma_\mu, \gamma_\nu] = -2i\sigma_{\mu\nu}$ to rewrite (7.3) as:

$$\overline{\psi}\gamma_{[\mu}(\not p - m)^{-1}\gamma_{\nu]}\psi = -2i\frac{m\overline{\psi}\sigma_{\mu\nu}\psi}{p^2 - m^2} + 2i\frac{\varepsilon_{\mu\nu\alpha\beta}\rho^{[\alpha}\overline{\psi}\gamma^{\beta]}\gamma^5\psi}{p^2 - m^2}. \tag{7.6}$$

We see therefore, that this generic expression contains both a second rank antisymmetric tensor $\overline{\psi}\sigma_{\mu\nu}\psi$ and a first rank *axial* vector $\overline{\psi}\gamma^\beta\gamma^5\psi$. Using chirality language, this means that $F_{\mathrm{eff}\,0\mu\nu} = F_{V\,\mathrm{eff}\,0\mu\nu} + F_{A\mathrm{eff}\,0\mu\nu}$ in (7.2) has both a vector (*V*) and an axial (*A*) term.

Let us now set aside the axial term $F_{A\mathrm{eff}\,0\mu\nu}$ and focus on the vector term $F_{V\,\mathrm{eff}\,0\mu\nu}$ in the $p^2 \to 0$ limit for which the propagators disappear and the interactions essentially occur at a point. We refer to, e.g., [23] at p. 257, for a similar analysis explaining how the Fermi coupling constant G_F really is a point-interaction manifestation of a *W* vector boson propagator $\left(g_{\mu\nu} - k_\mu k_\nu / M_W{}^2\right)/\left(k^2 - M_W{}^2\right)^{-1}$ in the $k^2 \to 0$ limit for which $G_F/\sqrt{2} = g_w{}^2/8M_W{}^2$, connecting the modern understanding of weak interactions with Fermi's original conception of *β*-decay modelled on electromagnetic interactions. Using the *V* portion of (7.6) in (7.2) for $p^2 \to 0$ allows us to now write this matrix as:

$$F_{V\,\mathrm{eff}\,0\mu\nu}=2\begin{pmatrix}\frac{\overline{\psi}_R\sigma_{\mu\nu}\psi_R}{m_R} & 0 & 0\\ 0 & \frac{\overline{\psi}_G\sigma_{\mu\nu}\psi_G}{m_G} & 0\\ 0 & 0 & \frac{\overline{\psi}_B\sigma_{\mu\nu}\psi_B}{m_B}\end{pmatrix}. \tag{7.7}$$

It is this matrix which is the theoretical point of departure for connecting with the electron rest mass in (3.1) and the various nuclear energies elaborated in sections 3 through 6 of this paper. So now, with the benefit of two years of retrospective perspective including the many empirical connections enumerated in section 6, we shall elucidate that connection which was originally uncovered in sections 11 and 12 of [1] between (7.7) and observational energy data.

As reviewed at the start of section 5, the energy of pure gauge fields in Yang-Mills theory may be deduces by taking $E=\iiint d^3x\,\frac{1}{2}\mathrm{Tr}F_{\mu\nu}F^{\mu\nu}$ in both outer and inner product traces. We now have an $F_{V\,\mathrm{eff}\,0\mu\nu}$ in (7.7) above which flows from the thesis that baryons are the chromo-magnetic monopoles of Yang-Mills and specifically from combining Maxwell and Yang-Mills and Dirac Theories and Fermi-Dirac-Pauli exclusion. So we shall use this to deduce the associated energy *E*.

First, based on (7.7), we form the outer product trace:

$$\begin{aligned}&\tfrac{1}{2}\mathrm{Tr}F_{V\,\mathrm{eff}\,0\mu\nu}\otimes F_{V\,\mathrm{eff}\,0}{}^{\mu\nu}\\ &=2\left(\begin{aligned}&\frac{\overline{\psi}_R\sigma_{\mu\nu}\psi_R}{m_R}\frac{\overline{\psi}_R\sigma^{\mu\nu}\psi_R}{m_R}+\frac{\overline{\psi}_G\sigma_{\mu\nu}\psi_G}{m_G}\frac{\overline{\psi}_G\sigma^{\mu\nu}\psi_G}{m_G}+\frac{\overline{\psi}_B\sigma_{\mu\nu}\psi_B}{m_B}\frac{\overline{\psi}_B\sigma^{\mu\nu}\psi_B}{m_B}\\ &+2\frac{\overline{\psi}_R\sigma_{\mu\nu}\psi_R}{m_R}\frac{\overline{\psi}_G\sigma^{\mu\nu}\psi_G}{m_G}+2\frac{\overline{\psi}_G\sigma_{\mu\nu}\psi_G}{m_G}\frac{\overline{\psi}_B\sigma^{\mu\nu}\psi_B}{m_B}+2\frac{\overline{\psi}_B\sigma_{\mu\nu}\psi_B}{m_B}\frac{\overline{\psi}_R\sigma^{\mu\nu}\psi_R}{m_R}\end{aligned}\right).\end{aligned} \tag{7.8}$$

It will be appreciated that this includes the inner product trace, which consists only of the top parenthetical line in the above:

$$\tfrac{1}{2}\mathrm{Tr}F_{V\,\mathrm{eff}\,0\mu\nu}\cdot F_{V\,\mathrm{eff}\,0}{}^{\mu\nu}=2\left(\frac{\overline{\psi}_R\sigma_{\mu\nu}\psi_R}{m_R}\frac{\overline{\psi}_R\sigma^{\mu\nu}\psi_R}{m_R}+\frac{\overline{\psi}_G\sigma_{\mu\nu}\psi_G}{m_G}\frac{\overline{\psi}_G\sigma^{\mu\nu}\psi_G}{m_G}+\frac{\overline{\psi}_B\sigma_{\mu\nu}\psi_B}{m_B}\frac{\overline{\psi}_B\sigma^{\mu\nu}\psi_B}{m_B}\right). \tag{7.9}$$

So the inner product has pure-color RR, GG and BB products while the outer product adds RG, GB and BR cross-color products.

Next, we refer to sections 7 and 8 of [1] as also reviewed in section 10 of [10] whereby for the proton, the RGB colors of quark are respectively assigned to and have the appropriate flavor generators for the duu flavors of quark and for the neutron these same colors are assigned to and have generators for the udd flavors of quark. Therefore, (7.7) is use to derive both a proton (P) and a neutron (N) field strength:

$$F_{V\,P\mathrm{eff}\,0\mu\nu} = 2\begin{pmatrix} \frac{\overline{\psi}_d \sigma_{\mu\nu} \psi_d}{m_d} & 0 & 0 \\ 0 & \frac{\overline{\psi}_u \sigma_{\mu\nu} \psi_u}{m_u} & 0 \\ 0 & 0 & \frac{\overline{\psi}_u \sigma_{\mu\nu} \psi_u}{m_u} \end{pmatrix}, \tag{7.10}$$

$$F_{V\,N\mathrm{eff}\,0\mu\nu} = 2\begin{pmatrix} \frac{\overline{\psi}_u \sigma_{\mu\nu} \psi_u}{m_u} & 0 & 0 \\ 0 & \frac{\overline{\psi}_d \sigma_{\mu\nu} \psi_d}{m_d} & 0 \\ 0 & 0 & \frac{\overline{\psi}_d \sigma_{\mu\nu} \psi_d}{m_d} \end{pmatrix}. \tag{7.11}$$

This is the first place at which the up and down current quark masses enter the picture. This means that the outer product traces:

$$\tfrac{1}{2}\mathrm{Tr} F_{V\,P\mathrm{eff}\,0\mu\nu} \otimes F_{V\,P\mathrm{eff}\,0}{}^{\mu\nu} = 2\left(\frac{\overline{\psi}_d \sigma_{\mu\nu} \psi_d}{m_d} \frac{\overline{\psi}_d \sigma^{\mu\nu} \psi_d}{m_d} + 4\frac{\overline{\psi}_u \sigma_{\mu\nu} \psi_u}{m_u} \frac{\overline{\psi}_d \sigma^{\mu\nu} \psi_d}{m_d} + 4\frac{\overline{\psi}_u \sigma_{\mu\nu} \psi_u}{m_u} \frac{\overline{\psi}_u \sigma^{\mu\nu} \psi_u}{m_u} \right), \tag{7.12}$$

$$\tfrac{1}{2}\mathrm{Tr} F_{V\,N\mathrm{eff}\,0\mu\nu} \otimes F_{V\,N\mathrm{eff}\,0}{}^{\mu\nu} = 2\left(\frac{\overline{\psi}_u \sigma_{\mu\nu} \psi_u}{m_u} \frac{\overline{\psi}_u \sigma^{\mu\nu} \psi_u}{m_u} + 4\frac{\overline{\psi}_u \sigma_{\mu\nu} \psi_u}{m_u} \frac{\overline{\psi}_d \sigma^{\mu\nu} \psi_d}{m_d} + 4\frac{\overline{\psi}_d \sigma_{\mu\nu} \psi_d}{m_d} \frac{\overline{\psi}_d \sigma^{\mu\nu} \psi_d}{m_d} \right). \tag{7.13}$$

And if we subtract (7.12) for the proton from (7.13) for the neutron, we find that the difference:

$$\tfrac{1}{2}\mathrm{Tr} F_{V\,N\mathrm{eff}\,0\mu\nu} \otimes F_{V\,N\mathrm{eff}\,0}{}^{\mu\nu} - \tfrac{1}{2}\mathrm{Tr} F_{V\,P\mathrm{eff}\,0\mu\nu} \otimes F_{V\,P\mathrm{eff}\,0}{}^{\mu\nu} = 2\left(3\frac{\overline{\psi}_d \sigma_{\mu\nu} \psi_d}{m_d} \frac{\overline{\psi}_d \sigma^{\mu\nu} \psi_d}{m_d} - 3\frac{\overline{\psi}_u \sigma_{\mu\nu} \psi_u}{m_u} \frac{\overline{\psi}_u \sigma^{\mu\nu} \psi_u}{m_u} \right). \tag{7.14}$$

It is (7.12) which eventually turns into $E_P = \left(m_d + 4\sqrt{m_u m_d} + 4m_u\right)/\left(2\pi\right)^{\frac{3}{2}}$ in (5.1), (7.13) which becomes $E_N = \left(m_u + 4\sqrt{m_u m_d} + 4m_d\right)/\left(2\pi\right)^{\frac{3}{2}}$ in (5.2), and finally, (7.14) which turns into $E_N - E_P = 3\left(m_d - m_u\right)/\left(2\pi\right)^{\frac{3}{2}} \equiv m_e$ (5.3) a.k.a. the primary relationship (3.1). One should closely compare all of this, because these is how the structure of the theory that baryons including protons and neutrons are the chromo-magnetic monopoles of Yang-Mills gauge theory bleeds through to (5.1), (5.2) and (5.3) which become the basis for all of the other empirical relationships heretofore reviewed.

