2.12 - The Four-Preon Flavor SU(4)xU(1) Unification of the Electromagnetic, Weak, Colorless Strong and Quantum Gravitational Interactions; and the Colorless Flavor Classification of Left and Right Handed Real Fermion and Boson Chiral Projections, for a Single Fermion Generation

The requirement of chiral left-right symmetry for the various flavor and color quantum numbers of Table 1.1, specifically, F, Q_u, L, Q, R, G, B, but not necessarily I^3, Y^8, B^{15}, for all flavors of particle, was stated in the introduction to be the particular constraint by which the chosen G.U.T. flavor gauge group (which we now know to be preonic flavor SU(4), with diagonalized generators (2.265)) was to be gauged with quantum gravitation. Put into other terms, this is to say the preonic SU(4) flavor theory developed in the preceding section must now be reconciled with quantum gravitation; on the one hand so as to ensure the chiral symmetry of the generators for the flavor interactions F, Q_u, L, Q; and on the other, so as to account for the established experimental fact of chiral asymmetry in the generators I^3 and Y^8, and perhaps B^{15}. To implement this principle as a gauge condition, it is helpful to return to the equations (2.275) relating the left-right symmetric F, Q_u, L, Q to the not necessarily symmetric I^3, Y^8, B^{15}. Including left and right handed chiral designations, these equations may be rewritten as:

$$Q_{uLR} = Q_{uL} = Q_{uR} = 3B^{15}{}_L = 3B^{15}{}_R \qquad (2.281)(a)$$

$$L_{LR} = L_L = L_R = B^{15}{}_L - 2Y^8{}_L = B^{15}{}_R - 2Y^8{}_R \qquad (2.281)(b)$$

$$Q_{LR} = Q_L = Q_R = Y^8{}_L + I^3{}_L = Y^8{}_R + I^3{}_R \qquad (2.281)(c)$$

$$F_{LR} = F_L = F_R = 4B^{15}{}_L - 2Y^8{}_L = 4B^{15}{}_R - 2Y^8{}_R \quad . \qquad (2.281)(d)$$

The equation for Q is of course the same one utilized in electroweak theory, as indeed it must be to ensure that the electroweak interaction is properly embedded in a subset of the larger flavor theory, and that the electromagnetic interaction remains symmetric with respect to chirality, and by implication, parity. The problem with the above starts in equation (2.281)(a), which because of the required chiral symmetry of Q_u, also suggests the left/right chiral symmetry of B^{15}. If this is so, then substitution of (2.281)(a) into either of (2.281)(b) of (2.281)(d), and further substitution into (2.281)(c), leads directly to the consequence that:

$$Y^8{}_L = Y^8{}_R \qquad (2.282)(a)$$

$$I^3{}_L = I^3{}_R \quad . \qquad (2.282)(b)$$

Prior to the discovery by Lee and Yang of parity non-conservation in weak interactions, this might have been an acceptable result. In light of this discovery however, there is little doubt that eqs. (2.282) above are directly contradicted by a well established body of experimental data. On the other hand, if we start with the a-priori knowledge that $Y^8{}_L \neq Y^8{}_R$, $I^3{}_L \neq I^3{}_R$ and substitute back into eqs. (2.281), continuing to insist at least on the left-right symmetry of Fermion number F, one concludes individually, for quark number and lepton number, that:

$$Q_{uL} = R_L + G_L + B_L \neq Q_{uR} = R_R + G_R + B_R \qquad (2.283)(a)$$

$$L_L \neq L_R \quad ; \qquad (2.283)(b)$$

even though

$$F_L = R_L + G_L + B_L + L_L = F_R = R_R + G_R + B_R + L_R \quad . \qquad (2.283)(c)$$

This too, appears to be clearly contradicted by experiment, as it would among other things result in a strong color interaction that is not left-right symmetric, ie., which violates parity, in violation of established results.

Consequently, to maintain on the one hand the left-right symmetry of the strong and electromagnetic interactions; and on the other the left-right asymmetry of the weak interaction, we are given no choice but to make some form of modification in eqs. (2.281), originally (2.275).

In electroweak theory, one faces a similar problem, insofar as the electromagnetic Q generator is left-right symmetric, but the I^3 generator is not. The problem is solved by introducing a new U(1) interaction $Y \equiv Q - I^3$ which is also not left-right symmetric, but which combines with I^3 so as to leave Q left-right symmetric. In preonic grand unification with quantum gravitation, a similar solution exists. In particular, and in addition to I^3, Y^8, B^{15}, we now introduce a fourth flavor interaction generator G^0, which is to be associated with the quantum gravitational interaction. Because gravitation is a symmetric (abelian) interaction, this new generator must be associated with a U(1) subgroup. The appropriate choice, given that I^3, Y^8, B^{15} form a preonic SU(4) flavor subgroup with generators given in (2.265), is arrived at by defining:

$$G^0 \equiv \tfrac{1}{2}\,(1/\sqrt{2})\;T^0 = \begin{pmatrix} \frac{1}{4} & 0 & 0 & 0 \\ 0 & \frac{1}{4} & 0 & 0 \\ 0 & 0 & \frac{1}{4} & 0 \\ 0 & 0 & 0 & \frac{1}{4} \end{pmatrix}, \qquad (2.284)(a)$$

where:

$$T^0 = 1/\sqrt{2} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \qquad (2.284)(b)$$

taken together with T^3,T^8,T^{15} from (2.264), forms the simple Lie group SU(4)xU(1). It is <u>this</u> group, and not SU(4) by itself, which is therefore most readily associated with the unification of electroweak, strong <u>and</u> quantum gravitational interactions. Because G^0 is related by a constant factor to the 4x4 unit flavor matrix, it will commute symmetrically with all of the remaining generators of SU(4)xU(1), and will therefore generate an abelian interaction. This is precisely the characteristic that must be required of the quantum gravitational interaction. Contrast this to the use of SU(2)xU(1) in electroweak theory.

