

2.13 - The Six-Preon Unification of Flavor  $SU(4) \times U(1)$  with High Energy Color  $SU(4) \times U(1)$  and Two Overlapping Degrees of Freedom; the Flavor and Color Classification of Real Fermions and Vector Bosons for a Single Generation; and the Derivation of Electroweak and Strong/Hyperweak Massless and Massive Neutral Current Vector Bosons

Let us step back at this point, to review briefly the progression of the last five sections. In Section 2.8, which was based in large part on the discussions of the prior sections 2.1-2.7, we began by examining the covariant classification of the various real and virtual polarization states, for ordinary massless and massive vector Bosons. We saw that the four covariant polarizations of ordinary real vector Bosons could be decomposed into two complex polarizations, of spin one-half, namely, the spinor states "spin up" and "spin down." As further indicated, this is closely related to the manner in which four real spacetime coordinates may be decomposed into two complex spinor coordinates, in the manner suggested in-depth by Penrose and Rindler. -2.15

In section 2.9, it was shown that a very similar sort of decomposition is possible, for the four real particles  $A^u$ ,  $W^{+u}$ ,  $W^{-u}$ ,  $W^{0u}$  ( $=Z^u$ ) of electroweak theory. Here, the complex spinor states, which were labelled as "preons," are referred to as "isospin up" and "isospin down," utilizing the weak isospin model of beta-decay that has long been known; and it is these states which are conversely composed into the four real (three independent) Bose particles of electroweak theory. Anticipating the introduction of additional preons, this isospin preon doublet was labelled as (C,D). The  $W^{\pm u}$  are composed by the mesonic combination of these C and D preons; while the very similar forms of quarkonic and leptonic beta decay are regarded to take place

whenever a C preon (or antipreon) is exchanged with a D preon (or antipreon), for either of these two forms of beta-decay. It is in this manner that an important redundancy, as between the quarkonic and leptonic forms of beta decay, is in some sense "reduced" to its fundamental components. The distinction between the quarks and the leptons themselves, is achieved by the addition of two additional preons, A and B, which are associated respectively with the quark and (anti)lepton charges. These may be further composed with C and D, in mesonic fashion, to produce the real fermionic flavors of particle (u,d) and ( $\bar{u}$ ,e). Ultimately, this must all be put together in such a manner as to ensure the correct conservation of flavor at all vertices, and so as to ensure the correct flavor quantum numbers for all of these particles, along the lines of the vertex diagram (2.252).

In sections 2.10, <sup>2.11</sup> this is shown in detail. By what Minkowski might perhaps have regarded as ~~on the basis of~~ "pre-established harmony" between theoretical physics and certain key branches of mathematics, it turns out that the flavor quantum numbers necessary to generate all of the known real <sup>(left-handed)</sup> fermions and bosons, and their various flavor quantum numbers, are supplied for the four preons, by an SU(4) flavor group that is linearly related to the SU(4) Lie algebra of pure mathematics, see eqs. (2.265) and (2.275). This SU(4) flavor group by itself however, was found to be insufficient, if one wished also to consider right-handed chiral projections.

By an even further harmony, section 2.12 demonstrates, in order to account simultaneously for the observed chiral symmetry of the electromagnetic and strong (with lepton number) interactions; and at the same time

→ projections, see eg., Table 2.7.

for the known chiral asymmetry in the weak (and hypercharge) interactions, that it is necessary to extend the flavor gauge group further. With this addition, it then became possible to account for right-handed to  $SU(4) \times U(1)$ . This new interaction associated with the  $4 \times 4$   $U(1)$  flavor generator, shown in eq. (2.284), is an abelian interaction; and it is most readily associated with the quantum gravitational interaction. A particular bonus resulting from the inclusion of  $U(1)$  quantum gravitation with the original  $SU(4)$  gauge group, was the fact noted, at the close of the prior section, eqs. (2.308), that the A and B preons now become the exclusive carriers of the quark and lepton charges; whereas without quantum gravitation, eqs. (2.276)(b) and (c), the quark and lepton charges would be non-exclusively distributed among all of the A, B, C, D preons. By confining quark and lepton numbers to the A and B preons, it becomes possible to examine certain additional symmetries as between these two preons, independently of the C and D preons of electroweak theory. The re-addition of the C and D preons, from this vantage point, serves to distinguish simply between the isospin-up and isospin-down states of quark and lepton; with the issue of quark vs. lepton already determined by whether one is utilizing an A or a B preon. Given the above characterization, it is convenient for future reference if we give specific names to the various preons. Particularly, we shall define:

- A ≡ Quark  $(Q_u = 1, L = 0, I^3 = 0)$  (2.309)(a)
- B ≡ Antilepton  $(Q_u = 0, L = -1, I^3 = 0)$  (2.309)(b)
- C ≡ Isospin Up  $(Q_u = 0, L = 0, I^3 = \frac{1}{2})$  (2.309)(c)
- D ≡ Isospin Down  $(Q_u = 0, L = 0, I^3 = -\frac{1}{2})$  (2.309)(d)

Even with all of this however, a number of limitations remain. One

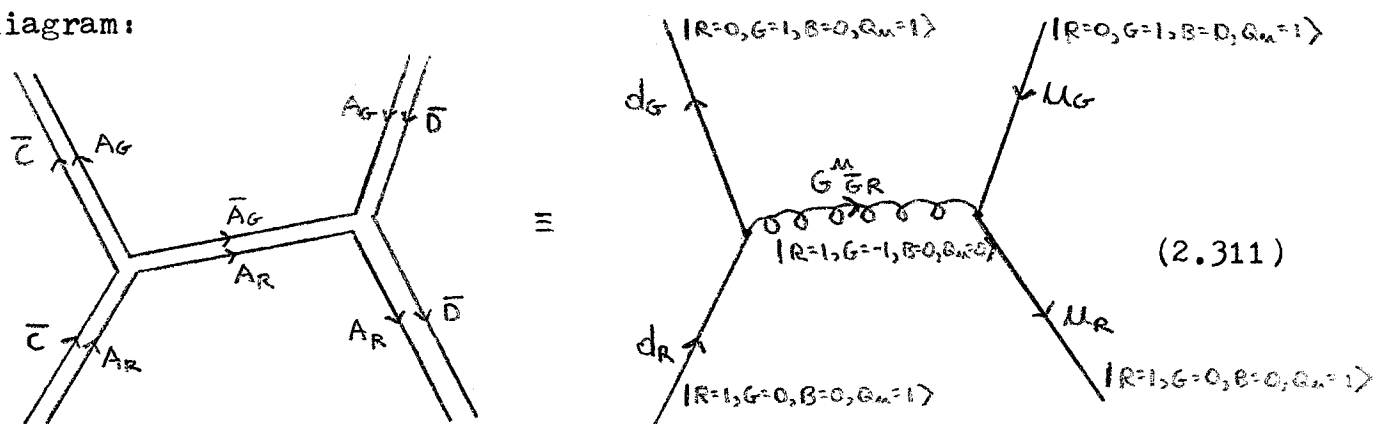
of the more notable limitations shows up, for example, in Table 2.10, where the quark interaction is designated as an abelian U(1) interaction. Clearly, this is not correct, and some modification needs to be made to reflect the fact that low-energy strong interactions actually obey a non-abelian SU(3) color symmetry. Additionally, the multiplicity of fermionic generations has not yet been addressed. At this point, we are now ready to address these remaining questions.

It has been noted on a number of occasions, and originally in equation 1.1 of the introduction, that the quark number  $Q_u$  for any given flavor of particle is related to its color quantum numbers R,G,B according to the rather simple:

$$Q_u = R + G + B . \quad (2.310)$$

Further, as shown in (2.309), and by virtue of the introduction of quantum gravitation in the preceding section, it is the A preon, with  $Q_u = 1$ , which is the exclusive carrier of the quark charge. All of the remaining preons, B,C,D, have  $Q_u = 0$ . From eqs. (2.297), we know that the A preon appears exclusively in the "up" and "down" quarks, <sup>and the (to this point) colorless gluon of (2.299)(b)</sup> the hyperweak  $X^{-1/3u}$  vector boson, <sub>A</sub> We also know, on the basis of well settled theory, that it is only the up and down quarks, <sup>and gluons,</sup> along with any charged hyperweak vector bosons, which indeed carry the strong color charge. Taking all of this into consideration, it now becomes apparant that the A preon, with  $Q_u = 1$ , is in fact to be regarded as the exclusive preonic carrier of the strong color charge interaction. Specifically, it appears that the A preon, which we have thus far regarded as a single flavor of preon, may in fact exist in any one of three color states, which, following the usual approach, we label as R, G, B for "red," "green" and "blue" respectively.