Specifically, as will now be reviewed, when we use (7.12) to (7.14) in $E=\iiint \frac{1}{2}\mathrm{Tr}F_{\mu\nu}F^{\mu\nu}d^3x$, carry out the integration, and then establish the normalization of the Dirac spinors by comparing the theoretical energy results to empirical data ("empirical normalization", see [1] after [11.29]), we uncover term mappings $\overline{\psi}_u\sigma_{\mu\nu}\psi_u\overline{\psi}_u\sigma_{\mu\nu}\psi_u / m_u{}^2 \Rightarrow m_u$, $\overline{\psi}_d\sigma_{\mu\nu}\psi_d\overline{\psi}_d\sigma_{\mu\nu}\psi_d / m_d{}^2 \Rightarrow m_d$ and $\overline{\psi}_u\sigma_{\mu\nu}\psi_u\overline{\psi}_d\sigma_{\mu\nu}\psi_d / m_u m_d \Rightarrow \sqrt{m_u m_d}$, together with the $(2\pi)^{\frac{3}{2}}=\sqrt{2\pi}^3$ divisor which emerges from the $E=\iiint \frac{1}{2}\mathrm{Tr}F_{\mu\nu}F^{\mu\nu}d^3x$ integral over three space dimensions. Let us now review how this is done.

All of (7.12), (7.13) and (7.14) when used as integrands in $E=\iiint \frac{1}{2}\mathrm{Tr}F_{\mu\nu}F^{\mu\nu}d^3x$ will yield one of three distinct terms: $\frac{1}{2}E_{uu}=\iiint d^3x\overline{\psi}_u\sigma_{\mu\nu}\psi_u\overline{\psi}_u\sigma_{\mu\nu}\psi_u / m_u{}^2$ which is a pure up / up term, $\frac{1}{2}E_{dd}=\iiint d^3x\overline{\psi}_d\sigma_{\mu\nu}\psi_d\overline{\psi}_d\sigma_{\mu\nu}\psi_d / m_d{}^2$ which is a pure down / down term, and $\frac{1}{2}E_{ud}=\iiint d^3x\overline{\psi}_u\sigma_{\mu\nu}\psi_u\overline{\psi}_d\sigma_{\mu\nu}\psi_d / m_u m_d$ which is a mixed up / down term. The factor of $\frac{1}{2}$ is to account for the overall factors of 2 in (7.12) through (7.14) so we are comparing energy numbers to energy numbers. These are then weighted within the overall energies $E=\iiint \frac{1}{2}\mathrm{Tr}F_{\mu\nu}F^{\mu\nu}d^3x$ via the constant coefficients (1, 3, 4) variously appearing in (7.12), (7.13) and (7.14). And these also become the "energy dosages" in the "toolkit" first referred to after (6.7) which physically, are emitted from nuclei during fusion events. So, for example, we earlier spoke after (6.8) of how nine (9) energy doses $9\sqrt{m_u m_d}/(2\pi)^{1.5}$ are emitted as energy when ^{4}He is fused with two protons to create ^{6}Li with the same number of nine (9) up / down quark pairs, and of how fifteen (15) energy doses $15\sqrt{m_u m_d}/(2\pi)^{1.5}$ are emitted when ^{8}Be is fused with two protons to create ^{10}Li with the same number of fifteen (15) up / down quark pairs. What we were really saying when more formally-specified in terms of the underlying theoretical physics, is that in the former case ${}^4_2He+2p\rightarrow {}^6_3Li+e^++\nu+\text{Energy}$ there are nine (9) and in the latter case ${}^8_4Be+2p\rightarrow {}^{10}_5B+e^++\nu+\text{Energy}$ there fifteen (15) simultaneous emissions of the energy dosage $\frac{1}{2}E_{ud}=\iiint d^3x\overline{\psi}_u\sigma_{\mu\nu}\psi_u\overline{\psi}_d\sigma_{\mu\nu}\psi_d / m_u m_d$, one such dosage associated with each pair of up and down quarks. So now, let us review how this connection gets made.

Let us start with the generic expression $\frac{1}{2}E=\iiint d^3x\overline{\psi}\sigma_{\mu\nu}\psi\overline{\psi}\sigma_{\mu\nu}\psi / m^2$ for a fermion wavefunction $\psi(\mathbf{x})$ and take this to be representative of the up or down quark, when used in the "pure" terms mentioned just above. Now, any spatial dependence for this integral over d^3x is contained in $\psi(\mathbf{x})$, so to go any further with this calculation we must make some supposition as to spatial-dependency of $\psi(\mathbf{x})$. We can choose from a range of possible functions, e.g., Lorentzian, exponential, Gaussian, etc. Indeed, any function may be used, whether or not it is radially symmetric, provided it is renormalizable and so finitely integrates when placed in

$\frac{1}{2}E=\iiint d^3x\overline{\psi}\sigma_{\mu\nu}\psi\overline{\psi}\sigma_{\mu\nu}\psi/m^2$. As an *ansatz* to be able to perform *some* numeric calculation, and without limitation as to any other *ansatz* that another may choose, the author at [9.9] of [1] chose the radially-symmetric Gaussian wavefunction $\psi(r)=u(p)\left(\pi/m^2\right)^{-.75}\exp\left(-\frac{1}{2}m^2\left(r-r_0\right)^2\right)$ where *m* generically needs to be a number with mass dimensionality and r_0 is the radial coordinate of the center peak of the Gaussian. Further, to give *m* some meaning in relation to the physics being studied, *m* is chosen in this *ansatz* to be equal to the rest mass of the fermion. Again, this is done simply to be able to do a calculation with the hope that energy numbers which makes sense in relation to something observed might emerge from this calculation; other exploratory choices for $\psi(\mathbf{x})$ are also possible.

Now, a Gaussian is the standard expression use to represent a minimum-uncertainty wave-packet and thus is associated with free particles. So, one may ask whether this "freedom" is suitable for quarks which are confined. But quarks *are* in fact *asymptotically free*, so aside from the "edge" region of a nucleon near $Q=\Lambda_{\text{QCD}}$ as discussed in section 2, a free-particle Gaussian would be a good approximation to an "approximately free" fermion such as an asymptotically-free quark. Also, wave-packets such as the foregoing Gaussian with a standard deviation comparable to their Compton wavelength $\lambda=\hbar m/c$ contain negative-energy amplitudes indicating the presence of antiparticles. But we know that nucleons are teeming with quark / antiquark states, exhibited no more clearly than through the manifold of $\overline{q}q$ meson jets emitted under any substantial scattering impact. Finally, the Compton wavelengths of the current quark masses are on the order of 40 Fermi for the down quark and 85 Fermi for the up quark, which exceeds ~ 2 Fermi length scale $r_\Lambda\equiv\hbar/c\Lambda_{\text{QCD}}=2.1780\text{ fm}$ of Λ_{QCD} by more than a full order of magnitude and so "bleeds out" from the proton and neutron even though the quarks are confined. But as noted after (6.16), see also the end of section 11 in [1], the constituent i.e. contributive quark masses have a standard deviation of less than 1 Fermi which places them well within the r_Λ length scale. And what we learn in sections 5 and 6 is that although the current quarks are confined, their mass values are the central drivers of the energies which do pass in and out of nuclides and nucleons during fusion and fission events. So while nucleons do confine quarks, *they do not confine energies*, and the energies they release are driven directly by the current quark masses. Thus one can acquire some qualitative comfort with a Compton wavelength that extends beyond r_Λ by over 1 order of magnitude given that the same wavelength drives the energies which also bleed out from the nucleons. So we set aside playing "Hamlet" over what $\psi(\mathbf{x})$ to use, we keep in mind that different $\psi(\mathbf{x})$ can be tried and that this might be an interesting exercise, and we go into $\frac{1}{2}E=\iiint d^3x\overline{\psi}\sigma_{\mu\nu}\psi\overline{\psi}\sigma_{\mu\nu}\psi/m^2$ with a radially-symmetric Gaussian and with the Compton wavelengths of the current quarks masses setting the spatial spread and see what comes out.

So, we set $\psi(r)=u(p)\left(\pi/m^2\right)^{-.75}\exp\left(-\frac{1}{2}m^2\left(r-r_0\right)^2\right)$ in $\frac{1}{2}E=\iiint d^3x\overline{\psi}\sigma_{\mu\nu}\psi\overline{\psi}\sigma_{\mu\nu}\psi/m^2$ four times which yields fourth powers of the terms in $\psi(r)$, and remove the space-independent terms from the integral. We then make use of the solution $\iiint d^3x \exp\left(-2m^2\left(r-r_0\right)^2\right)=\left(\pi/2\right)^{\frac{3}{2}}/m^3$ for the Gaussian integral, and finally reduce. Thus:

$$\begin{aligned}\tfrac{1}{2}E &= \iiint d^3x \frac{\overline{\psi}\sigma_{\mu\nu}\psi\overline{\psi}\sigma_{\mu\nu}\psi}{m^2} = \frac{1}{m^2}\left(\frac{\pi}{m^2}\right)^{-3}\overline{u}\sigma_{\mu\nu}u\overline{u}\sigma_{\mu\nu}u\iiint d^3x \exp\left(-2m^2\left(r-r_0\right)^2\right) \\ &= \frac{1}{m^2}\left(\frac{m^2}{\pi}\right)^3\left(\frac{\pi}{2}\right)^{\frac{3}{2}}\frac{1}{m^3}\overline{u}\sigma_{\mu\nu}u\overline{u}\sigma_{\mu\nu}u = \frac{m}{\left(2\pi\right)^{\frac{3}{2}}}\overline{u}\sigma_{\mu\nu}u\overline{u}\sigma_{\mu\nu}u\end{aligned} \qquad (7.15)$$