T^0, like T^3, T^8, T^{15} in (2.264), acquires the square root factor from the normalization condition: (see (2.266))

$$\mathrm{Tr}\ (T^0\ T^0) = 2 \quad , \tag{2.285)(a}$$

while: (see (2.267))

$$\mathrm{Tr}\ (G^0\ G^0) = \tfrac{1}{4} \quad . \tag{2.285)(b}$$

Combining this with (2.268) leads then to the overall sum of diagonalized traces of squares relationship:

$$\mathrm{Tr}\ (G^0 G^0) + \mathrm{Tr}\ (I^3 I^3) + \mathrm{Tr}(Y^8 Y^8) + \mathrm{Tr}(B^{15} B^{15}) = 1 \ , \tag{2.286}$$

which provides a particularly simple summarization of the preonic flavor generator matrices. Further, we can define a quantum gravitational charge: (see (2.269))

$$g_G \equiv g_0 \ , \tag{2.287}$$

and also the square coupling a_G, defined as (see 2.271)

$$a_G \equiv a_0 \quad . \tag{2.288}$$

utilizing (2.285)(b), these are related according to: (see (2.272))

$$a_G = 2\ \mathrm{Tr}\ ({G^0}_L {G^0}_L) = \tfrac{1}{2}\ g_G^2 \quad . \tag{2.289}$$

At the Planck mass M_P=1.22x10^{19} GeV, which we know a-priori must set the mass scale for unification with quantum gravitation, all four couplings must converge, and, with high order renormalization, this must take place at order $\frac{1}{2}$:

$$a_G = a_W = a_Y = a_B = \tfrac{1}{2}\hbar c. \tag{2.290}$$

At this point, we return to the diagonalized (2.265) along with (2.284)(a), and we again form the square matrix product of (2.274), but with the quantum gravitational generator G^0 included, ie.,

$$
\begin{aligned}
\tfrac{1}{2}T^{\delta}T_{\delta} &= \tfrac{1}{2}\begin{pmatrix} \frac{3}{\sqrt{6}}T^{15}_0 & 0 & 0 & 0 \\ 0 & \frac{2}{\sqrt{3}}T^8_0-\frac{1}{\sqrt{6}}T^{15}_0 & 0 & 0 \\ 0 & 0 & T^3_0-\frac{1}{\sqrt{3}}T^8_0-\frac{1}{\sqrt{6}}T^{15}_0 & 0 \\ 0 & 0 & 0 & -T^3_0-\frac{1}{\sqrt{3}}T^8_0-\frac{1}{\sqrt{6}}T^{15}_0 \end{pmatrix} \\
&+ \tfrac{1}{2}\begin{pmatrix} \frac{1}{\sqrt{2}}T^0_0 & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}}T^0_0 & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}}T^0_0 & 0 \\ 0 & 0 & 0 & \frac{1}{\sqrt{2}}T^0_0 \end{pmatrix} \\
&= \begin{pmatrix} G_0+3B_{15} & 0 & 0 & 0 \\ 0 & G_0+2Y_8-B_{15} & 0 & 0 \\ 0 & 0 & G_0+I_3-Y_8-B_{15} & 0 \\ 0 & 0 & 0 & G_0-I_3-Y_8-B_{15} \end{pmatrix} \\
&\equiv \begin{pmatrix} Q_u & 0 & 0 & 0 \\ 0 & -L & 0 & 0 \\ 0 & 0 & G_0+I_3-Y_8-B_{15} & 0 \\ 0 & 0 & 0 & G_0-Q-B_{15} \end{pmatrix} .
\end{aligned}
\tag{2.291}
$$

This is to say that we utilize the above matrix product, including the U(1) quantum gravitational generator G_0, in order to form the <u>ammended</u> linear definitions:

$$Q_u = G^0 + 3B^{15} \quad (=R+G+B) \tag{2.292)(a}$$

$$L = B^{15} - 2Y^8 - G^0 \tag{2.292)(b}$$

$$Q = Y^8 + I^3 \tag{2.292)(c}$$

$$F = Q_u + L = 4B^{15} - 2Y^8 \quad (= R + G + B + L) , \tag{2.292)(d}$$

which we shall henceforth use to replace (2.275). Note that it is only the Q_u and L definitions which are modified by the above, while Q and F are defined exactly as before. The addition of the G_0 term to the Q_u and L definitions has no impact on the already correct values of these quantum numbers for real left handed chiral projections, because $G_0=0$ for all of these projections, as we shall see shortly. This is <u>not</u> the case however, for right handed projections.

At this point, we return to the earlier discussion, and we rewrite eqs. (2.292) above to show explicit chiral components in the form of (2.281), ie.,

$$Q_{uLR} = Q_{uL} = Q_{uR} = G^0{}_L + 3B^{15}{}_L = G^0{}_R + 3B^{15}{}_R \qquad (2.293)(a)$$

$$L_{LR} = L_L = L_R = B^{15}{}_L - 2Y^8{}_L - G^0{}_L = B^{15}{}_R - 2Y^8{}_R - G^0{}_R \qquad (2.293)(b)$$

$$Q_{LR} = Q_L = Q_R = Y^8{}_L + I^3{}_L = Y^8{}_R + I^3{}_R \qquad (2.293)(c)$$

$$F_{LR} = F_L = F_R = 4B^{15}{}_L - 2Y^8{}_L = 4B^{15}{}_R - 2Y^8{}_R \qquad (2.293)(d)$$

