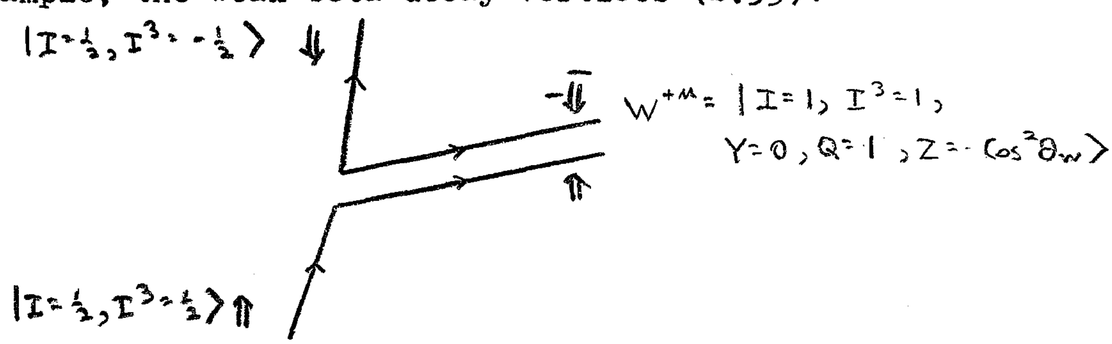
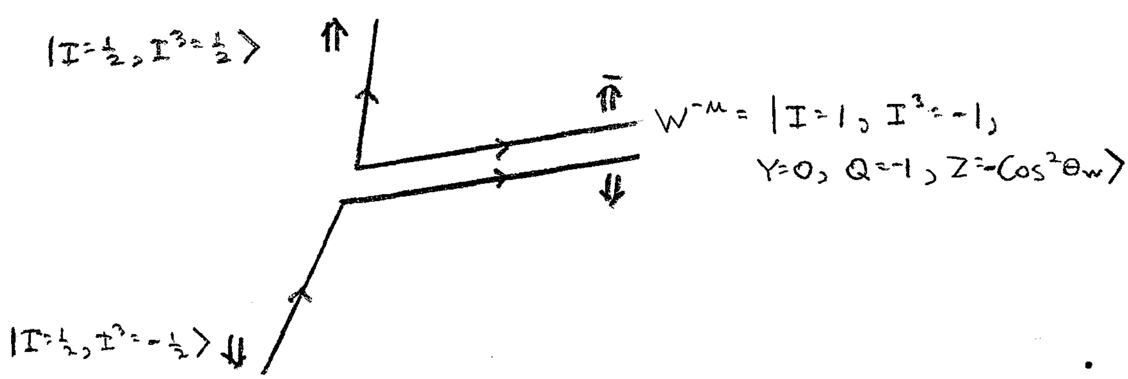
2.11 - The Four-Preon Flavor SU(4) Unification of the Electromagnetic, Weak and Colorless Strong Interactions Excluding Quantum Gravitation; and the Colorless Flavor Classification of Left Handed Real Fermion and Boson Chiral Projections, for a Single Fermion Generation

An in-depth analysis of weak interaction beta-decay, which was already started to some degree in Section 9, see, eg. the vertex diagrams E(2.230),(2.235), is one of the more fruitful means by which preonic grand unification may be developed. Consider for example, the weak beta-decay vertices (2.235):



(2.245)(a)



(2.245)(b)

Recalling also Table 1.1 from way back in the introduction, and Table 2.3, let us concentrate for the moment simply on the left-handed chiral projections of the various fermions and bosons (for reference, see eqs. (2.105)-(2.108) , and the electromagnetic (2.109)-(2.111),). See also eqs. (1.2) and (1.11). Additionally, referring again to Table 1.1, let us for the moment concern ourselves only with the flavor quantum numbers F, Q_u, L and Q; and not with any of the color quantum numbers R,G,B. Note again, that there is some degree of overlap between flavor and color quantum numbers, specifically as given by eq. (1.1):

$$F = Q_u + L = R + G + B + L \quad , \tag{2.246}$$

which is to say, out of the quantum numbers listed in Table 1.1, that F, Q_u and L are connected with <u>both</u> flavor and color; Q is connected exclusively with flavor; and R, G, B are connected exclusively with color. We shall discuss the above in more formalized terms shortly. In addition to the flavor quantum numbers F, Q_u, L and Q, which are all gauged so as to be left-right symmetric, we now wish to examine the various other flavor quantum numbers which are not necessarily left-right symmetric. Specifically, we wish to examine Y, I^3, and also, baryon number B. (Distinguish from B=Blue by context.) Thus, what we are presently concerned with, absent also, for the moment, any consideration of multiple fermionic generation, is the following flavor classification of left-handed fermionic chiral projections, and the vector Bosons which mediate their electroweak interactions: (Casimir I is also included) (See (2.233))

	I	I^3	Y	$Q=Y+I^3$	$Z=I^3-Q\sin^2\theta_W$	B	L	Q_u	$F=Q_u+L$
ν_L	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	0	1	0	1
e_L	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	-1	$-\frac{1}{2}+\sin^2\theta_W$	0	1	0	1
u_L	$\frac{1}{2}$	$\frac{1}{2}$	$1/6$	$2/3$	$\frac{1}{2}-(2/3)\sin^2\theta_W$	$1/3$	0	1	1
d_L	$\frac{1}{2}$	$-\frac{1}{2}$	$1/6$	$-1/3$	$-\frac{1}{2}+(1/3)\sin^2\theta_W$	$1/3$	0	1	1
W^{+u}	1	1	0	1	$\cos^2\theta_W$	0	0	0	0
A^{u}	1	0	0	0	0	0	0	0	0
W^{-u}	1	-1	0	-1	$-\cos^2\theta_W$	0	0	0	0
Z^{u}	$\cos^2\theta_W$	0	0	0	0	0	0	0	0

Table 2.4 - Flavor Quantum Number Classification of Elementary Real Left-Handed Chiral Fermionic Projections, and Electroweak Vector Bosons.

The above, it must again be stressed, describes <u>real</u> elementary particles. A similar table can be constructed for the <u>complex</u> isospin preons, $\Uparrow$, $\Downarrow$ and their "antipreon" counterparts $\bar{\Downarrow}$, $\bar{\Uparrow}$, again from Table 2.3: (B, L, Q_u, F not supplied for $\Uparrow$, $\Downarrow$ by $SU(2)\times U(1)$, and hence are equal to zero for these preons)

	I	I^3	Y	$Q=Y+I^3$	$Z=I^3-Q\sin^2\theta_W$	B	L	Q_u	$F=Q_u+L$
$\Uparrow$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	$\frac{1}{2}-\sin^2\theta_W$	0	0	0	0
$\Downarrow$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	0	0	0	0
$-\bar{\Downarrow}$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	0	0	0	0
$\bar{\Uparrow}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	-1	$-\frac{1}{2}+\sin^2\theta_W$	0	0	0	0

Table 2.5 - Flavor Quantum Number Classification of Elementary Complex Isospin Preons.