So we see how the this integration converts the pure terms and also injects a $\left(2\pi\right)^{\frac{3}{2}}$ divisor via $\overline{\psi}_u\sigma_{\mu\nu}\psi_u\overline{\psi}_u\sigma_{\mu\nu}\psi_u/m_u{}^2 \Rightarrow m_u/\left(2\pi\right)^{\frac{3}{2}}$ and $\overline{\psi}_d\sigma_{\mu\nu}\psi_d\overline{\psi}_d\sigma_{\mu\nu}\psi_d/m_d{}^2 \Rightarrow m_d/\left(2\pi\right)^{\frac{3}{2}}$. The $\left(2\pi\right)^{\frac{3}{2}}$ which was laced throughout the empirical calculations in sections 3 through 6 is seen to have its fundamental mathematical origins in $\iiint d^3x \exp\left(-.5Ax^2\right)=\left(2\pi/A\right)^{\frac{3}{2}}$ which is the three-space Gaussian integral. And we see that for some different, not-Gaussian normalizable $\psi\left(\mathbf{x}\right)$ with a fourth-power integral $\iiint d^3x f\left(\mathbf{x}\right)=M$, whatever factor appears in place of $\left(2\pi\right)^{\frac{3}{2}}$ would be driven by *M*. Beyond $m/\left(2\pi\right)^{\frac{3}{2}}$, because of the Dirac spinors being a function $u\left(m,\mathbf{p}\right)$, the remaining term $\overline{u}\sigma_{\mu\nu}u\overline{u}\sigma_{\mu\nu}u$ in (7.15) above is a function only of mass *m* and momentum **p**. The Dirac spinors are subject to normalization and this normalization can be *chosen*. So we should choose the spinor normalization such that the energy number in the resultant $\tfrac{1}{2}E=m/\left(2\pi\right)^{\frac{3}{2}}\cdot\overline{u}\sigma_{\mu\nu}u\overline{u}\sigma_{\mu\nu}u$ makes sense in relation to an observed energy or energies.

So we return to (7.14) which contain only pure up / up and down / down terms, and so can make use of (7.15). Specifically, combining (7.14) and (7.15) enables us to write:

$$\begin{aligned}E_\Delta &\equiv E_{V\,N\mathrm{eff}\,0}-E_{V\,N\mathrm{eff}\,0}=\iiint d^3x\tfrac{1}{2}\mathrm{Tr}F_{V\,N\mathrm{eff}\,0\mu\nu}\otimes F_{V\,N\mathrm{eff}\,0}{}^{\mu\nu}-\iiint d^3x\tfrac{1}{2}\mathrm{Tr}F_{V\,P\mathrm{eff}\,0\mu\nu}\otimes F_{V\,P\mathrm{eff}\,0}{}^{\mu\nu} \\ &=2\left[3\iiint d^3x\left(\frac{\overline{\psi}_d\sigma_{\mu\nu}\psi_d}{m_d}\frac{\overline{\psi}_d\sigma^{\mu\nu}\psi_d}{m_d}\right)-3\iiint d^3x\left(\frac{\overline{\psi}_u\sigma_{\mu\nu}\psi_u}{m_u}\frac{\overline{\psi}_u\sigma^{\mu\nu}\psi_u}{m_u}\right)\right] \\ &=2\left(\frac{3}{\left(2\pi\right)^{\frac{3}{2}}}m_d\overline{u}_d\sigma_{\mu\nu}u_d\overline{u}_d\sigma_{\mu\nu}u_d-\frac{3}{\left(2\pi\right)^{\frac{3}{2}}}m_u\overline{u}_u\sigma_{\mu\nu}u_u\overline{u}_u\sigma_{\mu\nu}u_u\right)\end{aligned} \qquad (7.16)$$

This E_Δ represents the energy difference between $E_{V\mathrm{eff}\,0}=\iiint d^3x\tfrac{1}{2}\mathrm{Tr}F_{V\mathrm{eff}\,0\mu\nu}\otimes F_{V\mathrm{eff}\,0}{}^{\mu\nu}$ for the neutron and proton vector (*V*), monopole-effective, zero-recursive-order pure field strengths (7.11) and (7.10). And it will be seen that if we normalize the Dirac spinors *such that* $\overline{u}_d\sigma_{\mu\nu}u_d\overline{u}_d\sigma_{\mu\nu}u_d=\overline{u}_u\sigma_{\mu\nu}u_u\overline{u}_u\sigma_{\mu\nu}u_u=\tfrac{1}{2}$, that (7.16) will reduce to:

$$E_\Delta = E_{V\,N\mathrm{eff}\,0} - E_{V\,P\mathrm{eff}\,0} = \frac{3}{(2\pi)^{\frac{3}{2}}}(m_d - m_u)\,, \tag{7.17}$$

which is (5.3) a.k.a. the primary relationship (3.1) upon which all of the empirical results from section 3 onward were based.

Now, as was stated after (5.3), and as may be reviewed in section 11 and specifically [11.21] of [1], the author first evaluated (7.16) and (7.17) using the PDG data $m_u = 2.3^{+0.7}_{-0.5}\,\mathrm{MeV}$ and $m_d = 4.8^{+0.5}_{-0.3}\,\mathrm{MeV}$ and its error bar ranges to deduce that $.286\,\mathrm{MeV}<\mathrm{E}_\Delta < .704\,\mathrm{MeV}$, with a median value of $\mathrm{E}_\Delta = .495\,\mathrm{MeV}$ which is only about 3% off from the electron rest mass based on PDG data with error bars much larger than 3%. The author then hypothesized for further confirmation which was subsequently successful in the other ways enumerated section 6, that *this energy* $E_\Delta = E_{V\,N\mathrm{eff}\,0} - E_{V\,P\mathrm{eff}\,0}$ *is in fact equal to the electron rest mass* because in the zero-recursion abelian limit where $G_\mu\left((0)\right)_0 = \left(k_\tau k^\tau - m^2 + i\varepsilon\right)^{-1} J_\mu$, all of the interaction which gives rise to the observed neutron minus proton mass difference has been turned off. Thus (7.17) is a relationship which contains only a "signal" for bare current quarks without "noise." And with only signal and no noise, it is sensible that the neutron "signal mass" would differ from the proton "signal mass" by precisely the mass of the electron.

So this data concurrence motivated the author to set $m_e \equiv E_\Delta = E_{V\,N\mathrm{eff}\,0} - E_{V\,P\mathrm{eff}\,0}$ by definitional hypothesis, which then mandates $\overline{u}_d \sigma_{\mu\nu} u_d \overline{u}_d \sigma_{\mu\nu} u_d = \overline{u}_u \sigma_{\mu\nu} u_u \overline{u}_u \sigma_{\mu\nu} u_u = \tfrac{1}{2}$ for normalization because this is what reduces (7.16) to (7.17) which then enables the empirically-accurate definition $m_e \equiv E_\Delta = E_{V\,N\mathrm{eff}\,0} - E_{V\,P\mathrm{eff}\,0}$. When we then calculate out the consequence of this "empirical normalization," we find in [11.29] of [1] that the quark normalization coefficient has the form $N^2 = \tfrac{1}{\sqrt{4!}}(E+m)/2m$, and specifically, that $N_u{}^2 = \tfrac{1}{\sqrt{4!}}(E_u + m_u)/2m_u$ and $N_d{}^2 = \tfrac{1}{\sqrt{4!}}(E_d + m_d)/2m_d$ for the up and down quark spinors respectively, based on the conventional definition $u^{(s)T} \equiv N\left(\chi^{(s)} \quad \chi^{(s)}\boldsymbol{\sigma}\cdot\mathbf{p}/(E+m)\right)$. It is also of interest as discussed in Figure 3 of [1] that by empirically matching up (7.17) with the electron via $m_e \equiv E_\Delta$ the deduced $4!$ constant in the divisor of the normalization coefficient happens to coincide with the precise number of fermions known in nature: 4=3+1 colors of quark plus lepton times 3 generations times 2 isospin states up and down.

So if $E_\Delta = E_{V\,N\mathrm{eff}\,0} - E_{V\,P\mathrm{eff}\,0}$ appears to produce a close empirical result, one might expect each of the neutron and proton signal energies $E_{V\,N\mathrm{eff}\,0}$ and $E_{V\,P\mathrm{eff}\,0}$ to also have some meaning in relation to something that is observed, so the next step is to study these energies. But as noted after (7.14), the mixed energy $\tfrac{1}{2}E_{ud} = \iiint d^3x \overline{\psi}_u \sigma_{\mu\nu}\psi_u \overline{\psi}_d \sigma_{\mu\nu}\psi_d / m_u m_d$ needs to now be calculated because these up / down mixed integrands appear in (7.12) and (7.13) for the proton and neutron field strengths. So similarly to (7.15), we use $\psi_{u,d}(r) = u_{u,d}(p)\left(\pi/m_{u,d}{}^2\right)^{-.75}\exp\left(-\tfrac{1}{2}m_{u,d}{}^2(r-r_0)^2\right)$ now explicitly quark-labelled because we need to distinguish up from down quarks to calculate the mixed energy. Here, after solving the Gaussian and reducing and separately isolating a term $\sqrt{m_u m_d}$ with mass dimensionality of +1 we obtain:

$$\begin{aligned}
\tfrac{1}{2}E_{ud} &= \iiint d^3x \frac{\overline{\psi}_u \sigma_{\mu\nu}\psi_u \overline{\psi}_d \sigma_{\mu\nu}\psi_d}{m_u m_d} \\
&= \frac{1}{m_u m_d}\left(\frac{\pi}{m_u^{\,2}}\right)^{-\frac{3}{2}}\left(\frac{\pi}{m_d^{\,2}}\right)^{-\frac{3}{2}} \overline{u}_u \sigma_{\mu\nu} u_u \overline{u}_d \sigma_{\mu\nu} u_d \iiint d^3x \exp\left(-\left(m_u^{\,2}+m_d^{\,2}\right)\left(r-r_0\right)^2\right) \\
&= \frac{1}{m_u m_d}\left(\frac{m_u^{\,2}}{\pi}\right)^{\frac{3}{2}}\left(\frac{m_d^{\,2}}{\pi}\right)^{\frac{3}{2}}\left(\frac{\pi}{m_u^{\,2}+m_d^{\,2}}\right)^{\frac{3}{2}} \overline{u}_u \sigma_{\mu\nu} u_u \overline{u}_d \sigma_{\mu\nu} u_d \\
&= \frac{\sqrt{m_u m_d}}{\pi^{\frac{3}{2}}}\left(\frac{m_u m_d}{m_u^{\,2}+m_d^{\,2}}\right)^{\frac{3}{2}} \overline{u}_u \sigma_{\mu\nu} u_u \overline{u}_d \sigma_{\mu\nu} u_d
\end{aligned} \quad . \tag{7.18}$$

To solve the Gaussian we start with the solution $\iiint d^3x \exp\left(-m^2\left(r-r_0\right)^2\right) = \pi^{\frac{3}{2}}/m^3$ and may then obtain $\iiint d^3x \exp\left(-\left(m_u^{\,2}+m_d^{\,2}\right)\left(r-r_0\right)^2\right) = \pi^{\frac{3}{2}}/\left(m_u^{\,2}+m_d^{\,2}\right)^{\frac{3}{2}}$ by the variable scaling substitution $m^2 \to m_u^{\,2}+m_d^{\,2}$ thus $m^3 \to \left(m_u^{\,2}+m_d^{\,2}\right)^{\frac{3}{2}}$. As a check on the calculation we see that in the special case where $m_u = m_d \equiv m$, the result in (7.18) will coincide identically that in (7.15).