Here however, one no longer runs into the contradictions with observation that were encountered in (2.282) and (2.283)(a),(b). Instead, one may rewrite the above simultaneously for right handed components as:

$$\begin{aligned} G^0{}_R &= \tfrac{1}{4}(Q_{uLR} - 3L_{LR} - 6(Q_{LR} - I^3_R)) \\ &= Q_{uLR} - \tfrac{3}{4}(F_{LR} + 2(Q_{LR} - I^3{}_R)) \end{aligned} \qquad (2.294)(a)$$

$$"I^3{}_R = I^3{}_R" \qquad (2.294)(b)$$

$$Y^8{}_R = Q_{LR} - I^3{}_R \qquad (2.294)(c)$$

$$\begin{aligned} B^{15}{}_R &= \tfrac{1}{4}(Q_{uLR} + L_{LR} + 2(Q_{LR} - I^3{}_R)) \\ &= \tfrac{1}{4}(F_{LR} + 2(Q_{LR} - I^3{}_R)) \quad . \end{aligned} \qquad (2.294)(d)$$

which is to say that we leave I^3_R undetermined, and solve for G^0_R, $Y^8{}_R$, $B^{15}{}_R$ in terms of the three (already known) left-right symmetric generators Q_{uLR}, L_{LR}, Q_{LR} (also F_{LR}) and the as yet unspecified $I^3{}_R$. We do know of course, that I^3 is the generator of ordinary weak interactions, and that these interactions are <u>not</u> left-right symmetric. In fact we know, that the "vector minus axial" weak interaction violates left-right symmetry maximally, insofar as it is left handed chiral projections only, which are involved in the weak interactions. Thus, following the well-established course of setting:

$$I^3_R = 0 \qquad (2.295)$$

for all right handed chiral projections (which bars these projections from participating in weak interactions), the remaining equations (2.294) reduce easily down to the following, wherein $(G^0,Y^8,B^{15})_R$ are given entirely in terms of the already known left right symmetric generators $(Q_u,L,Q)_{LR}$, and F_{LR}:

$$G^0{}_R = \tfrac{1}{4}(Q_u - 3L - 6Q) = Q_u - \tfrac{3}{4}(F + 2Q) \tag{2.296(a)}$$

$$Y^8{}_R = Q \tag{2.296(b)}$$

$$B^{15}_R = \tfrac{1}{4}(Q_u + L + 2Q) = \tfrac{1}{4}(F + 2Q) , \tag{2.296(c)}$$

wherein, because Q_u,L,Q,F are all explicitly gauged to exhibit left/right symmetry, we no longer need exhibit the $_{LR}$ subscript explicitly. The contradictions encountered in (2.282) and (2.283) are resolved, as it is now possible to ensure the left-right symmetry of all of Q_u,L,Q,F; and at the same time account for the known fact that I^3 and Y^8 are not left-right symmetric. The price we pay here however, is that the left-right asymmetry of the weak interaction propagates from I^3 and Y^8 to B^{15} and G^0. This is to say that the "baryon" interaction, B^{15}, to be very carefully distinguished from the "strong quark" interaction for which Q_u=R+G+B=G^0+3B^{15}, and the quantum gravitational interaction, G^0, which makes this distinction necessary, are themselves also not symmetric, with respect to left-right chiral projection. This is all the fault, as it were, of the weak interaction; and its origin lies in the original discovery of parity non-conservation in weak interactions, by Yang and Lee. Ultimately however, it is the quantum gravitational interaction, which is gauged with the remaining interactions so as to ensure that Q_u,L,Q,F (and color R,G,B) remain left-right symmetric, at the same time that the weak interaction I^3 generator remains maximally asymmetric (left handed V-A symmetry), which we are <u>forced</u> to introduce in order to account for all known facts about left-right chiral and parity conservation in electromagnetic and strong, and non-conservation in weak, interactions. If quantum gravitation did not exist, then one would be forced to invent it in order to account for chiral and parity non-conservation in weak interactions. The surprise is that quantum gravitation does not appear to conserve chirality or parity either. (Recall (2.135) and (2.153), which establish the close connection between parity and chiral symmetries, as each inverts the chiral dimension to produce L↔R transformations.

At this point, we have all of the basic ingredients required to arrive at a complete flavor classification of the elementary real Fermions and vector Bosons, in terms of their mesonic decomposition into complex preons, including both left and right handed chiral projections. We begin with the preonic flavor eigenvectors (2.262), which label the A,B,C,D preons. Henceforth, we must be concerned with both left and right handed chiral projections, which is to say that we should be concerned with both $(A,B,C,D)_L$ and $(A,B,C,D)_R$. It is more straightforward, if one concentrates particularly on the chiral projections of the various interaction generators. We start first with G^0, I^3, Y^8 and B^{15} from (2.284)(a) and (2.265), which we now know are to be associated with eigenvalues of the left handed projections, $G^0{}_L, I^3{}_L, Y^8{}_L, B^{15}{}_L$, and which may be formally regarded as the diagonalized 4x4 matrices associated via linear factors with the Lie flavor group SU(4)xU(1). We next utilize (2.292) to further arrive at the flavor projections for F, Q_u, L and Q. We insist as a gauge condition that these generators remain unchanged by a chiral transformation, ie., by any operation which takes L↔R. (Recall for example, (2.135), (2.153).) Thus, having already deduced these generators for left-handed projections, we automatically know their values for the right handed projections as well. We additionally insist as a further gauge condition, to account for what is observed experimentally, that the right handed weak interaction generator $I^3_R=0$, eq. (2.295). At this point, the three linearly independent gauge conditions $Q_{uL}=Q_{uR}$, $L_L=L_R$, $Q_L=Q_R$ imposed earlier allow us to use eqs. (2.292),(2.293) in an inverse capacity, (2.294)-(2.296), to arrive at an explicit determination of the remaining right handed generators $G^0{}_R, I^3{}_R, Y^8{}_R, B^{15}{}_R$. Pulling all of this together into a tabular form, showing only the particle states (flavor quantum numbers for antiparticles are of course the negative of those for particles), and noting again that $(Q_u,L,F,Q)_L=(Q_u,L,F,Q)_R$, one may arrive at the following: (contrast Table 2.6.)

