Addendum to Section 2.12

Throughout the discussion in Section 2.12, we have continuously referred to the flavor generator $G^{\underline{0}}$ as the generator of the quantum gravitational interaction. Of course gravitation, as one ordinarily understands it, is mediated by a graviton of spin 2. On the other hand, all of the particles that we have discussed to this point are either spin 1 vector Bosons or spin $\frac{1}{2}$ Fermions. Thus the reader at this point may well be asking, where's the graviton?

We note first of all, if $G^{\underline{0}}$ is to be associated with quantum gravitation, that there exists also a spin 1 vector Boson $G^{\underline{0}u}$ of quantum gravitation (which in its observable form will "mix" in some manner with other flavors of Boson component, see, ie., (2.379), (2.380) infra); and which, in detailed preonic decomposition, using the propagator term of (2.190)-(2.193), the form (2.182) for vector Boson polarizations, the quantum gravitational generator (2.284) and associated singlet (2.298), and the running quantum gravitational couplings (2.287)-(2.289), may be described by: (recall, underline denotes a flavor index):

$$G^{\underline{0}u} = \frac{-g_G}{q^\sigma q_\sigma - m^2}\bar{\chi}G^{\underline{0}}\gamma^u\chi = \frac{-\frac{1}{4}g_G}{q^\sigma q_\sigma - m^2}\left(\bar{A}\gamma^u A+\bar{B}\gamma^u B+\bar{C}\gamma^u C+\bar{D}\gamma^u D\right) \tag{2.308).(1}$$

$$\equiv \epsilon^{\underline{0}u}\cdot e^{-iq^\sigma x_\sigma} \quad .$$

The question then is, how might this spin 1 vector Boson be related to the spin 2 graviton which is normally associated with gravitation?

At this point, it helps to recall the discussion of vector Boson polarization in Section 2.7. Particularly, using the form of

(2.198), one notes that the two transverse polarizations of $\epsilon^{\underline{0u}}$ above may be written as:

$$\epsilon^{\underline{0u}}_{(1)} = (\ 0, -1,\ 0,\ 0) \qquad (2.308).(2)(a)$$

$$\epsilon^{\underline{0u}}_{(2)} = (\ 0,\ 0, -1,\ 0)\ . \qquad (2.308).(2)(b)$$

To obtain the corresponding transverse plane polarizations for the spin 2 graviton, one need merely form the tensor products:

$$\epsilon^{\underline{00uv}}_{I} = \epsilon^{\underline{0u}}_{(1)}\epsilon^{\underline{0v}}_{(2)} + \epsilon^{\underline{0v}}_{(1)}\epsilon^{\underline{0u}}_{(2)} \qquad (2.308).(3)(a)$$

$$\epsilon^{\underline{00uv}}_{II} = \epsilon^{\underline{0u}}_{(1)}\epsilon^{\underline{0v}}_{(1)} - \epsilon^{\underline{0u}}_{(2)}\epsilon^{\underline{0v}}_{(2)}\ , \qquad (2.308).(3)(b)$$

along with the further linear combinations for a circular plane wave:

$$\epsilon^{\underline{00uv}}_{(+2)} = \epsilon^{\underline{00uv}}_{I} + i\,\epsilon^{\underline{00uv}}_{II} \qquad (2.308).(4)(a)$$

$$\epsilon^{\underline{00uv}}_{(-2)} = \epsilon^{\underline{00uv}}_{I} - i\,\epsilon^{\underline{00uv}}_{II}\ , \qquad (2.308).(4)(b)$$

where (a) and (b) above respectively, designate the polarizations for gravitons of spin +2 and -2. (See for example, Ohanian, Section 4.1.) Consequently, it is the $G^{\underline{0u}}$ vector Boson, which is associated with the quantum gravitational generator $G^{\underline{0}}$, and with polarization vector $\epsilon^{\underline{0u}}$, which is composed into the tensor products of (3) and (4) above, so as to produce what is generally thought of as the spin 2 graviton of gravitation. In particular, the gravitational field variables ϕ^{uv}, h^{uv} may be generalized to include explicit flavor indices, and are then developed in the usual manner, ie., as:

$$\phi^{\underline{00uv}} = \epsilon^{\underline{00uv}}\, e^{iE^{\sigma}x_{\sigma}} = h^{\underline{00uv}} - \tfrac{1}{2}\eta^{uv}\, h^{\underline{00}}\ . \qquad (2.308).(5)$$