Now, the dimensionless term $\left(m_u m_d / \left(m_u^{\,2}+m_d^{\,2}\right)\right)^{\frac{3}{2}}$ from which we have separated the +1 dimensional $\sqrt{m_u m_d}$ looks a bit complicating at first. But we recall that in electroweak theory there are similar expressions of the form $m_u m_d / \left(m_u^{\,2}+m_d^{\,2}\right)$. Specifically, we recall that in electroweak theory $g_w \sin\theta_W = g_y \cos\theta_W = e$ where e is the electric charge, g_w the weak charge, g_y the weak hypercharge, and θ_W is of course the weak mixing angle. And we recall that in the course of calculating from this one arrives at $\sin\theta_W \cos\theta_W = g_w g_y / \left(g_w^{\,2}+g_y^{\,2}\right)$ where $g_Z^{\,2} \equiv g_w^{\,2}+g_y^{\,2}$ is the charge strength of the Z boson with a mass $M_Z = \tfrac{1}{2} v_F g_Z$ where v_F is the Fermi vev. So the $m_u m_d / \left(m_u^{\,2}+m_d^{\,2}\right)$ above seems suggestive that there is an analogous mixing angle rotating between the up and down quark masses. Let us now explore this connection *which the author has not presented explicitly in any earlier papers*. As the discussion of this angle proceeds, the reader may find it helpful to refer to Figure 3 following (8.15) below.

8. First Generation Quark Mass Mixing

Analogously to electroweak theory, we postulate a first generation quark mass mixing angle θ and mass m_1 *defined* such that:

$$m_d \sin\theta \equiv m_u \cos\theta \equiv m_1 . \tag{8.1}$$

So immediately, because $\tan\theta = m_u / m_d$, we may draw a right triangle with m_u on the leg opposite and m_d on the leg adjacent θ, and thus with $\sqrt{m_u{}^2 + m_d{}^2}$ on the hypotenuse. Therefore $\sin\theta = m_u / \sqrt{m_u{}^2 + m_d{}^2}$, $\cos\theta = m_d / \sqrt{m_u{}^2 + m_d{}^2}$ and thus:

$$\sin\theta\cos\theta = \frac{m_u m_d}{m_u{}^2 + m_d{}^2} = \frac{m_u m_d}{m_\zeta{}^2} \tag{8.2}$$

which is identical to the factor to the 3/2 power that appeared in (7.18). In the above we have defined $m_\zeta{}^2 \equiv m_u{}^2 + m_d{}^2$ simply for convenience, and used the Greek zeta to remind us of the analogy to the electroweak $g_Z{}^2 \equiv g_w{}^2 + g_y{}^2$. So we can use (8.2) to remove the masses from this factor, and instead express it in terms of θ, thus:

$$\tfrac{1}{2} E_{ud} = \iiint d^3x \frac{\overline{\psi}_u \sigma_{\mu\nu} \psi_u \overline{\psi}_d \sigma_{\mu\nu} \psi_d}{m_u m_d} = \frac{\sqrt{m_u m_d}}{\pi^{\frac{3}{2}}} \left(\sin\theta\cos\theta\right)^{\frac{3}{2}} \overline{u}_u \sigma_{\mu\nu} u_u \overline{u}_d \sigma_{\mu\nu} u_d . \tag{8.3}$$

If (3.3) and (3.4) are indeed the empirical $Q = 0$ quark masses in the EPN measurement scheme discussed section 4, then these can be used to deduce tan θ = 0.453 236 693, therefore the mixing angle θ = 24.381 777 8°. Additionally, $m_\zeta = \sqrt{m_u{}^2 + m_d{}^2} = 0.005\,783\,076\text{ u} = 5.386\,90110\text{ MeV}$ may be deduced.

At this point, we have all that we need to return to (7.12) and (7.13), use them as integrands in $E = \iiint d^3x \tfrac{1}{2}\mathrm{Tr}F_{\mu\nu} \otimes F^{\mu\nu}$ for each of the proton $F_{V\,P\mathrm{eff}\,0}$ and the neutron $F_{V\,N\mathrm{eff}\,0}$ and thereby calculate associated energies $E_{V\,P\mathrm{eff}\,0}$ and $E_{V\,P\mathrm{eff}\,0}$. Inserting (7.15) for both the up and down quarks and (8.3) into (7.13) and (7.14) we obtain:

$$\begin{aligned} E_{V\,P\mathrm{eff}\,0} &= \iiint d^3x \tfrac{1}{2}\mathrm{Tr}F_{V\,P\mathrm{eff}\,0\mu\nu} \otimes F_{V\,P\mathrm{eff}\,0}{}^{\mu\nu} \\ &= 2\iiint d^3x \left(\frac{\overline{\psi}_d \sigma_{\mu\nu} \psi_d}{m_d} \frac{\overline{\psi}_d \sigma^{\mu\nu} \psi_d}{m_d} + 4 \frac{\overline{\psi}_u \sigma_{\mu\nu} \psi_u}{m_u} \frac{\overline{\psi}_d \sigma^{\mu\nu} \psi_d}{m_d} + 4 \frac{\overline{\psi}_u \sigma_{\mu\nu} \psi_u}{m_u} \frac{\overline{\psi}_u \sigma^{\mu\nu} \psi_u}{m_u} \right) \\ &= 2\left(\frac{m_d}{(2\pi)^{\frac{3}{2}}} \overline{u}_d \sigma_{\mu\nu} u_d \overline{u}_d \sigma_{\mu\nu} u_d + 4 \frac{\sqrt{m_u m_d}}{\pi^{\frac{3}{2}}} \left(\sin\theta\cos\theta\right)^{\frac{3}{2}} \overline{u}_u \sigma_{\mu\nu} u_u \overline{u}_d \sigma_{\mu\nu} u_d + 4 \frac{m_u}{(2\pi)^{\frac{3}{2}}} \overline{u}_u \sigma_{\mu\nu} u_u \overline{u}_u \sigma_{\mu\nu} u_u \right) \end{aligned} ,\tag{8.4}$$

$$\begin{aligned} E_{V\,N\mathrm{eff}\,0} &= \iiint d^3x \tfrac{1}{2}\mathrm{Tr}F_{V\,N\mathrm{eff}\,0\mu\nu} \otimes F_{V\,N\mathrm{eff}\,0}{}^{\mu\nu} \\ &= 2\iiint d^3x \left(\frac{\overline{\psi}_u \sigma_{\mu\nu}\psi_u}{m_u}\frac{\overline{\psi}_u \sigma^{\mu\nu}\psi_u}{m_u} + 4\frac{\overline{\psi}_u \sigma_{\mu\nu}\psi_u}{m_u}\frac{\overline{\psi}_d \sigma^{\mu\nu}\psi_d}{m_d} + 4\frac{\overline{\psi}_d \sigma_{\mu\nu}\psi_d}{m_d}\frac{\overline{\psi}_d \sigma^{\mu\nu}\psi_d}{m_d} \right) \\ &= 2\left(\frac{m_u}{(2\pi)^{\frac{3}{2}}}\overline{u}_u\sigma_{\mu\nu}u_u\overline{u}_u\sigma_{\mu\nu}u_u + 4\frac{\sqrt{m_u m_d}}{\pi^{\frac{3}{2}}}(\sin\theta\cos\theta)^{\frac{3}{2}}\overline{u}_u\sigma_{\mu\nu}u_u\overline{u}_d\sigma_{\mu\nu}u_d + 4\frac{m_d}{(2\pi)^{\frac{3}{2}}}\overline{u}_d\sigma_{\mu\nu}u_d\overline{u}_d\sigma_{\mu\nu}u_d \right) \end{aligned} \quad .(8.5)$$