	G^0_L	I^3_L	Y^8_L	B^{15}_L	G^0_R	I^3_R	Y^8_R	B^{15}_R	Q	Q_u	L	F
A	$\frac14$	0	0	$\frac14$	$\frac14$	0	0	$\frac14$	0	1	0	1
B	$\frac14$	0	1/3	-1/12	$\frac14$	0	1/3	-1/12	1/3	0	-1	-1
C	$\frac14$	$\frac12$	-1/6	-1/12	$-\frac12$	0	1/3	1/6	1/3	0	0	0
D	$\frac14$	$-\frac12$	-1/6	-1/12	1	0	-2/3	-1/3	-2/3	0	0	0
$X^{-\frac13 u}$	0	0	-1/3	1/3	0	0	-1/3	1/3	-1/3	1	1	2
W^{+u}	0	1	0	0	-3/2	0	1	$\frac12$	1	0	0	0
u	0	$\frac12$	1/6	1/3	-3/4	0	2/3	7/12	2/3	1	0	1
d	0	$-\frac12$	1/6	1/3	3/4	0	-1/3	1/12	-1/3	1	0	1
ν	0	$\frac12$	$-\frac12$	0	-3/4	0	0	$\frac14$	0	0	1	1
e	0	$-\frac12$	$-\frac12$	0	3/4	0	-1	$-\frac14$	-1	0	1	1
$\bar{A}A$	0	0	0	0	0	0	0	0	0	0	0	0
$\bar{B}B$	0	0	0	0	0	0	0	0	0	0	0	0
$\bar{C}C$	0	0	0	0	0	0	0	0	0	0	0	0
$\bar{D}D$	0	0	0	0	0	0	0	0	0	0	0	0

Table 2.7 - Flavor Quantum numbers for Complex Preons and Real Fermions and Bosons, Left and Right Handed Chiral Projections

¶ In the mixed flavor current sector, the explicit definitions of each of the real fermion and boson flavors of particle and by conjugation, antiparticle shown in the above, out of the four preonic flavors (A,B,C,D) and anti-flavors ($\bar{A}$,$\bar{B}$,$\bar{C}$, $\bar{D}$), is given by the following: (cf. (2.280))

$$X^{-1/3u} = \bar{B}A \quad ; \quad X^{+1/3u} = \bar{A}B \tag{2.297(a)}$$

$$W^{+u} = \bar{D}C \quad ; \quad W^{-u} = \bar{C}D \tag{2.297(b)}$$

$$u = \bar{D}A \quad ; \quad \bar{u} = \bar{A}D \tag{2.297(c)}$$

$$d = \bar{C}A \quad ; \quad \bar{d} = \bar{A}C \tag{2.297(d)}$$

$$\nu = \bar{B}C \quad ; \quad \bar{\nu} = \bar{C}B \tag{2.297(e)}$$

$$e = \bar{B}D \quad ; \quad \bar{e} = \bar{D}B \ . \tag{2.297(f)}$$

This takes care of twelve of the sixteen logical meson combinations that can be formed out of four complex preons, though due to particle-antiparticle considerations, only six of these twelve states are independent. It is the <u>mesonic</u> construction of the real particles out of complex preons in particular, which further allows placement of real particles and antiparticles into the <u>same</u> gauge multiplet. This is an important prerequisite for grand unification.[-2.10] The above shows clearly the manner of overlap between quarkonic and leptonic beta-decay, insofar as each involves either a preon or an antipreon $C \leftrightarrow D$ transition, while the $W^{\pm u}$ bosons themselves, which mediate both of quarkonic and leptonic beta decay, also contain precisely the meson combination of C and D preons necessary to conserve flavor throughout the entire process. (See, ie., (2.278)) The $X^{\pm 1/3u}$, which are involved in proton decay (hyperweak) phenomena, see (2.279), simply occur through a perpendicular beta decay channel, contrast (2.278), (2.279). Note, from table 2.7, that these X^u bosons carry $F=\pm 2$, which as Pati points out, is a further prerequisite for the exotic vector bosons of grand unification.[-2.11]

In the <u>neutral</u> flavor current sector, the four remaining logical mesons, $\bar{A}A$, $\bar{B}B$, $\bar{C}C$, $\bar{D}D$, are expected to mix in some as yet unspecified manner, to produce four independent flavors of neutral current vector boson. Among these must be the photon A^u, the electroweak Z^u, and the (colorless) gluon G^u. The connection to colored gluons will be covered in the section following. The remaining Boson, X^{0u}, is a very massive neutral vector boson associated with the hyperweak interaction. The precise specification of this particle takes place during strong/hyperweak symmetry breaking, in much the same manner that the specification of the Z^u in electroweak theory comes about. In reality, the neutral current vector bosons will involve the mixing of the following, with $\chi=(A,B,C,D)$, and G^0, I^3, Y^8, B^{15} taken from (2.284)(a) and (2.265): ($\bar{\chi}G^0\chi$ is a quantum gravitational singlet.)