Now, it is tempting to identify the complex isospin preons in Table 2.5 with the real neutrino/electron doublet in Table 2.4 according to the association:

$$\begin{matrix} \nu_L \sim -\bar{\Downarrow} & \bar{e}_L \sim \Uparrow \\ e_L \sim \bar{\Uparrow} & \bar{\nu}_L \sim \Downarrow \end{matrix} \quad , \tag{2.247}$$

particularly because the I, I^3, Y, Q and Z flavor quantum numbers for these doublets are identical in all respects. In fact, this type of association is often made, though it is correct or more precisely, exact, only to a certain point. This is <u>not</u> a correct association insofar as the preons in Table 2.5 do not carry the necessary lepton and Fermion numbers F=L=1 to be fully associated with real electrons and neutrinos. To generate lepton number, one anticipates that an additional preon will be needed. Among other things, it will be necessary for this additional preon to carry F=L=1, in order to correctly generate <u>all</u> of the required quantum numbers for the electron/neutrino doublet, ~~particular~~ly Fermion number F=1, which is essential for any particle that is to be identified with an electron or a neutrino (or a Fermion). Additionally, there is the necessity of providing particles with the appropriate quark numbers $F=Q_u=1$ for the u_L, d_L fermions. Here too, the preons in Table 2.5 might permit a loose association of the form:

$$u_L \sim -\overline{\Downarrow} \qquad \bar{d}_L \sim \Uparrow$$
$$d_L \sim \overline{\Uparrow} \qquad \bar{u}_L \sim \Downarrow \tag{2.248}$$

because the I and I^3 flavor quantum numbers for these doublets are identical in all respects. Here however, not only does this fail to explain the fact that for these particles, $F=Q_u=1$, $L=0$; it also fails to explain the values of Y, Q and Z for the u_L and d_L quarks. Hence the above association (2.248) is even less exact than (2.247). To generate quark number, one anticipates yet another preon. This preon must carry $F=Q_u=1$ and $L=0$ among other things, and when combined appropriately with the "isospin up" and "isospin down" preons which we already know something about from our electroweak discussions, this preon must be capable of generating <u>all</u> of the correct (flavor) quantum numbers for the up/down doublet. Thus, to account for the real physical existence of both quarks and leptons, along with the (identical) form in which beta decay takes place within both the quark and lepton doublets, it is anticipated that precisely four preons will be required. The first of these preons, which we shall label as the "A" preon, will be defined as the carrier of the quark (and later, also the color) charge, ie., it will have $F=Q_u=R+G+B=1$, $L=0$. The second preon, which we shall label as "B," will be defined as the carrier of (negative, as discussed shortly) lepton number, $F=L=-1$, $Q_u=R+G+B=0$. The third and fourth preons, which we shall label "C" and "D," will be defined simply as the "isospin up", $\Uparrow$, and "isospin down", $\Downarrow$, preons that are already at hand, by virtue of Electroweak theory. As such, there are now a total of four flavor preons, (A,B,C,D). Let us now see if we can develop a more detailed picture of these preons.

First, let us carry out the relabelling of the "isospin up" and "isospin down" preons according to:

$$\Uparrow \xrightarrow{\text{relabel}} C \quad ; \qquad \Downarrow \xrightarrow{\text{relabel}} D \qquad (2.249)$$

with the consequence that the "flavor polarizations" of the $W^{\pm u}$ are similarly relabelled: (For ease of discussion, and since it will have later no substantive impact, we have removed the minus sign in front of $\bar{D}$)

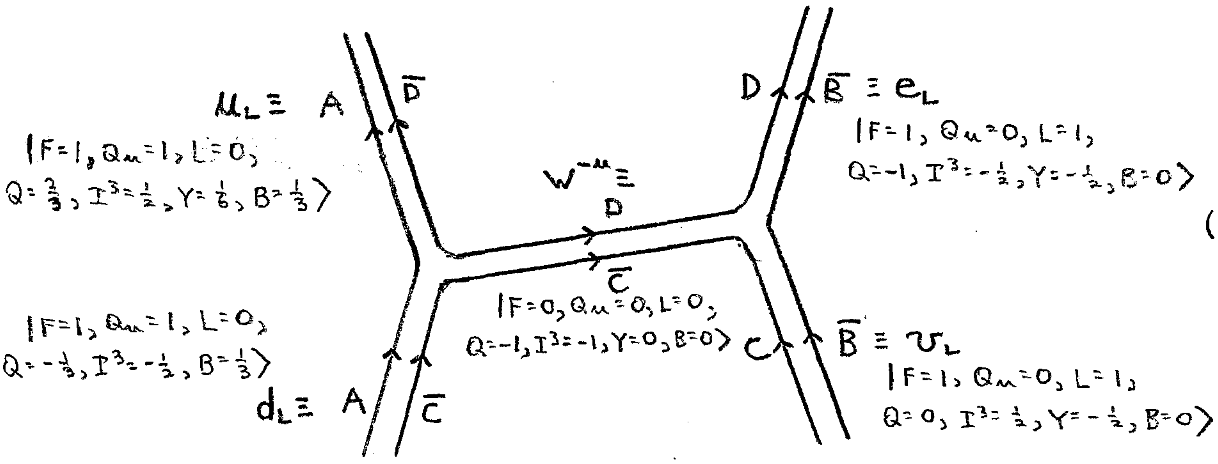
$$W^{+u} = B^{u}\,(-\bar{\Downarrow}\Uparrow) \xrightarrow{\text{relabel}} B^{u}\,(\bar{D}C) \qquad (2.250)(a)$$

$$W^{-u} = B^{u}\,(\bar{\Uparrow}\Downarrow) \xrightarrow{\text{relabel}} B^{u}\,(\bar{C}D) \;. \qquad (2.250)(b)$$

Using the new labels, we turn next to the analysis of ordinary left-hande (no Cabibbo mixing) nuclear beta-decay, of the form: (n=neutron, p=proton)

$$\begin{aligned}
& n_{LLL} \longrightarrow p_{LLL} + e_L + \bar{\upsilon}_L \\
& u_L d_L d_L \longrightarrow u_L u_L d_L + e_L + \bar{\upsilon}_L \\
\Longrightarrow \quad & d_L \longrightarrow u_L + e_L + \bar{\upsilon}_L \\
\Longrightarrow \quad & d_L + \bar{u}_L \longrightarrow W^{-u} = B^{u}(\bar{C}D) \longrightarrow e_L + \bar{\upsilon}_L \;.
\end{aligned} \qquad (2.251)$$

Now, as per our earlier discussion, the "A" preon must be defined to have $F=Q_u=1$, $L=0$; while the "$\bar{B}$" preon must have $F=L=1$, $Q_u=0$. When combined with the $\bar{C}$ and $\bar{D}$ isospin preons, the A preon must yield all of the correct flavor quantum numbers for the $(u,d)_L$ quark isospin doublet. When combined with the C and D preons, the $\bar{B}$ must similarly yield all of the correct flavor quantum numbers for the $(\upsilon,e)_L$ leptonic isospin doublet. Finally, when combined with one another, the C and $\bar{D}$, and D and $\bar{C}$ must yield all of the correct flavor quantum numbers for the $W^{\pm u}$, see (2.250), as was done for example in (2.234). Accounting for all of the above, one is led to <u>define</u> the following as the preonic decomposition of the Feynman for nuclear weak beta-decay without generation mixing, ie., the reaction (2.251): (See Table 2.4)



(2.252)