Next we apply the empirical normalization $\overline{u}_d\sigma_{\mu\nu}u_d\overline{u}_d\sigma_{\mu\nu}u_d = \overline{u}_u\sigma_{\mu\nu}u_u\overline{u}_u\sigma_{\mu\nu}u_u = \tfrac{1}{2}$ used after (7.17) to conform the deduced energy difference $E_\Delta = E_{V\,N\mathrm{eff}\,0} - E_{V\,P\mathrm{eff}\,0}$ with the electron rest mass via $m_e \equiv E_\Delta$ which results in $N_u{}^2 = \tfrac{1}{\sqrt{4!}}(E_u + m_u)/2m_u$ and $N_d{}^2 = \tfrac{1}{\sqrt{4!}}(E_d + m_d)/2m_d$. So this means that in the mixed term $N_{ud}{}^2 = \tfrac{1}{\sqrt{4!}}\sqrt{(E_u + m_u)(E_d + m_d)/(2m_u)(2m_d)}$ turns out to be the normalization which emerges from the square root of the product of these individual quark normalizations via (8.3), and this in turn means that there is a like-normalization $\overline{u}_u\sigma_{\mu\nu}u_u\overline{u}_d\sigma_{\mu\nu}u_d = \tfrac{1}{2}$ for the mixed term found in (8.3). Applying all of these normalizations in (8.4) and (8.5) now leads us to:

$$E_{V\,P\mathrm{eff}\,0} = \iiint d^3x \tfrac{1}{2}\mathrm{Tr}F_{V\,P\mathrm{eff}\,0\mu\nu} \otimes F_{V\,P\mathrm{eff}\,0}{}^{\mu\nu} = \frac{m_d}{(2\pi)^{\frac{3}{2}}} + 4\frac{\sqrt{m_u m_d}}{\pi^{\frac{3}{2}}}(\sin\theta\cos\theta)^{\frac{3}{2}} + 4\frac{m_u}{(2\pi)^{\frac{3}{2}}}, \quad (8.6)$$

$$E_{V\,N\mathrm{eff}\,0} = \iiint d^3x \tfrac{1}{2}\mathrm{Tr}F_{V\,N\mathrm{eff}\,0\mu\nu} \otimes F_{V\,N\mathrm{eff}\,0}{}^{\mu\nu} = \frac{m_u}{(2\pi)^{\frac{3}{2}}} + 4\frac{\sqrt{m_u m_d}}{\pi^{\frac{3}{2}}}(\sin\theta\cos\theta)^{\frac{3}{2}} + 4\frac{m_d}{(2\pi)^{\frac{3}{2}}}. \quad (8.7)$$

For the special case $\theta = \pi/4 = 45°$, we have $(\sin\theta\cos\theta)^{\frac{3}{2}} = 1/2^{\frac{3}{2}}$, and these will reduce to:

$$E_{V\,P\mathrm{eff}\,0} = \iiint d^3x \tfrac{1}{2}\mathrm{Tr}F_{V\,P\mathrm{eff}\,0\mu\nu} \otimes F_{V\,P\mathrm{eff}\,0}{}^{\mu\nu} = \frac{1}{(2\pi)^{\frac{3}{2}}}\left(m_d + 4\sqrt{m_u m_d} + 4m_u\right), \quad (8.8)$$

$$E_{V\,N\mathrm{eff}\,0} = \iiint d^3x \tfrac{1}{2}\mathrm{Tr}F_{V\,N\mathrm{eff}\,0\mu\nu} \otimes F_{V\,N\mathrm{eff}\,0}{}^{\mu\nu} = \frac{1}{(2\pi)^{\frac{3}{2}}}\left(m_u + 4\sqrt{m_u m_d} + 4m_d\right). \quad (8.9)$$

These are now identical with (5.1) and (5.2), which then led in (5.8) and (5.9) to the missing mass average $\tfrac{1}{2}(\Delta_P + \Delta_N) = 8.714\,9941\,\mathrm{MeV}$ at the empirical peak in the nuclear binding curve of Figure 2 and the 99.9710% match to the ^{56}Fe binding energy and an understanding of how this relates to quark confinement and nuclear binding and to the toolkit masses m_u, m_d, $\sqrt{m_u m_d}$ and the foregoing divided by $(2\pi)^{\frac{3}{2}}$. This then exploded into the plethora of empirical matches enumerated in section 6 culminating in the neutron minus proton mass difference in (3.2) which was then elevated into a primary relationship and used in combination with (3.1) to deduce the very precise up and down quark masses (3.3) and (3.4). And this further led once the Fermi vev v_F and the CKM mixing matrix are brought to bear, to the proton and neutron masses themselves *within all experimental errors*. So it is abundantly clear that (8.8) and (8.9) can be connected tightly with and indeed are the springboard to a whole wealth of nuclear energy data, and thus are empirically-accurate relationships to high degrees of precision. But there is only one problem: to get from (8.6) and (8.7) to the empirically-validated (8.8) and (8.9) we employed $\theta = \pi / 4 = 45°$. But from the definitions (8.1) and (8.2) and the quark masses (3.3) and (3.4) which are one of the consequences of (8.8) and (8.9), we found that θ = 24.381 777 8°, not 45°. So what do we do?

We *defined* θ in (8.1) in a manner which ensured based on the current quark masses (3.3) and (3.4) that it would be equal to θ = 24.381 777 8°. But as we see from (8.8) and (8.9) and all the development in sections 5 and 6, it is $\theta = \pi / 4 = 45°$ which in fact matches the empirical data. So if θ so-defined *does not match* the empirical data, but if we also now know that the up and down quark masses *do mix* over a circle with a hypotenuse radius $m_\zeta = \sqrt{m_u{}^2 + m_d{}^2} = 5.386\,90110\,\text{MeV}$ and that $\sqrt{m_u m_d}$ *is in general multiplied by the factor* $(\sin\theta\cos\theta)^{\frac{3}{2}}$ which specializes to $(\sin\theta\cos\theta)^{\frac{3}{2}} = 1/2^{\frac{3}{2}}$ for $\theta = \pi / 4 = 45°$, then that means that we need to retain the mass mixing over the circle with mass radius m_ζ but change (rotate) the definition of our angle to match the empirical data. That is, the empirical data suggests that we are correct that there is a mixing of the up and down masses via a mixing angle, but are incorrect about how we defined this angle in (8.1). So we now need to redefine our angle to match the empirical data. How?

In addition to θ, let us introduce a new angle ϕ, defined such $\phi = 0$ when the current quark masses are (3.3) and (3.4). That is, we define $\phi \equiv 0$ to be the mixing angle associated with the $Q = 0$ current quark masses (3.3) and (3.4). So likewise by implication, $\phi = 0$ is the associated angle for all of the empirical data developed and enumerated in sections 3 through 6. Then, because (8.2) and (8.3) teach that there *is* a rotation occurring between the up and down quark masses which maintains a $m_\zeta = 5.386\,9011\,\text{MeV}$ hypotenuse, we shall define ϕ by way of the mixing relationship:

$$\begin{pmatrix} m'_u = m_u(Q) \\ m'_d = m_d(Q) \end{pmatrix} \equiv \begin{pmatrix} \cos\phi(Q) & \sin\phi(Q) \\ -\sin\phi(Q) & \cos\phi(Q) \end{pmatrix} \begin{pmatrix} m_u(0) \\ m_d(0) \end{pmatrix}. \tag{8.10}$$

As specified, for $\phi = 0$ this definition produces $m'_u = m_u$ and $m'_d = m_d$ which are also the Q=0 quark masses. *This now replaces the definition of θ in (8.1), which we now withdraw in favor of (8.10).* There is, of course, still a rotation between the quark masses of the exact same form produced by (8.1), and $m_\zeta = 5.386\,9011\,\text{MeV}$ is still maintained as the hypotenuse of rotation. But we are no longer tied to a tan θ = 0.453 236 693 and θ = 24.381 777 8° which is a mismatch with the empirical data. In fact, as we indicate above and will shortly elaborate after some further mathematical development, both θ and ϕ need to be understood not as fixed angles, but as *variable angles with run with Q*, i.e., as $\theta(Q)$ and $\phi(Q)$, which thus help to specify the behaviors of *all* of the empirical data previously developed as a function of Q for Q>0, including the running. Most directly, $m'_u = m_u(Q)$ and $m'_d = m_d(Q)$ now specify the running of the up and down quarks masses as a function of the renormalization scale / impact energy *Q*.

Now, with the definitions (8.1) and thus the constraint *θ* = 24.381 777 8° no longer in force, we revert to (8.6) and (8.7) keeping in mind that $\theta = \pi/4 = 45°$ leads to (8.8) and (8.9) and many correct empirical matches. So we now define $\theta \equiv \pi/4 + \phi$ as the general relationship between *θ* and *ϕ* in each of (8.6) and (8.7), which is to say, we simply define *ϕ* to be equal to *θ* less 45 degrees. Via basic trigonometric angle addition formulae we find that $\sin(\pi/4+\phi) = \frac{1}{\sqrt{2}}(\cos\phi + \sin\phi)$ and $\cos(\pi/4+\phi) = \frac{1}{\sqrt{2}}(\cos\phi - \sin\phi)$ and therefore that $\sin\theta\cos\theta = \sin(\pi/4+\phi)\cos(\pi/4+\phi) = \frac{1}{2}(\cos^2\phi - \sin^2\phi)$. Consequently, we may use $(\sin\theta\cos\theta)^{\frac{3}{2}} = (1/2^{\frac{3}{2}})(\cos^2\phi - \sin^2\phi)^{\frac{3}{2}}$ in (8.6) and (8.7) to write:

$$E_{V\,P\mathrm{eff}\,0} = \iiint d^3x \tfrac{1}{2}\mathrm{Tr}F_{V\,P\mathrm{eff}\,0\mu\nu} \otimes F_{V\,P\mathrm{eff}\,0}{}^{\mu\nu} = \frac{1}{(2\pi)^{\frac{3}{2}}}\left(m_d + 4\sqrt{m_u m_d}\left(\cos^2\phi - \sin^2\phi\right)^{\frac{3}{2}} + 4m_u\right), \tag{8.11}$$

$$E_{V\,N\mathrm{eff}\,0} = \iiint d^3x \tfrac{1}{2}\mathrm{Tr}F_{V\,N\mathrm{eff}\,0\mu\nu} \otimes F_{V\,N\mathrm{eff}\,0}{}^{\mu\nu} = \frac{1}{(2\pi)^{\frac{3}{2}}}\left(m_u + 4\sqrt{m_u m_d}\left(\cos^2\phi - \sin^2\phi\right)^{\frac{3}{2}} + 4m_d\right). \tag{8.12}$$

Now the empirically-supported (8.8) and (8.9) are more transparently visible, and when $\phi = 0$, these will reduce identically to (8.8) and (8.9).