$$\bar{\chi}G^0\chi = \tfrac{1}{4}(\bar{A}A + \bar{B}B + \bar{C}C + \bar{D}D) \tag{2.298)(a}$$

$$\bar{\chi}I^3\chi = \tfrac{1}{2}(\bar{C}C - \bar{D}D) \tag{2.298)(b}$$

$$\bar{\chi}Y^8\chi = \frac{1}{3}\bar{B}B - \frac{1}{6}\bar{C}C - \frac{1}{6}\bar{D}D \tag{2.298)(c}$$

$$\bar{\chi}B^{15}\chi = \tfrac{1}{4}\bar{A}A - \frac{1}{12}\bar{B}B - \frac{1}{12}\bar{C}C - \frac{1}{12}\bar{D}D \quad . \tag{2.298)(d}$$

In particular, one will wish as a further gauge condition to break the symmetry in the neutral current by ensuring that the photon A^u, and the (colorless) gluon G^u both remain massless. The electroweak Z^u (which is probably better labelled as W^{0u}), and the hyperweak X^{0u}, are both to be explicitly determined during the actual mixing of interactions, and can be anticipated to involve one or more Weinberg/Glashow type mixing angles. This is to say that: (See (2.292), (2.298)), (2.233(e)).)

$$A^u \equiv \bar{\chi}Q\chi = \frac{1}{3}\bar{B}B + \frac{1}{3}\bar{C}C - \frac{2}{3}\bar{D}D \tag{2.299)(a}$$

$$G^u \equiv \bar{\chi}Q_u\chi = \bar{A}A \tag{2.299)(b}$$

$$(W^{0u}=)Z^u \equiv \bar{\chi}Z\chi \equiv \bar{\chi}(I^3 - Q\,\mathrm{Sin}^2\theta_W)\chi = \tfrac{1}{2}(\bar{C}C-\bar{D}D) - \left(\tfrac{1}{3}\bar{B}B + \tfrac{1}{3}\bar{C}C - \tfrac{2}{3}\bar{D}D\right)\mathrm{Sin}^2\theta_W \tag{2.299)(c}$$

$$(X^{0u}=)L^u \equiv \text{Defined via Strong/Hyperweak Color Mixing.} \tag{2.299)(d}$$

Note that Z^u is a left-handed projection. Until we have discussed strong color symmetry in more detail, we do not yet have all of the information required to precisely specify X^{0u}. $Z^u(=W^{0u})$ however, remains the same as in Electroweak theory. For right handed Z^{u}, $I^{3}=0$, hence the $\frac{1}{2}(\bar{C}C-\bar{D}D)$ Factor is removed From (c) above. All other definitions are chiral symmetric.

In total, Table 2.7 along with (2.297) and (2.299) depicts sixteen (4x4) distinct flavors of real particle, of which exactly ten are fully conjugally independent, independently of chirality considerations. These four independent flavors of fermion, and six of vector boson, may be summarized thus:

Fermions

Quarks

(u,d)

Leptons

(ν,e)

Bosons

Electroweak

A^{u} (massless); W^{0u}, $W^{\pm u}$ (massive)

Strong/Hyperweak

G^{u} (massless); X^{0u}, $X^{\pm\frac{1}{3}u}$ (massive)

Table 2.8 - The Ten Elementary Flavors of Fermion and Vector Boson.

All other real particles, are to be regarded as composite configurations of the above elementary real flavors of particle. And, as we now know, these ten independent real particle flavors are in turn composed out of four independent complex flavors of preon. Thus, the essential simplicity of the preonic approach to grand unification lies in the fact that ten distinct flavors of real elementary particle can be even further reduced, down to but four distinct flavors, (A,B,C,D), of complex elementary preon. (Note, $10=\frac{1}{2}(4)(4+1)$) This decomposition, as was indicated on a number of prior occasions, is closely analogous to the decomposition of four real spacetime dimensions (Three conjugally independent) into two complex spinors, (Note, $3=\frac{1}{2}(2)(2+1)$) which is the reason that the monographs by Penrose and Rindler are of such particular interest.-2.12

We turn from the elementary particle flavors, to the various flavor interaction quantum numbers in Table 2.7. These of course, are linearly interconnected according to (2.292), ie.,

$$Q_u = G^0 + 3B^{15} \tag{2.300)(a}$$

$$L = B^{15} - 2Y^8 - G^0 \tag{2.300)(b}$$

$$Q = Y^8 + I^3 \tag{2.300)(c}$$

$$F = Q_u + L = 4B^{15} - 2Y^8 \,. \tag{2.300)(d}$$

These relationships, as we now know, are invariant with respect to the chiral transformation $L \leftrightarrow R$; however, in order to arrive at these chiral invariant relationships, it was necessary to introduce the U(1) abelian generator G^0 of quantum gravitation, and to utilize the above, rather than (2.281), as the basis for forming Q_u, L, Q, F. Eq. (2.281) does work for left handed particles, because $G^0_L=0$ for all real particles (see Table 2.7). However, $G^0_R \neq 0$ necessarily, for the right handed real particles; and hence, eqs. (2.281) are not chiral invariant, while (2.300) are so. From a generalized viewpoint, one now notes that electroweak/strong flavor unification, absent quantum gravitation, is not chiral symmetric. By starting at the outset with the requirement that the laws of nature respect the fifth-dimensional chiral symmetry, one is automatically led to introduce quantum gravitation, as a means of "reabsorbing" chiral non-symmetries in the remaining interactions. Quantum gravitation is, by itself however, chiral asymmetric, see Table 2.7. In fact, just as the weak interaction couples only left-handed projections, the quantum gravitational interaction couples only right-handed projections. The hyperweak $X^{\pm 1/3u}$ are the only real particles that exhibit chiral symmetry under all interactions, including G^0, I^3, Y^8, B^{15}. This is because the X^u charged bosons are composed out of the A and B preons, which also exhibit chiral symmetry under all interactions. This is not the case for the C and D preons, which are just the "isospin up" and "isospin down" preons of electroweak theory, see Section 2.9. It is this chiral asymmetry in the C and D preons which is the root of similar asymmetries in those real particles which contain either of the C and/or D preons.