From the above, it is apparant that all flavors of real particle, whether spin $\frac{1}{2}$ fermion or spin 1 boson, achieve their own particular flavor polarizations through the meson-like combination of a preon/antipreon polarization. To simplify notation, let us henceforth denote any particular flavor of particle simply by specifying its flavor polarization, ie., by specifying its preonic composition. From (2.252) above, this means that we make the definitions:

$$\begin{pmatrix} u \\ d \end{pmatrix}_L \equiv \begin{pmatrix} \bar{D}A \\ \bar{C}A \end{pmatrix} = \begin{pmatrix} A \\ A \end{pmatrix} + \begin{pmatrix} \bar{D} \\ \bar{C} \end{pmatrix} \qquad (2.253)(a)$$

$$\begin{pmatrix} \nu \\ e \end{pmatrix}_L \equiv \begin{pmatrix} \bar{B}C \\ \bar{B}D \end{pmatrix} = \begin{pmatrix} \bar{B} \\ \bar{B} \end{pmatrix} + \begin{pmatrix} C \\ D \end{pmatrix} \qquad (2.253)(b)$$

for fermions, along with

$$\begin{aligned} W^{+u} &= \bar{D}C \\ W^{-u} &= \bar{C}D \end{aligned} \quad , \qquad (2.254)$$

for bosons which we already know from electroweak theory. For Bosons, a more detailed definition is developed using (2.191), (2.192). From (2.253), the reason for the similarity between quarkonic and leptonic beta decay, and for the similar isospin properties of the quark doublet $(u,d)_L$ and the lepton doublet $(\nu,e)_L$ becomes manifestly apparant. In each case, weak β-decay results from the same C⟺D preon transition. Examination of (2.252)-(2.254) together shows why the $\bar{B}$ preon, not B, carries L=+1: given the choice that A carries Q_u=+1 (this is by convention), given the reversal of the C and D flavor lines as between the two fermion lines in (2.252), and given the requirement for a meson-like flavor structure for fermions and bosons, it becomes apparant that it is the antipreon $\bar{B}$, and not the preon B, which must carry L=+1. We also note from (2.253), how the essential difference between quark and lepton lies in that between the A and $\bar{B}$ preons.

Given the above, let us now set out to determine what values each of the preons A,B,C,D must individually carry, of each of the flavor quantum numbers F, Q_u, L, Q, I^3, Y and B in order to be consistent with all known real particles. These are some of the flavor numbers listed in Tables 2.4, 2.5. Note also, as discussed at length in the introduction 1.1, that F, Q_u, L, Q must be left-right symmetric for all particles; and that I^3, Y and B need not necessarily obey this same symmetry. Referring back to Table 2.4, and also to the definitions (2.252), (2.253), let us summarize what we now know about the fermions, as follows:

$$u_L \equiv \bar{D}A = |F=1, Q_u=1, L=0, Q=2/3, I^3=\tfrac{1}{2}, Y=1/6, B=1/3\rangle \qquad (2.255)(a)$$

$$d_L \equiv \bar{C}A = |F=1, Q_u=1, L=0, Q=-1/3, I^3=-\tfrac{1}{2}, Y=1/6, B=1/3\rangle \qquad (2.255)(b)$$

$$\nu_L \equiv \bar{B}C = |F=1, Q_u=0, L=1, Q=0, I^3=\tfrac{1}{2}, Y=-\tfrac{1}{2}, B=0\rangle \qquad (2.255)(c)$$

$$e_L \equiv \bar{B}D = |F=1, Q_u=0, L=1, Q=-1, I^3=-\tfrac{1}{2}, Y=-\tfrac{1}{2}, B=0\rangle . \qquad (2.255)(d)$$

we also know of course, that: (From (2.254))

$$W^{+u} \equiv \bar{D}C = |F=0, Q_u=0, L=0, Q=1, I^3=1, Y=0, B=0\rangle \qquad (2.256)(a)$$

$$W^{-u} \equiv \bar{C}D = |F=0, Q_u=0, L=0, Q=-1, I^3=-1, Y=0, B=0\rangle . \qquad (2.256)(b)$$

As we shall see shortly, not all of these quantum numbers are linearly independent. We already know for example, that $F=Q_u+L$ and $Q=Y+I^3$. In fact, due to linear interdependence, it is really only necessary to consider three of the above, specifically, I^3, Y and B. To ascertain the value of each of these three flavor quantum numbers, for each of the four preons A,B,C,D, it becomes necessarily to uniquely determine twelve (3x4) independent quantum number (eigen)values. Let us now reduce this uniquely.

First, one may impose the condition that for each and every one of I^3, Y and B, and hence for all linear combinations of these, that the sum (trace) of the eigenvalues is equal to zero, when all four preons A,B,C,D are considered. This condition, which in essence ensures the tracelessness of the generators for the fundamental flavor representation, may be stated as such:

$$I^3(A) + I^3(B) + I^3(C) + I^3(D) = 0 \tag{2.257)(a}$$

$$Y(A) + Y(B) + Y(C) + Y(D) = 0 \tag{2.257)(b}$$

$$B(A) + B(B) + B(C) + B(D) = 0 \quad . \tag{2.257)(c}$$

this reduces from twelve to nine, the number of independent eigenvalues. To reduce this down to six, consider the W^{-u} weak vector boson. As $W^{-u} = B^u(\bar{C}D) = |I^3=-1, Y=0, B=0\rangle$, see (2.256)(b), one imposes the further constraint: (Quantum numbers for anti-preons are the negatives of those for corresponding preons.)

$$I^3(D) - I^3(C) = -1 \tag{2.258)(a}$$

$$Y(D) - Y(C) = 0 \tag{2.258)(b}$$

$$B(D) - B(C) = 0 \quad . \tag{2.258)(c}$$

Next, we consider a lepton, say, ν_L. As $\nu = f(\bar{B}C) = |I^3=\frac{1}{2}, Y=-\frac{1}{2}, B=0\rangle$, see (2.255)(c), we impose the further constraint: ($f(\bar{B}C)$ designates a fermion with $\bar{B}C$ flavor polarization)

$$I^3(C) - I^3(B) = \tfrac{1}{2} \tag{2.259)(a}$$

$$Y(C) - Y(D) = -\tfrac{1}{2} \tag{2.259)(b}$$

$$B(C) - B(D) = 0 \quad , \tag{2.259)(c}$$

which leaves us with three unknown eigenvalues. These final three are determined, and hence a solution may be uniquely specified, by considering one of the quarks, say, u_L. Because $u_L = f(\bar{D}A) = |I^3=\frac{1}{2}, Y=1/6, B=1/3\rangle$, see (2.255)(a), one may also impose the constraint:

$$I^3(A) - I^3(D) = \tfrac{1}{2} \tag{2.260)(a}$$

$$Y(A) - Y(D) = 1/6 \tag{2.260)(b}$$

$$B(A) - B(D) = 1/3 \quad . \tag{2.260)(c}$$

At this point, solving the twelve equations (2.257)-(2.260) for the twelve unknown vaules of I^3,Y,B for each of A,B,C,D, one arrives at the unique solutions:

$$I^3(A) = 0 \;;\; I^3(B) = 0 \;;\; I^3(C) = \tfrac{1}{2} \;;\; I^3(D) = -\tfrac{1}{2} \qquad (2.261)(a)$$

$$Y(A) = 0 \;;\; Y(B) = 1/3 \;;\; Y(C) = -1/6 \;;\; Y(D) = -1/6 \qquad (2.261)(b)$$

$$B(A) = \tfrac{1}{4} \;;\; B(B) = -1/12 \;;\; B(C) = -1/12 \;;\; B(D) = -1/12 \;. \qquad (2.261)(c)$$

More neatly, if we define the eigenvector preon solutions:

$$A \equiv \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \;;\; B \equiv \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \;;\; C \equiv \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} \;;\; D \equiv \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} , \qquad (2.262)$$

then I^3,Y and B may be put into the diagonalized matrix form:

$$I^3 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & -\frac{1}{2} \end{pmatrix} \;;\; Y^8 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1/3 & 0 & 0 \\ 0 & 0 & -1/6 & 0 \\ 0 & 0 & 0 & -1/6 \end{pmatrix} \; B^{15} = \begin{pmatrix} \frac{1}{4} & 0 & 0 & 0 \\ 0 & -1/12 & 0 & 0 \\ 0 & 0 & -1/12 & 0 \\ 0 & 0 & 0 & -1/12 \end{pmatrix} , \qquad (2.263)$$

(reasons for the "8" and "15" to be discussed shortly.),
and (2.261) then denotes the eigenvalues of the matrices (2.263) corresponding to each of the eigensolutions (2.262). One may be concerned that in (2.259) and (2.260), we only used ν_L and u_L, and not e_L and d_L. Hence, one might expect, if e_L and d_L were also used, that the solution will be <u>overdetermined</u>. This is <u>not</u> so however, since u_L differs from d_L, and ν_L from e_L, simply by a W^u boson, which is already accounted for in (2.258). Hence, (2.261)-(2.263) really is a unique solution, given the I^3, Y and B values for all of u_L,d_L, ν_L,e_L and W^u in (2.255) and (2.256), and the required vertices in (2.252).