Now that we have simply use a different angle *ϕ* rotated clockwise by 45° from *θ* in the formulae for $E_{V\,P\mathrm{eff}\,0}$ and $E_{V\,N\mathrm{eff}\,0}$ to translate (8.6) and (8.7) into the more-transparent (8.11) and (8.12) we could, if we wish, go back to reintroduce the withdrawn definition (8.1) slightly differently, by defining yet a third angle η in the form of $m_d \sin\eta \equiv m_u \cos\eta \equiv m_1$, with the consequence that $\tan\eta = m_u / m_d$ and *η* = 24.381 777 8°, compare after (8.3). This *η* is a different angle from $\theta \equiv \pi/4+\phi$, and it does specify the empirical m_u / m_d ratio for the *Q*=0 up and down current quark masses. Then, if we wanted to ask how this *η* definition transforms as function of $\theta \equiv \pi/4+\phi$ both of which run as function of *Q* and indeed are *parameterizations of Q*, we would transform $m_d \sin\eta \equiv m_u \cos\eta \equiv m_1$ to $m'_d \sin\eta' \equiv m'_u \cos\eta' \equiv m'_1$ and use (8.10) to substitute m'_u, and m'_d. Thus:

$$\frac{m_u m_d}{m_u{}^2 + m_d{}^2} = \frac{m_u m_d}{m_\zeta{}^2} = \sin\eta\cos\eta \tag{8.13}$$

now replaces (8.2), and *η* = 24.381 777 8° which is the magnitude previously assigned to *θ* from the initial definition (8.1). To relate back to the redefined angle *θ* we may then also use $\phi = \theta - \pi/4$, apply the angle difference identities and consolidate. All this teaches that:

$$\begin{aligned} & m_d \sin\eta = m_u \cos\eta \Rightarrow m'_d \sin\eta' = m'_u \cos\eta' \\ &= \left(m_d \cos\phi - m_u \sin\phi\right)\sin\eta' = \left(m_u \cos\phi + m_d \sin\phi\right)\cos\eta' \\ &= \tfrac{1}{\sqrt{2}}\left(\left(m_d - m_u\right)\sin\theta + \left(m_d + m_u\right)\cos\theta\right)\sin\eta' = \tfrac{1}{\sqrt{2}}\left(\left(m_d + m_u\right)\sin\theta - \left(m_d - m_u\right)\cos\theta\right)\cos\eta' \end{aligned} \tag{8.14}$$

Therefore, the mass ratio angle η transforms $\eta \to \eta'$ with changing ϕ and θ and so also runs with Q according to:

$$\tan\eta = \frac{m_u}{m_d} \to \tan\eta' = \frac{m'_u}{m'_d} = \frac{m_u \cos\phi + m_d \sin\phi}{m_d \cos\phi - m_u \sin\phi} = \frac{\left(m_u - m_d\right)\cos\theta + \left(m_d + m_u\right)\sin\theta}{\left(m_d + m_u\right)\cos\theta + \left(m_d - m_u\right)\sin\theta} \tag{8.15}$$

All of the foregoing assignments of the angles ϕ, θ and η and their interrelationships of these angles with one another as well as with the quark masses m_u and m_d and the circle radius $m_\zeta = \sqrt{m_u{}^2 + m_d{}^2}$ and the renormalization energy Q as will be discussed further momentarily, are illustrated in Figure 3 below:

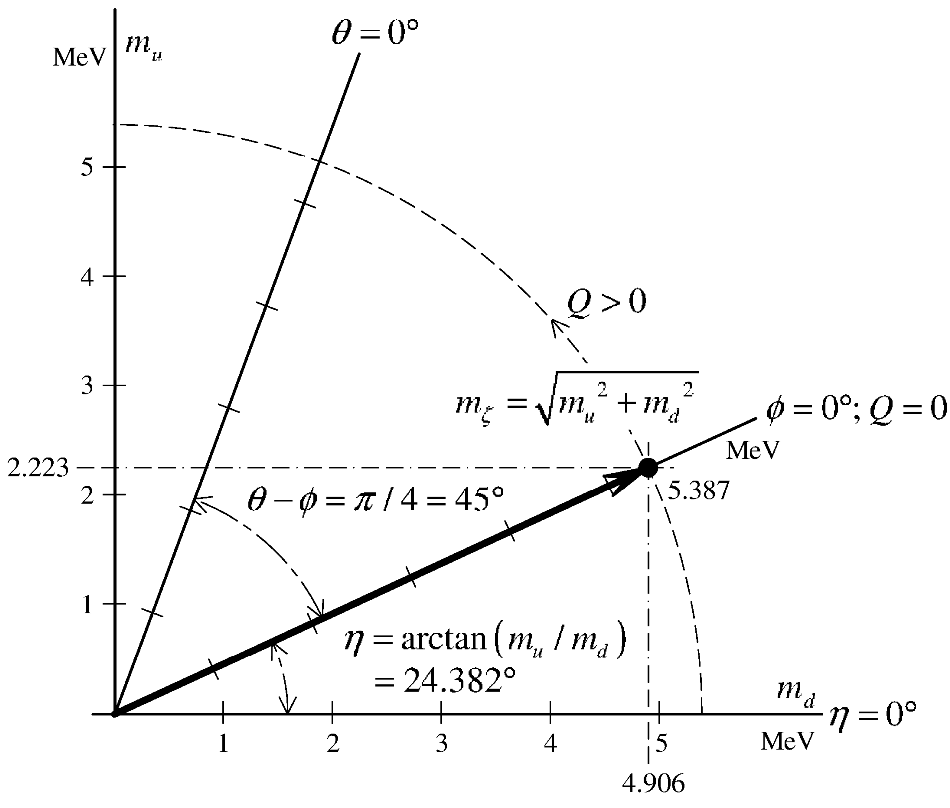

Figure 3: First Generation Quark Mass Mixing

Finally, to complete this development so we may turn from mathematics to physics, we may also use (8.10) in (8.11) and (8.12) to represent the transformation of the proton and neutron energies with Q, $E_{V\,P\mathrm{eff}\,0}(0) \to E'_{V\,P\mathrm{eff}\,0} = E_{V\,P\mathrm{eff}\,0}(Q)$ and $E_{V\,N\mathrm{eff}\,0}(0) \to E'_{V\,N\mathrm{eff}\,0} = E_{V\,N\mathrm{eff}\,0}(Q)$:

$$\begin{aligned}
&E_{V\,P\text{eff}\,0}(0)=\frac{1}{(2\pi)^{\frac{3}{2}}}\left(m_d+4\sqrt{m_u m_d}\left(\cos^2\phi-\sin^2\phi\right)^{\frac{3}{2}}+4m_u\right)\\
&\to E'_{V\,P\text{eff}\,0}=E_{V\,P\text{eff}\,0}(Q)=\frac{1}{(2\pi)^{\frac{3}{2}}}\left(m'_d+4\sqrt{m'_u m'_d}\left(\cos^2\phi-\sin^2\phi\right)^{\frac{3}{2}}+4m'_u\right)\\
&=\frac{1}{(2\pi)^{\frac{3}{2}}}\left(\begin{array}{l}m_d\left(\cos\phi+4\sin\phi\right)+4\sqrt{m_u m_d\left(\cos^2\phi-\sin^2\phi\right)+\left(m_d{}^2-m_u{}^2\right)\sin\phi\cos\phi}\left(\cos^2\phi-\sin^2\phi\right)^{\frac{3}{2}}\\+m_u\left(4\cos\phi-\sin\phi\right)\end{array}\right)
\end{aligned},\tag{8.16}$$

$$\begin{aligned}
&E_{V\,N\text{eff}\,0}(0)=\frac{1}{(2\pi)^{\frac{3}{2}}}\left(m_u+4\sqrt{m_u m_d}\left(\cos^2\phi-\sin^2\phi\right)^{\frac{3}{2}}+4m_d\right)\\
&\to E'_{V\,N\text{eff}\,0}=E_{V\,N\text{eff}\,0}(Q)=\frac{1}{(2\pi)^{\frac{3}{2}}}\left(m'_u+4\sqrt{m'_u m'_d}\left(\cos^2\phi-\sin^2\phi\right)^{\frac{3}{2}}+4m'_d\right)\\
&=\frac{1}{(2\pi)^{\frac{3}{2}}}\left(\begin{array}{l}m_u\left(\cos\phi-4\sin\phi\right)+4\sqrt{m_u m_d\left(\cos^2\phi-\sin^2\phi\right)+\left(m_d{}^2-m_u{}^2\right)\sin\phi\cos\phi}\left(\cos^2\phi-\sin^2\phi\right)^{\frac{3}{2}}\\+m_d\left(4\cos\phi+\sin\phi\right)\end{array}\right)
\end{aligned}.\tag{8.17}$$

Similarly, we may even examine how the electron rest mass $m_e=E_\Delta$ in (7.17) a.k.a. (5.3) a.k.a. the primary relationship (3.1) transforms $m_e\to m'_e$ with ϕ. Here, we just use (8.10) in (7.17):

$$\begin{aligned}
m_e(0)&=\frac{3}{(2\pi)^{\frac{3}{2}}}\left(m_d-m_u\right)\to m'_e=m_e(Q)=\frac{3}{(2\pi)^{\frac{3}{2}}}\left(m'_d-m'_u\right)\\
&=\frac{3}{(2\pi)^{\frac{3}{2}}}\left(m_d\left(\cos\phi-\sin\phi\right)-m_u\left(\sin\phi+\cos\phi\right)\right)
\end{aligned},\tag{8.18}$$

So now we can finally go directly to the relationships (5.1), (5.2) and (3.1) which were the springboard for all of the other empirical connections outlined earlier. We start with m_u and m_d which by definition are the $Q=0$ quark masses which also by the definition (8.10) correspond to $\phi=0$. So we first ask: what happens when we set $\phi=0$? By (8.10) $m'_u=m_u$ and $m'_d=m_d$ and so (8.16) through (8.18) immediately reduce to:

$$E'_{V\,P\text{eff}\,0}=E_{V\,P\text{eff}\,0}=\frac{1}{(2\pi)^{\frac{3}{2}}}\left(m_d+4\sqrt{m_u m_d}+4m_u\right),\tag{8.19}$$

$$E'_{V\,N\text{eff}\,0}=E_{V\,N\text{eff}\,0}=\frac{1}{(2\pi)^{\frac{3}{2}}}\left(m_u+4\sqrt{m_u m_d}+4m_d\right),\tag{8.20}$$

$$m'_e=m_e=\frac{3}{(2\pi)^{\frac{3}{2}}}\left(m_d-m_u\right).\tag{8.21}$$

These are the foundational relationships upon which all of the empirical connections in sections 5 and 6 are based. But there is still a rotation which can occur through a non-zero angle ϕ which first appeared in (8.3) as $\theta = \pi/4 + \phi$ and in the more general case, the $Q = 0$ quark masses rotate via (8.10) through a circle with a mass hypotenuse m_ζ, the proton and neutron and electron energies transform via (8.16) through (8.18), and the mass ratio angle η transforms via (8.15). Now let's briefly review what we learn from (8.1) through (8.21), and then let's talk about the broader physics within which all of this fits.