Finally, it is helpful to refer back to equations (2.105)-(2.109), which describe the relationships among left and right handed, and vector and axial chiral projections. Eqs. (2.109),(2.110), which we repeat here in the general form,

$$T_V = \tfrac{1}{2}\,(T_R + T_L) \qquad (2.301)(a)$$

$$T_A = \tfrac{1}{2}\,(T_L - T_R)\ , \qquad (2.301)(b)$$

with the inverses:

$$T_R = T_V + T_A \qquad (2.301)(c)$$

$$T_L = T_V - T_A\ , \qquad (2.301)(d)$$

where T designates any and all of the generators G^0, I^3, Y^8, B^{15} or the linear combinations Q_u, L, Q, F, (and also R,G,B in color theory). This is to say that the above relationships among the R,L,V,A generator projections are perfectly general, and can be applied to develop a precise measure of the degree of chiral symmetry or asymmetry, in each of the various interaction generators. In particular, one may again return to Table 2.7. Because this table already provides the numeric values for each of the various flavor quantum numbers, for each of the various complex preons and real fermions and bosons, for each of the left and right handed chiral projections; it is further possible, using (2.301)(a) and (b) in particular, to develop a similar tabularization for the vector and axial generator projections. For the left-right symmetric generators Q, Q_u, L, F, (2.301) further reduces to: (see (2.111))

$$T_V = T_R = T_L \qquad (2.302)(a)$$

$$T_A = 0 \ . \qquad (2.302)(b)$$

This is not a prediction, but rather a consistency check on our calculation, as these particular generators were explicitly designed (gauged) so as to be left-right symmetric for all particles. The V and A generators which are therefor of particular interest here are those which do <u>not</u> manifest chiral symmetry for all particles, namely, G^0, I^3, Y^8, B^{15}. The V and A components for these generators, particularly G^0 and B^{15} (I^3 and Y^8 are already known for most particles in standard electroweak theory), are indeed predictions of the theory, and provide one basis upon which experimental predictions of preonic G.U.T. might perhaps be predicated. This Table is as follows:

	G^0_V	I^3_V	Y^8_V	B^{15}_V	G^0_A	I^3_A	Y^8_A	B^{15}_A	Q_{uV}	L_V	Q_V	F_V	Q_{uA}	L_A	Q_A	F_A
A	$\frac{1}{4}$	0	0	$\frac{1}{4}$	0	0	0	0	1	0	0	1	0	0	0	0
B	$\frac{1}{4}$	0	1/3	- 1/12	0	0	0	0	0	-1	1/3	-1	0	0	0	0
C	-1/8	$\frac{1}{4}$	1/12	1/24	-3/8	$-\frac{1}{4}$	$\frac{1}{4}$	1/8	0	0	1/3	0	0	0	0	0
D	5/8	$-\frac{1}{4}$	-5/12	- 5/24	3/8	$\frac{1}{4}$	$-\frac{1}{4}$	-1/8	0	0	-2/3	0	0	0	0	0
$X^{-\frac{1}{3}u}$	0	0	-1/3	1/3	0	0	0	0	1	1	-1/3	2	0	0	0	0
W^{+u}	-3/4	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{4}$	-3/4	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{4}$	0	0	1	0	0	0	0	0
u	-3/8	$\frac{1}{4}$	5/12	11/24	-3/8	$-\frac{1}{4}$	$\frac{1}{4}$	1/8	1	0	2/3	1	0	0	0	0
d	3/8	$-\frac{1}{4}$	-1/12	5/24	3/8	$\frac{1}{4}$	$-\frac{1}{4}$	-1/8	1	0	-1/3	1	0	0	0	0
ν	-3/8	$\frac{1}{4}$	$-\frac{1}{4}$	1/8	-3/8	$-\frac{1}{4}$	$\frac{1}{4}$	1/8	0	1	0	1	0	0	0	0
e	3/8	$-\frac{1}{4}$	-3/4	- 1/8	3/8	$\frac{1}{4}$	$-\frac{1}{4}$	-1/8	0	1	-1	1	0	0	0	0
AA	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
BB	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
CC	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
DD	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Table 2.9 - Vector and Axial Flavor Quantum Numbers for Complex Preons and Real Fermions and Bosons

One well settled manner in which the above may be utilized is to determine the C_V and C_A coefficients for the electroweak neutral current $Z=I^3 - Q\,\mathrm{Sin}^2\theta_W$. In particular, referring to Tables 2.7, 2.9 and eqs. (2.301)-(2.302), it is possible to write:

$$
\begin{aligned}
Z &= I^3 - Q\,\mathrm{Sin}^2\theta_W \\
&= (I_V^3 - Q_V\,\mathrm{Sin}^2\theta_W) + \gamma^5(I_A^3 - Q_A\,\mathrm{Sin}^2\theta_W) \\
&= (\tfrac{1}{2}I_L^3 - Q_L\,\mathrm{Sin}^2\theta_W) + \gamma^5(-\tfrac{1}{2}I_L^3) \\
&\equiv \tfrac{1}{2}(C_V - C_A\gamma^5) \quad .
\end{aligned}
\tag{2.303}
$$