One can show explicitly that this solution is indeed unique, and can check the correctness of the solution by starting with the flavor quantum numbers of (2.261)-(2.263) and, using the definitions (2.252)-(2.256), by working backwards to reconstruct the quantum numbers for the real fermions $(u,d)_L$, $(\nu,e)_L$ and bosons $W^{\pm u}$ out of the preonic quantum numbers. Keep in mind, for all linear (non-Casimir) quantum numbers, that the antiparticle preons should have quantum numbers which are precisely opposite those of the corresponding preons, see discussion following (2.100). Thus, using (2.261)-(2.263) and the definitions (2.252)-(2.256), including Q, these results may be summarized in the tabular form:

	I^3	Y^8	B^{15}	$Q=Y^8+I^3$
A	0	0	$\frac{1}{4}$	0
B	0	1/3	-1/12	1/3
C	$\frac{1}{2}$	-1/6	-1/12	1/3
D	$-\frac{1}{2}$	-1/6	-1/12	-2/3
$\bar{A}$	0	0	$-\frac{1}{4}$	0
$\bar{B}$	0	-1/3	1/12	-1/3
$\bar{C}$	$-\frac{1}{2}$	1/6	1/12	-1/3
$\bar{D}$	$\frac{1}{2}$	1/6	1/12	2/3
$X^{-\frac{1}{3}u} \equiv \bar{B}A$	0	-1/3	1/3	-1/3
$X^{+\frac{1}{3}u} \equiv \bar{A}B$	0	1/3	-1/3	1/3
$d_L \equiv \bar{C}A$	$-\frac{1}{2}$	1/6	1/3	-1/3
$d_L \equiv \bar{A}C$	$\frac{1}{2}$	-1/6	-1/3	1/3
$u_L \equiv \bar{D}A$	$\frac{1}{2}$	1/6	1/3	2/3
$u_L \equiv \bar{A}D$	$-\frac{1}{2}$	-1/6	-1/3	-2/3
$\nu_L \equiv \bar{C}B$	$-\frac{1}{2}$	$\frac{1}{2}$	0	0
$\nu_L \equiv \bar{B}C$	$\frac{1}{2}$	$-\frac{1}{2}$	0	0
$e_L \equiv \bar{D}B$	$\frac{1}{2}$	$\frac{1}{2}$	0	1
$e_L \equiv \bar{B}D$	$-\frac{1}{2}$	$-\frac{1}{2}$	0	-1
$W^{+u} \equiv \bar{D}C$	1	0	0	1
$W^{-u} \equiv \bar{C}D$	-1	0	0	-1
$\bar{A}A$	0	0	0	0
$\bar{B}B$	0	0	0	0
$\bar{C}C$	0	0	0	0
$\bar{D}D$	0	0	0	0

Table 2.6 - Flavor Quantum Numbers of A,B,C,D Preons and Antipreons, and of Real Fermions and Bosons, Based Upon Preonic Decomposition.

Note in the above a new Boson $X^{\pm 1/3\ u}$ utilizing the $\bar{A}B$, $\bar{B}A$ combinations. As will be shown shortly, this Boson carries Fermion number F=2, and is consequently associated with the high energy hyperweak (proton-decay) interaction. Also shown are the four neutral current components $\bar{A}A$, $\bar{B}B$, $\bar{C}C$, $\bar{D}D$. In the completed theory, one anticipates neutral current mixing among these various components so as to produce four mutually orthogonal observable neutral current Bosons, some massive, others not, similar to the mixing of the $\bar{C}C$, $\bar{D}D$ states that we've already seen in

Electroweak theory, to compose the two mutually orthogonal Vector Bosons A^u and Z^u, see eq., (2.243)(b), (d), with relabelling (2.249). Whereas electroweak theory, which utilizes two preons, C and D, produces three mutually independent real particle flavors W^u, A^u, Z^u (not four, since $W^{\pm u}$ are the antiparticles of one another, and therefore are not independent), the proposed theory which utilizes four preons A,B,C,D produces a total of ten mutually independent real particle flavors. This is composed of the six "mixed current" particles $(u,d)_L$, $(\nu,e)_L$, $X^{\pm u}$, W^u (four fermions, two bosons) and four neutral current boson flavors, which must mix (ie., will be gauged so as to mix) the $\bar{A}A, \bar{B}B, \bar{C}C, \bar{D}D$ states in such a way as to produce the A^u and Z^u of electroweak theory, along with a (colorless) strong interaction gluon G^u, and a new hyperweak neutral gluon $\equiv X^{ou}$. If we follow Pati and Salam, and consider at hyperweak (high) energies that lepton number behaves like a fourth color of quark, then X^{ou} may be regarded as a neutral current (massive) gluon which carries leptonic color/anticolor $\bar{B}B$, mixed in some manner with quark color/anti color $\bar{A}A$. The mass of this neutral current gluon will be set by the grand unification mass scale (which is at least 10^{15} GeV., and by the arguments presented in Section 1.1, may be equivalent with 1.22×10^{19} GeV); hence it is produced by the same energies necessary to produce the mixed current Hyperweak $X^{\pm u}$ bosons. As it is this gloun which is ultimately responsible for Cabibbo mixing, if any, among the leptonic fermions, one expects that at low energies the production of these X^{ou} Bosons will be virtually nil. Therefore, leptonic Cabibbo mixing will be non-existent at low energies, and the directly related detection of a neutrino rest mass will be just as impossible. This will all be discussed shortly in detail.

What is important at this juncture, is to note that in <u>two</u>-preon electroweak theory, (C,D), it is possible to construct exactly <u>three</u>

independent flavors of real particle, the mixed W^u and the neutral A^u, Z^u, which are all Bosons. In four-preon grand unification, (A,B,C,D), it is possible to construct exactly ten independent flavors of real particle. This includes four mixed fermions $(u,d)_L$, $(\nu,e)_L$, and six Bosons, two of which are mixed, W^u, $X^{\pm u}$; and four of which are (flavor) neutral, A^u, Z^u, G^u, X^{0u}. We have to date said nothing explicitly about the right handed chiral projections; nor have we yet been explicit about color or generation symmetry. Our concern to this point has been the classification of particles strictly by flavor symmetry; and then, only for left handed chiral projections. Significantly, the four-preon theory incorporates both fermions and bosons into the same flavor multiplet; which is to be anticipated by noting that at any three-worldline vertex, if two of the three worldlines are fermionic, that the third one must be bosonic, see, eg. (2.252).-2.3 Thus, insofar as flavor (as opposed to spin) quantum numbers are concerned, in order to conserve flavor at any given 2-fermion,1-boson vertex, it should not be surprising that fermions and bosons do in fact end up in the same flavor multiplet.