By noticing that the $m_u m_d / \left(m_u^{\ 2} + m_d^{\ 2}\right)$ term which first emerged in (7.18) is analogous to a like-term $\sin\theta_W \cos\theta_W = g_w g_y / \left(g_w^{\ 2} + g_y^{\ 2}\right)$ which emerges in electroweak theory once we specify $g_w \sin\theta_W = g_y \cos\theta_W = e$, we are noticing that there is a similar type of mixing occurring between m_d and m_u via some angle θ as there is between g_w and g_y as there is via the electroweak mixing angle θ_W in electroweak theory. In (8.3) we see how this mixing enters in the form of the $\left(\sin\theta\cos\theta\right)^{\frac{3}{2}}$ factor. But we see in (8.8) and (8.9) that $\theta = \pi/4 = 45°$ is the specific angle which matches the empirical data, which contradicts the definition (8.1) from which we deduce θ = 24.381 777 8° from all of the empirical evidence reviewed earlier. So something must give, and in science, empirical validation certainly takes precedence over how we first define an angle.

To reconcile both ends of this seeming contradiction, we separate the appearance of $\sin\theta\cos\theta$ in (8.6) and (8.7) from its connection (8.2) to the quark masses because the empirically-accurate results differ from (8.6) and (8.7) simply by a rotation in the definition of the mixing angle against the quark masses. In other words, we treat $\sin\theta\cos\theta$ as being independent of its original moorings in (8.2), and allow it to be redefined so long as the redefinition takes place somewhere on the circle of radius $m_\zeta = \sqrt{m_u^{\ 2} + m_d^{\ 2}}$ which we now know exists mathematically. So we retain the rotations with radius m_ζ which we are tipped off about per above, and use a new angle $\phi \equiv \theta - \pi/4$ to define rotations from the observed current quark masses via (8.10) which then enters (8.11) and (8.12) in a fashion that is more transparent in relation to the empirical nuclear springboards (8.8) and (8.9). The original $m_u m_d / \left(m_u^{\ 2} + m_d^{\ 2}\right)$ which tipped us off to all of this now is redefined in (8.13) in terms of a new η = 24.381 777 8° angle. But let us see what we surmise about these angles themselves, because there is some interesting physics here, and because this bring us back full circle to the start of this paper when we first asked whether there was some sensible way to define Q=0 masses for the up and down current quarks when the current quarks are confined and so can never be directly observed without applying a Q>0, and indeed, a $Q > \Lambda_{\mathrm{QCD}}$. We established how this could be done with the Electron, Proton and Neutron (EPN) scheme in section 4, but have never gotten to the question – even with Q=0 masses properly established – of how these masses might run as we move up the Q scale.

When we first defined θ in (8.1), we were defining a simple ratio $\tan\theta = m_u / m_d$ of the up quark to the down quark mass at Q=0. There was nothing in this definition which might tell us how these masses run with Q. But we also saw in (8.3) and especially (8.6) and (8.7) that there is some mass mixing going on. And we know that in the two other known instances of mass mixing – via the weak mixing angle θ_W and via the CKM quark and lepton mixing matrices (see (6.13)) – these angles are understood to be running functions of Q. So we should suspect that the angle θ in (8.6) and (8.7) is a function of Q as well, and need to be alert for ways that this mixing might enter these equations. The empirically-driven need to withdraw the definition (8.1) and replace it with (8.10) solves two problems at once, because it enables the angles to be defined in relation to the masses so as to match up with the empirical data and at the same time it takes advantage of the rotation first noticed from $m_u m_d / \left(m_u{}^2 + m_d{}^2\right)$ to explicitly start with the EPN-defined $m_u(0)$ and $m_d(0)$ quark masses and then rotate them to $m'_u = m_u(Q)$ and $m'_d = m_d(Q)$ and thus gives us a way to understand how these masses and indeed all of the empirical data might run with the energy scale Q. This is highlighted especially by (8.15) in which we have defined η to replace what was the original role of θ right after (8.1) as the arctangent of the up-to-down mass ratio. We see in (8.15) that η is a running ratio of the quark masses, but is not the driving parameter as to running with Q. Rather, it is $\phi(Q)$ and $\theta(Q) = \pi/4 + \phi(Q)$ which directly drive the running. So the redefinition to match the empirical data also spawned a running ratio angle η which runs with Q but is not the underlying parameter for running, and θ and ϕ which are in lockstep with one another and are the underlying driving parameters for the running of everything else. We do not in this paper seek to ascertain how, precisely, these angles θ and ϕ run with Q. We merely wish make clear that they do.

One other point needs to be noted as well. The fact that the up and down quark masses are rotated via (8.10) as a function of $\phi(Q)$ suggests that $m_\zeta = \sqrt{m_u{}^2 + m_d{}^2} = 5.386\,90110\ \text{MeV}$ is an invariant of this rotation, i.e., that $m'_\zeta = m_\zeta(Q) = m_\zeta(0)$ at all Q. And we have mentioned on several occasions in this section that m_ζ is the hypotenuse of this rotation, i.e., the radius of the circle of rotation. But we need to be very careful, because out discussion here is limited to the first quark generation with contains the up and down quarks. When we expand our view to the second and third generations and the CKM mixing of these generations, we must keep in mind that the CKM angles $\theta_{12}(Q)$, $\theta_{13}(Q)$, $\theta_{23}(Q)$ and phase $\phi(Q)$ are also expected to run with Q, and can also shift mass from one generation to another. So if we rewrite m_ζ by $m_{\zeta 1}$ to denote that this is the mass radius / hypotenuse for the first generation rotation, one should consider the prospect that there are two other $m_{\zeta 2}$ and $m_{\zeta 3}$ radii for the second and third generation with some presently unknown relationships among all of them. (See, however, section 3 of [6] which discusses the Koide relationships which provide the best insights known to date for how to characterize the inter-generational empirical fermion masses, and relates these to matrices displayed here in (5.1) and (5.2) which are also another way to express (8.19) and (8.20).) And one should expect that as Q increases, not only does the angle η = 24.381 777 8° change, but so too does the $m_{\zeta 1}$ radius. Thus, as among the three generations, we might envision three circles of radii $m_{\zeta 1}$, $m_{\zeta 2}$ and $m_{\zeta 3}$ such that as the angles η_1, η_2 and η_3 are rotated, so to do the radii shift, and as one or two of the radii expand, the third one contracts, all in some presently unknown interrelationship. So these are less circles than spirals, which likely converge in some way at GUT and higher-*Q* scales.

9. Conclusion: A Century and a Half after Maxwell, Protons and Neutrons and other Baryons are Finally Understood to be Yang-Mills Chromo-Magnetic Monopoles

What we have detailed in sections 8 and 9 is that (7.2) for $F_{\text{eff}\,\mu\nu 0}$, which is obtained as a direct deductive consequence of the thesis that baryons are the chromo-magnetic monopoles of Yang-Mills gauge theory, is the theoretical expression which provides the "interface" to be able to make empirical predictions. One then uses (7.2) in $E = \iiint \frac{1}{2}\mathrm{Tr}F_{\mu\nu}F^{\mu\nu}d^3x$ to be able to deduce energies, and after a full test calculation using a Gaussian *ansatz* explained after (7.14), and the discovery and interpretation of inter-generational mixing between the up and down current quark masses reviewed in section 8, one arrives at (8.19) through (8.21) which form the basis for the broad range of empirically-accurate relationships developed and enumerated in sections 5 and 6. This is how the theoretical results captured in $F_{\text{eff}\,\mu\nu 0}$ connect to formulations which can be used for empirical validation via certain predicted energies driven by the current quark masses. So in effect, this paper has now shown the manner in which (7.2) for $F_{\text{eff}\,\mu\nu 0}$ leads to multiple empirical concurrences with a range of nuclear energies which have never been known before. So now, working backwards, we come to the final question as to the theoretical origins and foundations for $F_{\text{eff}\,\mu\nu 0}$ in (7.2).

The fundamental starting point is to recognize that in classical Yang-Mills theory, there is inherently a non-vanishing net flux $\oint\!\!\oint F \neq 0$ of a "magnetic field" across closed surfaces, as first communicated in [5.6] of [1] and thereafter reiterated in [3.3] of [10]. This is in contrast to electrodynamics for which $\oint\!\!\oint F = 0$ and so there is no net magnetic flux across closed surfaces, so that while electric fields terminate at an electric charge, magnetic fields are aterminal closed loops. As was initially made clear in [2.4] and [2.5] of [10], when expressed in differential forms, just as *ddA* = 0 in electrodynamics where *A* is the vector potential / photon one-form, *DDG* = 0 in Yang-Mills theory where *G* is Yang-Mills vector potential one-form which in chromodynamics becomes associated with the gluon fields. So formally speaking there are still no *elementary* magnetic monopoles in Yang-Mills theory either. But when taken in the integral formulations of Gauss and Stokes, there is a non-vanishing "faux" monopole $P' = -id[G,G] = -i[dG,G]$ which arises exclusively as a composite object via the non-commuting nature of Yang-Mills theory which does not exist in electrodynamics ([10] states that $P' = -idGG$; this is an error which will be corrected before this paper goes to formal publication). So when expressed in integral form there is also a non-vanishing $\oint\!\!\oint F = -i\oint\!\!\oint [G,G] = -i\iiint [dG,G] \neq 0$, and so these magnetic field analogs *do net flow across closed surfaces*. In electrodynamics everything commutes, so the analogous expression

$\oiint F = -i\oiint [A, A] = -i\iiint [dA, A] = 0$, and that is why classical Yang-Mills theory gives us $\oiint F \neq 0$ while electrodynamics gives us $\oiint F = 0$.