As a consequence of the above, one deduces for all particles, since $Q_L = Q_R$ while $I_R^3=0$, that: -2.13

$$C_V = I^3 - 2\cdot Q\,\mathrm{Sin}^2\theta_W \tag{2.304)(a}$$

$$C_A = I^3 \quad . \tag{2.304)(b}$$

The usual factor of $\frac{1}{2}$ used in $\frac{1}{2}(1\pm\gamma^5)$ is therefore already included in the definitions of the V and A generators in Table 2.9. While the above (2.303)(2.304) applies only to the electroweak neutral current, it is possible to engage in a similar exercise, and to derive similar sorts of results, for the remaining generators, and further linear neutral current combinations thereof, from Table 2.9. Referring to Tables 2.7 and 2.9, it is the prediction of a non-zero baryon number for right-handed electron and neutrino projections, which appears to be particularly intriguing. This suggests that right-handed leptons <u>do</u> carry a baryonic charge (but <u>not</u> a quark or color charge - be careful about this distinction) which is due to quantum gravitation, since $Q_m = G^6 + 3B^{15}$, and that such a charge might be physically observable during appropriate experimentation. Note too, that this chiral asymmetry in the baryonic interaction is due to, and ultimately compensated for, by the quantum gravitational interaction, see (2.300)(a). Therefore, observation of a chiral asymmetry in the baryonic interaction would at the same time provide indirect evidence for the existance of a quantum gravitational interaction which does not respect chiral symmetry either.

It is also illustrative to examine how the various flavor interactions "sit across" one another, as this greatly simplifies understanding of the overall theory. In the $SU(2)_I xU(1)_Y$ electroweak interaction, the $U(1)_{em} \equiv U(1)_Q$ interaction is said to sit across the $SU(2)_L \equiv SU(2)_I$ weak isospin and the $U(1)_Y$ hypercharge interaction. This is just a fancy way of saying that the electrostatic charge generator Q, which is associated with a U(1) abelian gauge group, is composed out of a linear combination of the non-abelian SU(2) weak interaction isospin generator I^3 and an abelian U(1) hypercharge interaction generator Y, in particular, because of the already familiar $Q = Y + I^3$. We have left the ^{8} superscript off of Y for reasons that will become apparant shortly. At an impact energy on the order of the Fermi coupling constant $mc^2 = \sqrt{\hbar c^5/G_F} \approx 10^2$ GeV, the symmetry of the $SU(2)_I xU(1)_Y$ electroweak interaction is "spontaneously broken" down to that of the $U(1)_Q$ electromagnetic interaction, which is most readily observed at lower energies. Of course, only two of the three interaction generators I^3, Y and Q are linearly independent. We may select for instance, Q and I^3, thereby regarding the overall $SU(2)_I \times U(1)_Y$ theory as a unification of the electromagnetic and weak flavor interactions.

In preonic flavor grand unification, a similar, though somewhat extended set of considerations apply. If we again examine (2.300)(a)-(c) for Q, L and Q_u, one may write without superscripts:

$$Q = Y + I \qquad (2.305)(a)$$

$$L = B - 2Y - G \qquad (2.305)(b)$$

$$Q_u = G + 3B \quad . \qquad (2.305)(c)$$

This suggests, in the same manner that the electromagnetic interaction "sits across" the hypercharge and weak isospin interactions in electroweak theory, that lepton number and its associated interaction sits among all three of the baryonic, hypercharge and quantum gravitational interactions; while quark number, and its associated interaction (which we shall shortly be able to associate with the strong color interaction) sits across the baryonic and quantum gravitational interactions, in preonic grand unification with quantum gravitation. One may also note that Q, L and Q_u all obey the left-right chiral symmetry (by design), while I, Y, B and G do not, see, eg., Table 2.9. Thus, one may summarize the relationships among the various interactions with the following interaction diagram.