As a consequence of the above, the gravitational potentials g^{uv} apparantly need to be generalized to include flavor indices, in the form:

$$g^{uv} \equiv g^{\underline{00uv}} = \eta^{uv} + \sqrt{5}\kappa h^{\underline{00uv}}\ , \qquad (2.308).(6)$$

where $\kappa = 8\pi G/c^4$. Thus, what one ordinarily regards as the gravitational potentials, are in fact just the $\underline{00}$ components of a larger flavor tensor which involves all interactions, and which, for SU(4)xU(1) flavor unification, contains the general flavor indices $\underline{UV}$, $\underline{U},\underline{V}=0,\ldots 15$. That is, what is ordinarily thought of as g^{uv} is in fact but one of 16x16=256 overall flavor components which involve all interactions. Based upon the linear approximation $-\kappa T^{uv}=(h^{uv} - \frac{1}{2}h^{uv}h)^{;\sigma}{}_{;\sigma} = \phi^{uv;\sigma}{}_{;\sigma}$ and upon (5) and (6) above, one anticipates likewise an extended form for the non-linear field equation, ie.,

$$-\kappa T^{\underline{UV}U}{}_{V} = R^{\underline{UV}U}{}_{V} - \tfrac{1}{2}\delta^{U}{}_{V}\, R^{\underline{UV}} \quad , \qquad (2.308).(7)$$

where the spacetime indeces u,v are also generalized to include chirality, ie., so as to be given by U,V=0,1,2,3,5. Consequently, the gravitational field equation also, is but a single flavor component, namely for $T^{\underline{00}U}{}_{V}$, of a 256 component flavor tensor equation that includes all flavors of interaction.

If we utilize $T^{\underline{U}}$, $\underline{U}=0,\ldots 15$ to designate all 16 of the SU(4)xU(1) flavor generators, including the four diagonalized generators $G^{\underline{0}}, I^{\underline{3}}, Y^{\underline{8}}, B^{\underline{15}}$ and the twelve non-diagonalized generators, it is intriguing to note that the symmetric and antisymmetric flavor structure constants, $s^{\underline{UV}}{}_{\underline{W}}$ and $a^{\underline{UV}}{}_{\underline{W}}$ respectively, may be defined according to: (contrast (2.24), (2.22))

$$s^{\underline{UV}} \equiv \tfrac{1}{2}\left(T^{\underline{U}}T^{\underline{V}} + T^{\underline{V}}T^{\underline{U}}\right) \equiv s^{\underline{UV}}{}_{\underline{W}}T^{\underline{W}} \qquad (2.308).(8)(a)$$

$$i\,a^{\underline{UV}} \equiv \tfrac{1}{2}\left[T^{\underline{U}}T^{\underline{V}} - T^{\underline{V}}T^{\underline{U}}\right] \equiv i a^{\underline{UV}}{}_{\underline{W}}T^{\underline{W}} . \qquad (2.308).(8)(b)$$

These flavor structure constants in turn, may be utilized along the lines of (2.30) to form a 16x16 flavor tensor, which is itself used to determine the strength of abelian vs. non-abelian characteristics for each interaction. This of course, is utilized in the renormalization group equations, to determine the running behaviour of each interaction at various energy (impact) scales, see for example, note 1.24. From eqs. (2.308).(3)-(7) however, it appears as though the symmetric and antisymmetric flavor structure constants (2.308).(8) will also figure into the formation of, among others, g^{uv} and T^{uv}, particularly because of the products (2.308).(3), and in light of the connections (2.308).(1) between the quantum gravitational polarization vector $\epsilon^{\underline{0}u}$ and the quantum gravitational generator $G^{\underline{0}}$. In general, this (eq.(2.308).(1)) established the connection between vector polarizations $\epsilon^{\underline{U}u}$ and flavor generators $T^{\underline{U}}$, U=0,...15, while it is the $T^{\underline{U}}$ which in turn, are used in the symmetric and antisymmetric products (2.308).(8). If this line of approach can be developed further and in more detail, then it may well be that the non-linear field equation (2.308).(7) can be of help in formulating higher order, non-linear corrections to the renormalization group equations. This in turn, should be of assistance in the prediction of the grand unification mass scale which, if quantum gravitation has truly been properly incorporated into the present theory, should work out within experimental errors to be of order 1.22 x 10^{19} GeV, once all higher order renormalization corrections have been fully accounted for.