Before proceeding to discuss such topics as the right handed chiral projections, and color and generation symmetry, a few final points about basic left-handed, 4-preon flavor unification, involving the solutions (2.261), are in order. First, as noted in the discussion following (2.256), it is possible to reduce the larger set of quantum numbers in (2.255),(2.256) and Tables 2.4, 2.5 down simply to three linearly independent quantum numbers, for which I^3, Y^8 and B^{15} are appropriate choices. This should now be justified explicitly.

To do this, and to place the preonic set of flavor quantum numbers in (2.261), (2.263) and Table 2.6 on a firmer mathematical footing, it is important to recognize at this point that the diagonalized generators of the (simple) mathematical Lie group SU(4), designated T^3, T^8, T^{15}, are given by:

$$T^3 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} ; \quad T^8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} ; \quad T^{15} = \frac{1}{\sqrt{6}} \begin{pmatrix} 3 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} , \tag{2.264}$$

and that the preonic flavor generators (2.263) are related to the above directly, by a linear factor. Specifically, contrasting the above with (2.263), one may easily write:

$$I^3 = \tfrac{1}{2} \quad T^3 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & -\frac{1}{2} \end{pmatrix} \qquad (2.265)(a)$$

$$Y^8 = \tfrac{1}{2} \frac{1}{\sqrt{3}} T^8 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1/3 & 0 & 0 \\ 0 & 0 & -1/6 & 0 \\ 0 & 0 & 0 & -1/6 \end{pmatrix} \qquad (2.265)(b)$$

$$B^{15} = \tfrac{1}{2} \frac{1}{\sqrt{6}} T^{15} = \begin{pmatrix} \frac{1}{4} & 0 & 0 & 0 \\ 0 & -1/12 & 0 & 0 \\ 0 & 0 & -1/12 & 0 \\ 0 & 0 & 0 & -1/12 \end{pmatrix} . \qquad (2.265)(a)$$

This is why we earlier introduced the ^{8} and ^{15} superscripts for Y and B in (2.263). Note, the source of the square root factors in (2.264) is due to the normalization condition:

$$\mathrm{Tr}\,(T^3 \cdot T^3) = \mathrm{Tr}\,(T^8 \cdot T^8) = \mathrm{Tr}\,(T^{15} \cdot T^{15}) = 2 \ , \tag{2.266}$$

(similarly so for the non-diagonalized SU(4) generators), while a similar set of "trace of squares" relationships involving (2.265) reveals that:

$$\mathrm{Tr}\ (I^3 \cdot I^3) = \tfrac{1}{2} \tag{2.267)(a}$$

$$\mathrm{Tr}\ (Y^8 \cdot Y^8) = 1/6 \tag{2.267)(b}$$

$$\mathrm{Tr}\ (B^{15} B^{15}) = 1/12 \quad , \tag{2.267)(c}$$

while the sum of these traces is given by:

$$\mathrm{Tr}\ (I^3\ I^3) + \mathrm{Tr}\ (Y^8\ Y^8) + \mathrm{Tr}\ (B^{15} B^{15}) = 3/4\ . \tag{2.268}$$

These trace relationships will be very useful in the later discussions, and are intimately involved in the prediction of the grand unified mass scale (which again, is at least 10^{15} GeV. and perhaps as large as 1.22×10^{19} GeV.) , and of how the various interaction running couplings vary as a function of impact parameter q^2. (see (2.178)(b)). Particularly, if g_3, g_8 and g_{15} are used to designate running charges associated with each of the flavor generators (2.265) respectively, and if we define:

$$g_W \equiv g_3 \ ;\ g_Y \equiv g_8 \ ;\ g_B \equiv g_{15} \tag{2.269}$$

whereby, see eq. (1.9),

$$\frac{1}{e^2} \equiv \frac{1}{g_Q^2} = \frac{1}{g_Y^2} + \frac{1}{g_W^2} \rightarrow 137.036 \text{ as } q^2 \rightarrow 0 \tag{2.270}$$

is the inverse of the Coulomb (fine-structure) coupling; and if we further designate the square couplings:

$$a_W \equiv a_3 \ ;\ a_Y \equiv a_8 \ ;\ a_B \equiv a_{15} \quad , \tag{2.271}$$

then (2.269) and (2.271) are related via the trace of squares relationships (2.267). Specifically, (note again however, only left handed chiral projections are reflected here)

$$a_W = 2\ \mathrm{Tr}\ (I^3\ I^3) = g_W^2 \tag{2.272)(a}$$

$$a_Y = 2\ \mathrm{Tr}\ (Y^8\ Y^8) = \frac{1}{3} g_Y^2 \tag{2.272)(b}$$

$$a_B = 2\ \mathrm{Tr}\ (B^{15} B^{15}) = \frac{1}{6} g_B^2 \quad . \tag{2.272)(c}$$

At the grand unification mass M_G, whatever this mass may be, it is the above square couplings which must all converge, ie., at M_G:

$$a_W = a_Y = a_B \quad . \tag{2.273}$$

Hence, the trace relationships (2.267) determine the Clebsch-Gordon ratios among the three g_W, g_Y, g_B interaction charges at $q^2 = M_G^2$.

Returning then, to the question of how the various remaining flavor quantum numbers such as F, Q_u, L and Q are to be linearly constructed by combining I^3, Y^8 and B^{15} appropriately, it is very helpful to form the following square matrix product using (2.265), and to make the following definitions, with $\sigma = 3, 8, 15$, and diagonalized $\overset{SU(4)}{\wedge}$ indeces raised and lowered with $\delta^3{}_3 = \delta^8{}_8 = \delta^{15}{}_{15} = 1$ (cf. (2.237) for electroweak SU(2)xu(1) structure, and (2.25)(a));

$$\frac{1}{2}T^{\sigma}T_{\sigma} = \frac{1}{2}\left[\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & T_3 & 0 \\ 0 & 0 & 0 & -T_3 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{2}{\sqrt{3}}T_8 & 0 & 0 \\ 0 & 0 & -\frac{1}{\sqrt{3}}T_8 & 0 \\ 0 & 0 & 0 & -\frac{1}{\sqrt{3}}T_8 \end{pmatrix} + \begin{pmatrix} \frac{3}{\sqrt{6}}T_{15} & 0 & 0 & 0 \\ 0 & -\frac{1}{\sqrt{6}}T_{15} & 0 & 0 \\ 0 & 0 & -\frac{1}{\sqrt{6}}T_{15} & 0 \\ 0 & 0 & 0 & -\frac{1}{\sqrt{6}}T_{15} \end{pmatrix}\right]$$

$$= \begin{pmatrix} 3B^{15} & 0 & 0 & 0 \\ 0 & 2Y^8 - B^{15} & 0 & 0 \\ 0 & 0 & I^3 - Y^8 - B^{15} & 0 \\ 0 & 0 & 0 & -I^3 - Y^8 - B^{15} \end{pmatrix} \qquad (2.274)$$

$$\equiv \begin{pmatrix} Q_u & 0 & 0 & 0 \\ 0 & -L & 0 & 0 \\ 0 & 0 & I^3 - Y^8 - B^{15} & 0 \\ 0 & 0 & 0 & -Q - B^{15} \end{pmatrix} .$$

That is to say, we utilize this square matrix product to form the linear definitions:

$$Q_u = 3B^{15} \quad (= R + G + B) \qquad (2.275)(a)$$

$$L = B^{15} - 2Y^8 \qquad (2.275)(b)$$

$$Q = Y^8 + I^3 \quad , \qquad (2.275)(c)$$

and we continue to utilize:

$$F = Q_u + L = 4B^{15} - 2Y^8 \quad (\mp \ R + G + B + L) \qquad (2.275)(d)$$

As with eq. (2.220) for spin, and eq. (2.236) for Electroweak flavor, the above again shows the significance of this sort of square matrix product.