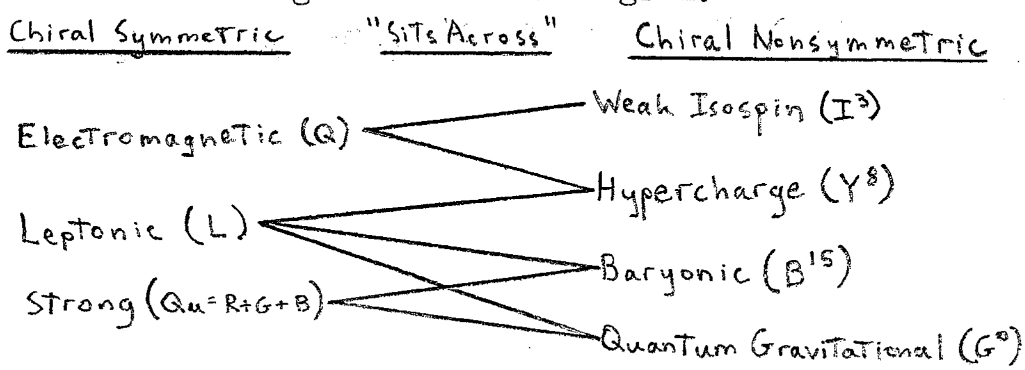

Figure 2.5 - Summary of Preonic G.U.T. Flavor Interactions

While the above shows a total of seven interactions, we know from (2.305) that only four of these are linearly independent. We may choose, for instance, the Q, I, Q_u and G generators, and thereby regard the above as a unified (quantum mechanical) theory of the electromagnetic, weak, strong (once we discuss color) and gravitational interactions. At quantum gravitational (Planck) energies, which one anticipates to be of the Cavendish coupling order $mc^2 = \sqrt{\hbar c^5/G} = 1.22 \times 10^{19}$ GeV., and which, as indicated in Section 1.1, should be independently predicted by the G.U.T., one begins now with an $SU(4)_B \times U(1)_G$ symmetry group. The quark interaction, Qu, which will shortly become associated with the strong color interaction, sits across the baryonic and gravitational interactions, and is manifest when the $SU(4)_B \times U(1)_G$ interaction symmetry is spontaneously broken below the Planck energy 1.22×10^{19} GeV. At this point, one reduces to a $SU(3)_Y \times U(1)_B \times U(1)_{Q_u}$ interaction symmetry. At some intermediate energy scale $mc^2 = \sqrt{\hbar c^5/G_I}$, set by an intermediate coupling constant G_I, (which may turn out to be associated with an energy of $\approx 10^{15}$ GeV) the lepton interaction first becomes apparent, which is to say that lepton number, which above the G_I scale is regarded merely as a fourth color of quark, now becomes distinct from the remaining three colors R,G,B of quark. Recall that lepton number sits among all of the baryonic, hypercharge and gravitational interactions. At this point, one further reduces to a $SU(2)_I \times U(1)_Y \times U(1)_{Q_u} \times U(1)_L$ symmetry group, which is closely related to the $SU(3)_{\text{color}} \times SU(2)_I \times U(1)_Y$ flavor group which, as is well established, must be incorporated at low energies into any phenomenologically correct grand unified theory. Finally, below the Fermi coupling energy $mc^2 = \sqrt{\hbar c/G_F} \approx 10^2$ GeV., the $SU(2)_I \times U(1)_Y$ electroweak gauge group is reduced even further, to the low energy symmetry group which we shall designate by $U(1)_Q \times U(1)_{Q_u} \times U(1)_L$. Once color is accounted for, this will be more fully represented as $SU(3)_{\text{color}} \times U(1)_{\text{lepton}} \times U(1)_Q$. All of this is tabularized below:

Gauge Group	Broken Below
$SU(4)_{B15} \times U(1)_{G0}$	G ($\approx 10^{19}$ GeV)
$SU(3)_{Y8} \times U(1)_{B0} \times U(1)_{Q_u}$	G_I ($\approx 10^{15}$ GeV)?
$SU(2)_{I3} \times U(1)_{Y0} \times U(1)_L \times U(1)_{Q_u}$	G_F ($\approx 10^2$ GeV)
$U(1)_Q \times U(1)_L \times U(1)_{Q_u}$	Low Energy Group

Table 2.10- Stages of Spontaneous Flavor Symmetry Breaking

Note in the above, that B^{15} "migrates" to B^0 below $\approx 10^{19}$ GeV., while Y^8 migrates to Y^0 below what may be $\approx 10^{15}$ GeV. This is the reason that some of the numeric superscripts were explicitly omitted in some of the earlier discussion, ie., eq. (2.305).

It is also worth noting, now that we know about both the left and right handed chiral projections, see Table 2.7, to form the various trace relationships corresponding to (2.285)(b) and (2.267), which account for both left and right handed chiral projections. In the preonic representation, which uses the values of $(G^0, I^3, Y^8, B^{15})_{L,R}$ for each of the A,B,C,D preons in Table 2.7, one finds that:

$$\mathrm{Tr}\,(G^0_L\,G^0_L) + \mathrm{Tr}\,(G^0_R\,G^0_R) = \tfrac{1}{4} + 11/8 = 13/8 \qquad (2.306)(a)$$

$$\mathrm{Tr}\,(I^3_L\,I^3_L) + \mathrm{Tr}\,(I^3_R\,I^3_R) = \tfrac{1}{2} + 0 = \tfrac{1}{2} \qquad (2.306)(b)$$

$$\mathrm{Tr}\,(Y^8_L\,Y^8_L) + \mathrm{Tr}\,(Y^8_R\,Y^8_R) = 1/6 + 2/3 = 5/6 \qquad (2.306)(c)$$

$$\mathrm{Tr}\,(B^{15}_L B^{15}_L) + \mathrm{Tr}\,(B^{15}_R B^{15}_R) = 1/12 + 5/24 = 7/24\,. \qquad (2.306)(d)$$

Of particular interest is the Electroweak Clebsch-Gordon coefficient C, which may be deduced in particular, from (2.306)(b) and (c) above. This is given by:

$$C^2 \equiv \frac{\mathrm{Tr}(Y^8_L\,Y^8_L) + \mathrm{Tr}\,(Y^8_R\,Y^8_R)}{\mathrm{Tr}(I^3_L\,I^3_L) + \mathrm{Tr}\,(I^3_R\,I^3_R)} = 5/3 \qquad (2.307)$$

It is encouraging to note that this is precisely the same value for C that is predicted by the full Georgi-Glashow SU(5) grand unified theory.[2.14] It is also intriguing that the value of C^2 turns out this way, since color and generation symmetry has not yet been considered. One should expect therefore that many of the numerical predictions of SU(4)xU(1) preonic flavor unification will correspond closely with those of the standard SU(5) model, particularly insofar as the value of C^2 is a relevent parameter in any given calculation.

Finally, referring back once again to Table 2.7 we now note, by virtue of the introduction of the quantum gravitational interaction, that all of the A,B,C,D preons now possess all of the correct quantum (flavor) numbers originally specified in the discussion following eq. (2.254), including quark and lepton numbers. Specifically,

$$Q_u(A) = 1; \quad Q_u(B) = 0; \quad Q_u(C) = 0; \quad Q_u(D) = 0 \qquad (2.308)(a)$$

$$L\,(A) = 0; \quad L\,(B) = -1; \quad L\,(C) = 0; \quad L\,(D) = 0\;, \qquad (2.308)(b)$$

rather than the $\pm\, 3/4$ and $\pm\,\frac{1}{4}$ values that appeared in the initial discussions without quantum gravitation, see (2.276)(b),(c). This particular feature of preonic grand flavor unification with quantum gravitation will serve us particularly well in the discussions of color and generation symmetry, to follow.