So if one believes in Maxwell and one believes in Yang-Mills as correct, empirically-validated theories of nature, then because their logical combination inexorably leads to a faux magnetic charge density $P' = -id[G, G] = i[G, dG] \neq 0$ and an associated $\oiint F \neq 0$ which do not appear in Maxwell's theory alone, one must believe that these $P' \neq 0$ and $\oiint F \neq 0$ exhibit *some manifestation in the physical universe*. The only question is how these are manifest. The author's fundamental thesis is that $\iiint [dG, G] \neq 0$ manifests as a baryons, and $\oiint F = -i\oiint [G, G]$ manifest as the meson and energy fluxes in and out of baryons, for example, through all of the nuclear binding and fusion energies reviewed in section 6 here. It is the field strength F appearing in $\oiint F \neq 0$ which eventually becomes the $F_{\text{eff}\,\mu\nu 0}$ for which we then calculate energies $E = \iiint \frac{1}{2}\mathrm{Tr}F_{\mu\nu}F^{\mu\nu}d^3x$ for both the proton and neutron. And it is from these energies that the empirical connections elaborated throughout this paper ultimately then emerge.

So now the question becomes how to "populate" these non-vanishing faux monopole entities $\oiint F = -i\oiint [G, G] = -i\iiint [dG, G] \neq 0$ with quarks and show that they manifest as baryons. Referring back to section 7 here, while the a) Maxwell and b) Yang-Mills get us to these net-flowing magnetic fields $\oiint F \neq 0$, it is c) Dirac theory and d) Dirac-Fermi-Pauli Exclusion which when deductively combined with a) and b) demonstrates that these entities have the correct color attributes of baryons and mesons. This was originally communicated in section 5 of [1] and was later elaborated in section 9 of [10] so as to establish all of the non-linear features of these monopoles, and at the same time show the monopole behaviors in the abelian limit as discussed following (7.1) here.

Briefly, while the classical field equations for Yang-Mills *electric* charges $*J = D*F = D*DG$ ordinarily express the current density J as a function of the gauge field G namely $J(G)$, it is desirable to invert this field equation to instead express G as a function of J, i.e., as a function $G(J)$. In this way, by what is effectively a merging of *both of Maxwell's classical field equations into a single equation* ("merged Maxwell"), one can then advance the monopole entities to $\oiint F = -i\oiint \left[G(J), G(J)\right] = -i\iiint \left[dG(J), G(J)\right] \neq 0$. But by Dirac, we know that current densities may in turn be expressed in terms of fermion wavefunctions $J(\psi)$, via $J^\sigma = \overline{\psi}\gamma^\sigma\psi$. So now $\oiint F = -i\oiint \left[G(\psi), G(\psi)\right] = -i\iiint \left[dG(\psi), G(\psi)\right] \neq 0$, and the monopole entities contain fermions. How many fermions? In the abelian linear approximation, each faux monopole entity contains precisely *three* fermion eigenstates. At bottom, this emerges from the fact that the faux magnetic charge density P' is a differential *three-form*. So if this monopole "system" contains precisely three fermion eigenstates, then by the Exclusion Principal, we must place these fermions into three distinct eigenstates. So we use the gauge group SU(3) to enforce exclusion, and now the only question is what to name these distinct eigenstates. So we choose R, G and B, call this color, and now the $SU(3)_C$ color group of chromodynamics naturally emerges *as a corollary to merely combing Maxwell, Yang-Mills, Dirac and the Exclusion Principle together all at once.* The rank-3 of the monopole three form converts over into the dimension-3 of the gauge group, and $SU(3)_C$ is seen not as a fundamental theory but as a corollary theory rooted in Merged-Maxwell-Yang-Mills-Dirac-Exclusion.

Once color is assigned, as first communicated in section 5 of [1] and thereafter in section 10 of [10], the faux monopole three form P' has the $R \wedge G \wedge B$ color symmetry of a baryon and the $\mathrm{Tr}\Sigma i F_{\mathrm{eff}\,\mu\nu}\left((0)\right)_0 = \mathrm{Tr}\Sigma\left[G_\mu, G_\nu\right]\left((0)\right)_0$ entity has the color wavefunction $\overline{R}R + \overline{G}G + \overline{B}B$ of a meson. And in equation [10.4] of [10] for $F_{\mathrm{eff}\,\mu\nu}\left((0)\right)_0$ where this $\overline{R}R + \overline{G}G + \overline{B}B$ meson wavefunction first becomes clear, reproduced earlier as equation (7.1) here (see also [5.6] of [1]), we also obtain the starting point for connecting the theory to its means of empirical confirmation by calculating the energies $E = \iiint \frac{1}{2}\mathrm{Tr}F_{\mu\nu}F^{\mu\nu}d^3x$. The very same equation which reveals to us the mesons which flow in and out of baryons and hold together the nuclei, also gives us the basis for quantitatively studying the energies which fuse and bind the nucleons into nuclei.

The one other important finding which emerges in the process of all this, is that because of the non-linear features of Yang-Mills gauge theory, when we attempt to express *G* as a function of *J*, we are unable to obtain a simple $G(J)$ *except in the abelian limit of Yang-Mills gauge theory*. In general, *G* is a function not only of *J*, but also of itself, $G(J,G)$. So if we are looking for an expression $G(J)$ which does not self-feed via $G(J,G)$, then as first detailed in section 8 of [10], we need to treat $G(J,G)$ *recursively*. We feed $G(J,G)$ into itself as many times as we wish – anywhere from zero times to an infinite number of times – and then cut off any further feeds by setting a perturbation *V* to zero. Doing this "zero times" expresses the abelian limit. In the other hand, self-feeding an infinite number of times is the behavior ascribed to nature. For human beings and their computers doing non-linear calculations to some acceptable level of precision, one would recurse a finite number of times, whether 1 or 2 or 5 or 10, etc. and then study those results. So this recursive approach enables us to as detailed in section 9 of [10] to describe these baryon monopoles in terms of their natural condition with infinite recursion, and to also take the abelian limit of zero recursion, as well as to do in-between calculation and analysis. The empirical connections we have developed here to nuclear binding energies are all developed from the zero-recursion limit, which informs us that the observed nuclear binding and fusion energies are expressing abelian "signals" from the nucleons which need to be "decoded" as in sections 5 and 6 to teach us about the "nuclear genome." On the other hand, the complete proton and neutron masses and the constituent / contributive quark masses discussed in see point 11 in section 6 tell us about all of the non-abelian "noise" which then overlays upon these signals in the infinite recursion limit to exhibit the observed properties of nucleons as complete nucleons.

It will be appreciated that all of the foregoing makes use only of the classical Yang-Mills theory. We have not yet discussed or resorted to *quantum* Yang-Mills theory, which because Merged-Maxwell-Yang-Mills-Dirac-Exclusion implies $SU(3)_C$, means we have no yet resorted to

QCD, but only to classical chromodynamics. So while one might approach the empirical questions we have laid out in sections 5 and 6 here under the assumption that they cannot be explained except by a quantum field theory, the result here reveal this to be a false assumption. All of the empirical results enumerated in sections 5 and 6 are based on *classical*, not *quantum* Yang-Mills theory! Classical field theory has more "juice" than it is given credit for in this day and age. But when we finally do wish to study *quantum* Yang-Mills theory which via Merged-Maxwell-Yang-Mills-Dirac-Exclusion means quantum chromodynamics, the recursion just discussed is an *indispensable* element. For, when we finally bring Feynman-path integration into the mix as laid out in point e) near the start of section 7, we run into the long-standing *mathematical problem* of how to analytically and exactly calculate a path integral for a non-linear classical field theory, which in the context of scalar fields is the so-called φ^4 problem. As demonstrated in section 11 of [10], this recursion is the precise aspect of Yang-Mills theory which enables us to finally overcome this important problem and perform an analytically exact path integration to prove the existence of a non-trivial quantum Yang–Mills theory on R^4 for any simple gauge group G, see [24] page 6.

Once this is achieved, it is possible to obtain the quantum field equations of Yang-Mills QCD which are [13.21] of [10], and thereafter, to derive the running QCD curve of Figure 1 within all experimental errors, see section 18 and especially Figure 14 of [10]. So in the simplest terms, QCD may now be thought of as no more and no less than Merged-Maxwell-Yang-Mills-Dirac-Exclusion-Feynman, where it is Feynman via path integration that finally takes a classical chromodynamic theory which properly explains a wide range of nuclear energy data including confinement when expressed in terms of nuclear energies as in point 1 of section 5, over to a quantum QCD theory which explains the running QCD curve which is the fundamental quantum evidence of confinement. All of this, combines to provide overwhelming evidence that the non-vanishing flows $\oiint F \neq 0$ of chromo-magnetic fields across closed spatial surfaces in Yang-Mills gauge theory, are in fact synonymous with the existence of baryons, including the protons and neutrons from which all of the atomic nuclei are constructed.

During the century and a half since Maxwell and Heaviside first taught that there are no magnetic monopoles in electrodynamics, these monopoles have been an endless source of fascination for physicists wondering whether the natural world contains some form of magnetic monopoles, and if so, what form those monopoles might take. At the same time, although Rutherford and Chadwick established the existence of protons and neutrons almost a century ago, and while protons and neutrons and their other baryon cousins have been well-characterized since, there remains to date no convincing *theoretical* explanation of *what a baryon actually is* beyond it being some confining bound state of three quarks teeming with gluons and highly-non-linear quantum interactions. Rabi's immortal quip, "who ordered that?" remains an unanswered question for protons and neutrons, to this very date.

The answer to Rabi's question is that the protons and neutron and other baryons were ordered by a deductive combination of Merged-Maxwell-Yang-Mills-Dirac-Exclusion-Feynman, with the exclusion principle being the combined effort of Fermi-Dirac-Pauli. The cast of characters who placed this order, and the highly-settled and thoroughly-validated nature of the theories which they used to do so, make clear that the author's thesis that baryons are Yang-Mills chromo-magnetic monopoles is a highly conservative thesis, grounded in a combination of some of the most fundamental, widely-accepted and extensively-tested scientific theories. To believe and accept this thesis requires nothing more than a belief that all of these theories are correct, and a belief that when mathematics is correctly applied to combine input component theories which themselves are also correct, the result of that mathematical combination will be equally correct.

So it is with great irony that when future generations look back on the century and a half from Maxwell's time to the present time during which scientists passionately pursued magnetic monopoles, they may chuckle in irony over the fact these monopoles were mocking our efforts and hiding in plain sight all along, as the protons and neutrons at the heart of the material universe.

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