Why choose these particular definitions? Because they work; which is to say, using I^3, Y^8, B^{15} from Table 2.6 for all ten of the (left-handed) real fermions and bosons, as constructed out of the four complex preons listed in that same table, that one arrives via. eqs. (2.275), in an identical manner at the correct Q_u, L and Q quantum numbers for each of these particles. In particular, one arrives precisely at the correct set of quantum numbers as listed in The original Table 2.4, for all real particles. There is however, an important limitation here. While (2.275) do indeed provide the correct relationship among the various flavor quantum numbers for left handed fermions, they do not quite work out when one examines right handed chiral projections. As we shall see in the next section, this requires that we gauge the three flavor SU(4) generators (2.265) with a fourth U(1) interaction generator, which, as the generator of an ultra high energy abelian interaction, is most naturally associated with quantum gravitation. When this is properly done, in a manner similarly to that in which SU(2)xU(1) is used to develop electroweak interactions, but using the flavor generators (2.265) as part of an SU(4)xU(1) high energy flavor interaction, Then the right handed chiral states can also be fully accounted for. Again, this is the primary emphasis of the following section. Another, closely related difficulty with (2.275) as they stand, while they do provide the correct Q_u, L, Q, hence F for the real left handed projections, is that they lead to the following quantum numbers for the complex preons:

$$F(A)=1 \quad ; \quad F(B)=-1 \quad ; \quad F(C)=0 \quad ; \quad F(D)=0 \qquad (2.276)(a)$$

as anticipated, but that

$Q_u(A)=3/4$; $Q_u(B)=-\frac{1}{4}$; $Q_u(C)=-\frac{1}{4}$; $Q_u(D)=-\frac{1}{4}$ (2.276)(b)

$L\ (A)=\frac{1}{4}$; $L\ (B)=-3/4$; $L\ (C)=\frac{1}{4}$; $L\ (D)=\frac{1}{4}$, (2.276)(c)

contrary to what was anticipated following (2.251), namely, that $Q_u(A)=1$; $L(B)=-1$, and that all other preons have $Q_u=0$ and/or $L=0$. While this by itself might not seem to be too much of a difficulty, since the correct Q_u and L values are nevertheless generated for all real particles, this is tied closely to the difficulties obtaining a correct description of right handed quantum numbers, and to the fact that the quantum gravitational interaction has not yet been accounted for.

Finally, it is possible to place our discussion here to date on a somewhat more graphical footing. I^3,Y^8 and B^{15}, which respectively denote the mutually orthogonal flavor interactions associated with weak isospin, hypercharge and baryon numbers, can be examined using a flavor diagram which is an extension of the flavor diagram Figure 2.2 for Electoweak preon theory, with B^{15} drawn perpendicularly to I^3 and Y^8. In the I^3, Y^8 flavor plane (B^{15} suppressed) Table 2.6 can be cast in the more graphical $4 \otimes \overline{4} = 15 \oplus 1$ decomposition, as such: (NC≡Neutral Current)

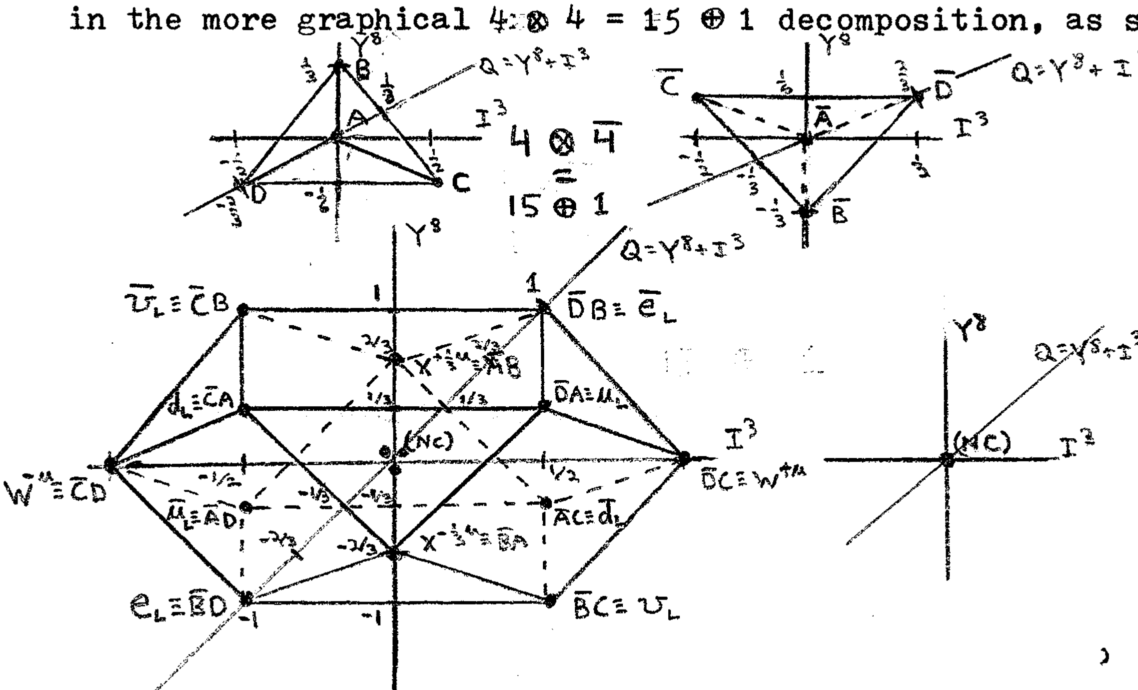


Figure 2.3- Flavor Decomposition of Real Left Handed Fermions and Bosons into Preons, I^3,Y^8 Plane, B^{15} suppressed

while in the Y^8,B^{15} flavor plane, one composes the flavor diagram:

(I^3 suppressed)

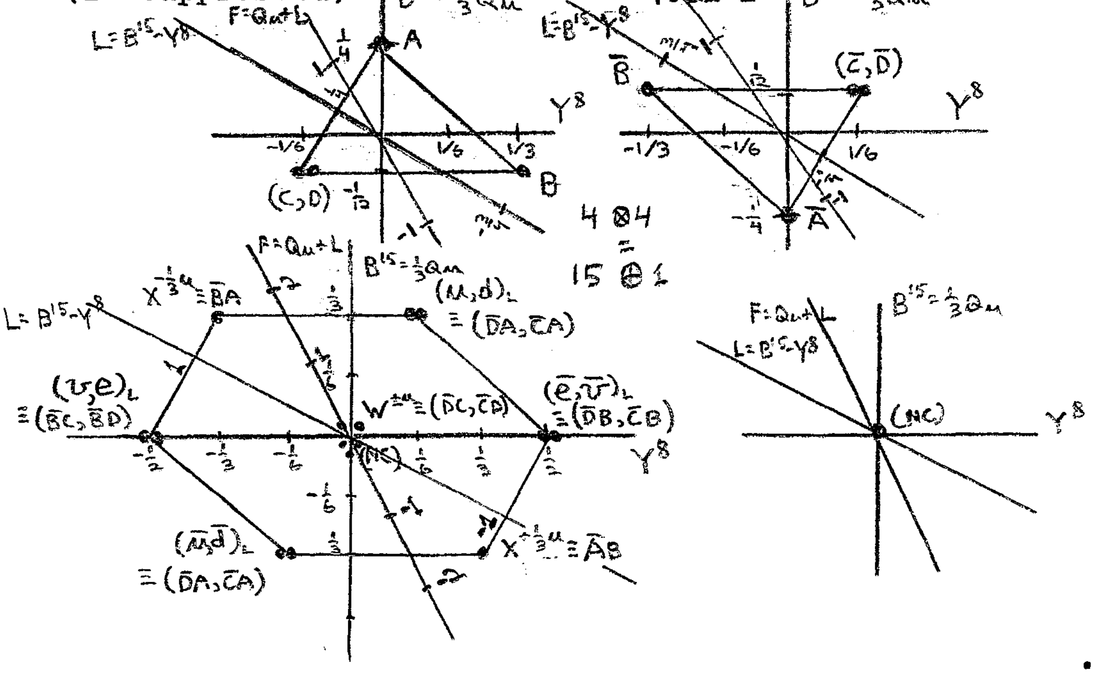
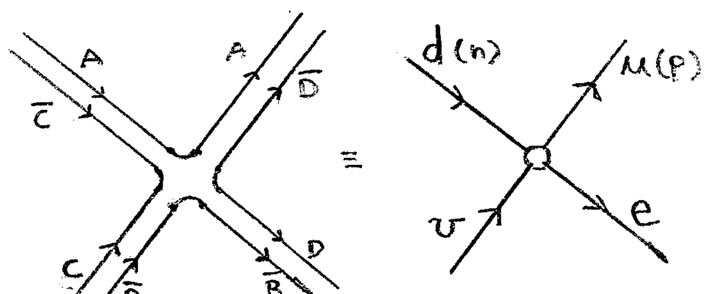


Figure 2.4- Flavor Decomposition of Real Left Handed Fermions and Bosons into Preons, Y^8,B^{15} Plane, I^3 Suppressed

Of course, any three of I^3,Y^8,B^{15},Q,Q_u and L, using (2.275), is sufficient to uniquely specify the above diagrams. Thus, the above composition describes three linearly independent quantum mechanical interactions. If we choose Q, I^3 and Q_u=$3B^{15}$=R+G+B, then one might say that the above describes the (colorless, generationless) left handed fermions and bosons, along with the electroweak and (colorless) strong interactions, as a unified set of interactions. One could of course, have conducted all of this discussion in the reverse direction. That is, starting simply with the standard particle table values for I^3,B and Y for the W^u and all left handed fermions, one could develop quite readily the type of 15⊕1 plot shown in Figures 2.3,2.4. Once this is done, if one has a basic understanding of how to construct and compose weight diagrams for the higher order non-abelian gauge groups such as SU(3) and SU(4), it becomes readily apparant that these Figures 2.3,2.4 are in fact based on an SU($\bar{4}$)xSU(4) type of composition, aside from the linear factors shown in (2.265). As such, it is virtually impossible to escape the conclusion that real Fermions and Bosons really are composed of preons. This approach, which is more direct, but which may not reveal as much physical content as the approach followed here, was in fact utilized by the author in the initial derivation of these results.

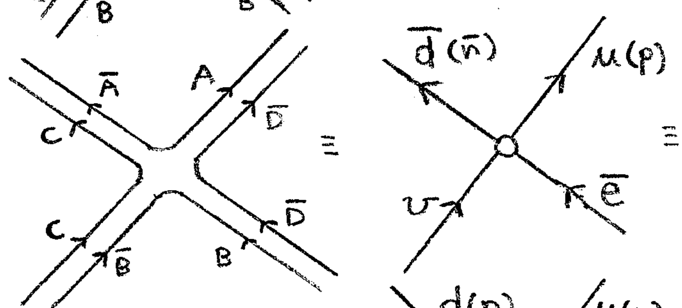
It is also useful to examine decay vertices involving all four preons, as these provide valuable insights into both nuclear beta-decay, and the hyperweak interactions of the grand unified part of the theory. Using the s,t,u channel processes described at the outset of Section 2.7, it is particularly illustrative, following our initial definition of preonic beta decay (2.252), to examine the β-decay processes: (related processes involving p=uud and n=udd in parentheses)

s:



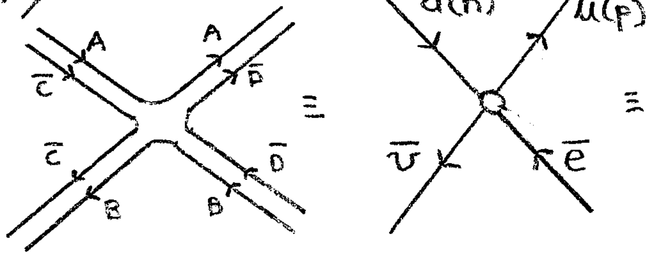
$$\equiv d(n)+\nu \rightarrow u(p)+e \qquad (2.277)(a)$$

t:



$$\equiv \bar{e}+\nu \rightarrow u(p)+\bar{d}(\bar{n}) \qquad (2.277)(b)$$

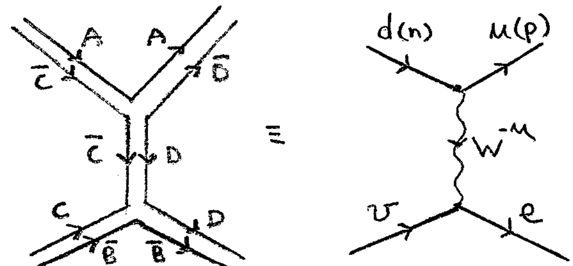
u:



$$\equiv d(n)+\bar{e} \rightarrow u(p)+\bar{\nu} \qquad (2.277)(c)$$

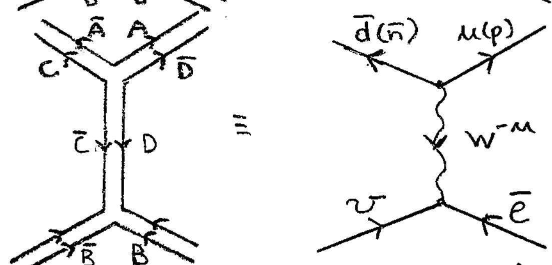
Using Table 2.6, the ~~"vertical" diagrams~~ are ordinary β-decay diagrams, as one might expect:

s:



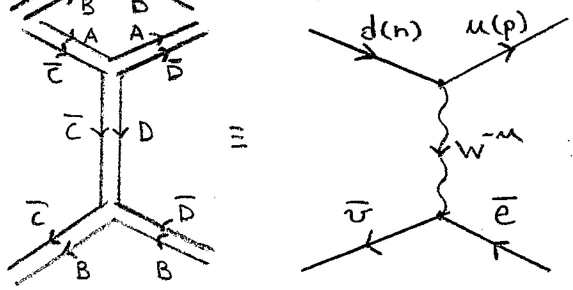
(2.278)(a)

t:



$\equiv \nu + \bar{e} \to W^{+\mu} \to \bar{d}(\bar{n}) + \mu(p)$ (2.278)(b)

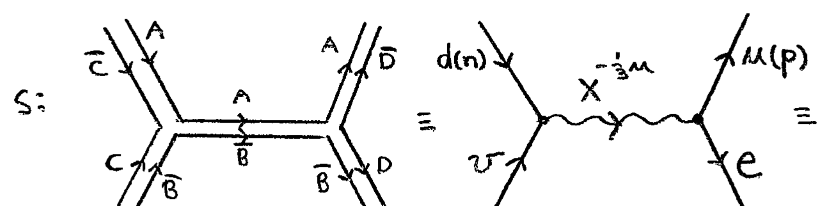
μ:



(2.278)(c)

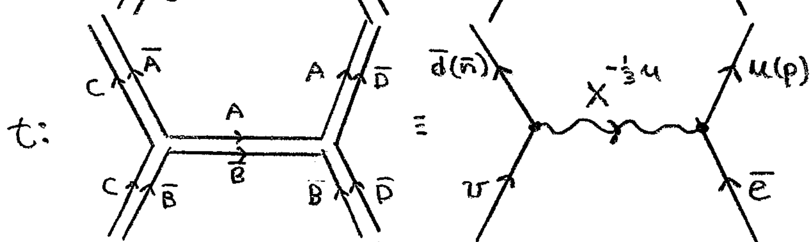
however, the "horizontal" diagrams might not be anticipated:

s:



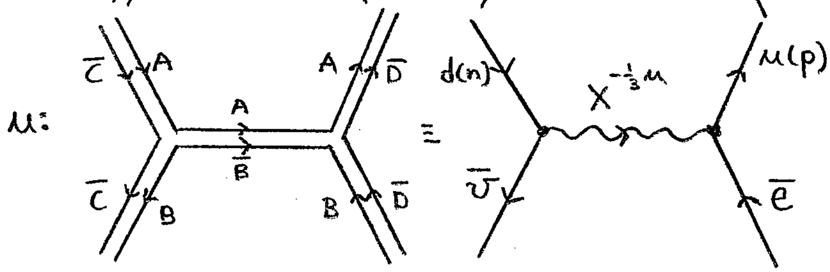
$\equiv d(n) + \nu \to X^{-\frac{1}{3}\mu} \to \mu(p) + e$ (2.279)(a)

t:



(2.279)(b)

μ:



(2.279)(c)

From Table 2.6, along with (2.275), one finds that the $X^{-1/3\,u}$ boson carries Q_u=1, L=1, hence F=2. Such particles, which decay quarks into (anti) leptons, as Pati has clearly pointed out, are the hallmarks by which any particular grand unified field theory is to be distinguished.[2.9] In the theory proposed here, the X^u exotic bosons of $Q=\pm 1/3$ are very closely related to the W^u bosons of electroweak theory, as is seen by contrasting (2.278) with (2.279). The process is the same; one merely examines different Feynman diagrams. Of course, the $X^{\frac{1}{3}u}$ bosons are expected to be many orders of magnitude more massive than the W^u bosons; hence the amplitude for their production will be extremely small at observable energies. Nevertheless, this serves to unify the weak and hyperweak interactions, as the hyperweak interaction here is but a high energy form of ordinary nuclear beta-decay. It is important to note, even without preon theory, that these hyperweak interactions are unavoidably implied by ordinary weak β-decay.

Finally, for mnemonic purposes, it is also convenient to describe the preonic composition of Table 2.6, Figures 2.3, 2.4 and (2.277)-(2.279) in the matrix format:

$$\begin{pmatrix}\bar{A}\\ \bar{B}\\ \bar{C}\\ \bar{D}\end{pmatrix}(A\ B\ C\ D) = \left(\begin{array}{cc|cc} \bar{A}A & \bar{A}B & \bar{A}C & \bar{A}D \\ \bar{B}A & \bar{B}B & \bar{B}C & \bar{B}D \\ \hline \bar{C}A & \bar{C}B & \bar{C}C & \bar{C}D \\ \bar{D}A & \bar{D}B & \bar{D}C & \bar{D}D \end{array}\right)$$

$$\equiv \left(\begin{array}{cc|cc} \bar{A}A & X^{+\frac{1}{3}u} & \bar{d}_L & \bar{u}_L \\ X^{-\frac{1}{3}u} & \bar{B}B & \nu_L & e_L \\ \hline d_L & \bar{\nu}_L & \bar{C}C & W^{-u} \\ u_L & \bar{e}_L & W^{+u} & \bar{D}D \end{array}\right) \qquad (2.280)($$

$$= \left(\begin{array}{c|c} \begin{array}{c}\text{Spin 1}\\ \text{(Colorless) Gluons,}\\ \text{Hyperweak Bosons}\\ (F=0,\pm 2)\end{array} & \begin{array}{c}\text{Spin } \frac{1}{2}\\ \text{Quarks, Leptons}\\ (F=\pm 1)\end{array} \\ \hline \begin{array}{c}\text{Spin } \frac{1}{2}\\ \text{Quarks, Leptons}\\ (F=\pm 1)\end{array} & \begin{array}{c}\text{Spin 1}\\ \text{Electroweak}\\ \text{Vector Bosons}\\ (F=0)\end{array} \end{array}\right).$$

In this format, the relationship between spin ½ fermions and spin 1 bosons also becomes particularly apparant. We are leaving the neutral current open for the moment, though the ultimate form here involves

Boson mixing similar to that which arises in Electroweak theory. This will be addressed in more detail during the discussion of Strong color, in Section 2.14, infra.

Thus, we conclude the initial set of discussions on preonic grand unification of the electroweak and (colorless) strong interactions. Table 2.6 and Figures 2.3,2.4 in particular, (also (2.280)) demonstrate the manner in which ten independent real left-handed chiral particle flavor projections, six Boson and four Fermion, may be decomposed easily into simply four complex flavors of preonic particle. The flavor quantum numbers I^3, Y^8, B^{15} for these preons, and the further quantum numbers F, Q_u, L and Q which may be composed from these by the linear combinations (2.275), are in turn related directly through a linear factor, as shown in eqs.(2.265), to the diagonalized generators of the strictly mathematical (simple) Lie Group SU(4). The primary decay channels through which particle flavor is redistributed are those highlighted in (2.277)-(2.279). The diagrams (2.278) show clearly in preonic terms <u>why</u> the left-handed quark and lepton projections undergo similar forms of weak beta decay; and hence, why the quarks and leptons, insofar as their isospin doublet structure, are so similar. The diagrams (2.279) show how in this approach, hyperweak proton decay can be viewed as but another, albeit ultra high energy diagram, for ordinary weak nuclear beta decay. Consequently, two areas of redundency are eliminated: that which exists as between the quarkonic and leptonic doublets and their respective forms of beta decay, and that which exists as between ordinary medium energy weak nuclear beta decay, and exotic high energy hyperweak proton decay. It should be noted that the decay mode shown here differs to some degree from that of the Georgi-Glashow SU(5) standard model. This elimination of redundencies through the preonic decomposition of nuclear beta decay should ultimately be manifested by a larger predicted value for the grand unified mass scale. Whether this mass scale, with all orders of renormalization considered, may yet turn out to be quantitatively equivalent with the long known magnitude of the Newton gravitational coupling energy 1.22×10^{19} GeV. is still an important question to be considered. First of course, one must make sure that quantum gravitation is included in the theory.

We close at this juncture by noting three outstanding, and closely related shortcomings, of the preonic Flavor theory as presented to this point. First, while this theory does unify the electroweak and (colorless) strong interactions, see Figures 2.3, 2.4 and discussion following, it does not as of yet have anything to say about the quantum gravitational interaction. Second, while this theory does work out correctly for left-handed chiral projections, it does not as of yet provide a basis for discussing right-handed chiral projections. Third, and this is more of a mathematical clue that points the way toward the resolution of the first two shortcomings, is the fact that the quark and lepton numbers for the various preons do not quite work out as expected. As shown in (2.276), (b), (c) they assume eigenvalues of $\pm\frac{1}{4}$ and $\pm 3/4$, rather than ± 1 and 0. This suggests that somehow we are "missing" a U(1) 4x4 unit matrix, multiplied by the factor of $\frac{1}{4}$. (See also the sum in (2.268), which is equal to 3/4.) This will all be discussed in depth in the ensuing section.

Also, aside from keeping an eye on the fact that $F = Q_u + L = R + G + B + L$ can be used to relate certain of the flavor quantum numbers, to certain of the color quantum numbers, we have not yet said anything explicit about color. Nor have we as of yet done anything explicit about the multiplicity of fermionic generations, These too must be addressed, and they will comprise a significant part of the discussion two and three sections hence.