Thus, it will be useful from here on, to label the A preon with an additional subscript to denote color, specifically,  $A_R, A_G, A_B$ . Regardless of the particular color state of a given A preon, the quark number  $Q_u = 1$ , by virtue of eq. (2.310), in which all three colors enter symmetrically. With the introduction of color, one now has six preons rather than four. These may be labelled as  $A_R, A_G, A_B, B, C, D$ . From here, it is very straightforward to model the strong color interaction on the basis of a preonic decomposition similar to that in the diagram (2.252). For example, for a red up quark ( $u_R$ ) exchanging color with a green down quark ( $d_G$ ), one utilizes the flavor definitions (2.297), supplemented by three colors of A preon, in order to draw the strong color interaction diagram:



One notes immediately that this also provides us with the preonic interpretation of the strong interaction gluons. The Gluons,  $G^u$ , are composed out of an  $\bar{A}A$  meson <sup>neutral flavor current</sup> combination of preons, in any one of  $3 \otimes 3 = 8 \oplus 1$  color combinations, according to all of the usual rules of strong color interaction theory. <sup>(cf. (2.297)(b))</sup> The  $G^u_{GR} \equiv A_{\bar{G}} A_R$  gluon in (2.311), is but one of the eight bicolored gluons which can be used to exchange color between two colored particles, while the ninth gluon is a singlet state which carries no net color. Again, this

can all be achieved without making any modifications whatsoever, to settled SU(3) strong interaction color theory. The reason this works, is because the A preon is the exclusive carrier of quark number, see eg., (2.309); and the reason that the A preon is the exclusive carrier of quark number, is <sup>because of</sup> the introduction of quantum gravitation. Consequently, all of these discussions are tied together rather closely. In fact, while the introduction of quantum gravitation in the prior section was motivated by the desire to maintain chiral symmetry in the strong and electromagnetic interactions, while at the same time accounting for observed chiral asymmetries in the weak (and consequently hypercharge) interactions, it could just as well have been motivated by the requirement that preonic G.U.T. must somehow reproduce exactly, everything that is known about the SU(3) strong color interaction; and specifically, the existence of three colors of quark. The way to do this is to isolate all of the quark charge  $Q_u = R + G + B$  into the A preon, <sup>which requires quantum gravitation,</sup> and then to embed the SU(3) color symmetry into the definition of the A preon, by providing three colors of the A flavor of preon,  $A_R, A_G, A_B$ . These preonic colors must be chosen so as to be synonymous with the exact, unbroken SU(3) colors and color symmetry of the strong interaction. Ordinarily, when one does not consider preons, these colors must be directly associated with the real quarks. Utilizing the preonic flavor decomposition (2.297), these colors are associated more directly with the complex A flavor of preon. The further connection to the real colors of real quarks of which real nucleons are composed, may be reproduced by the inverse utilization of the flavor recompositions (2.297). This is a particularly significant

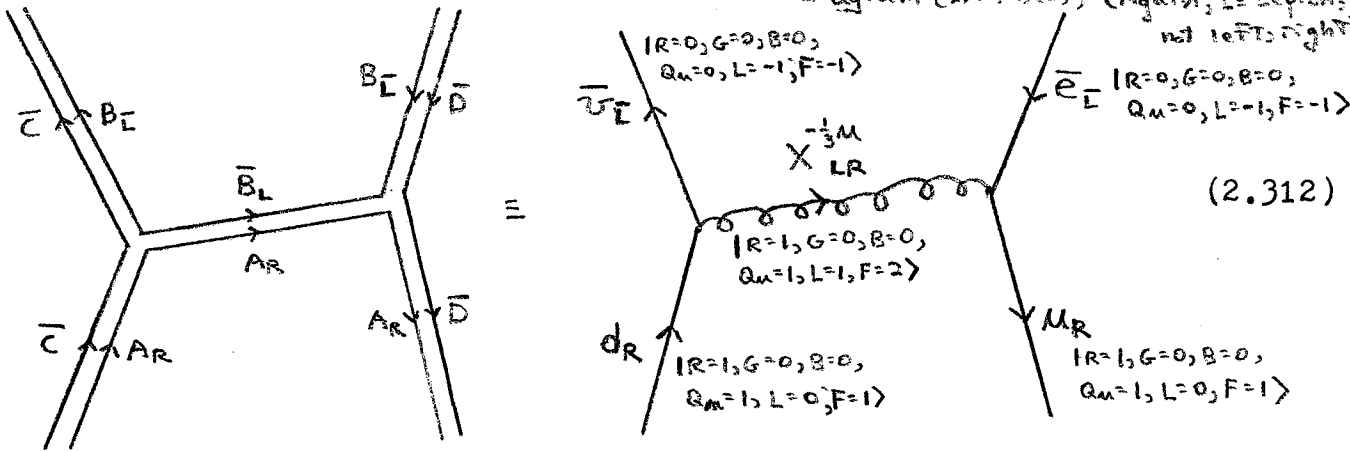
example of the extraordinary simplification which may be achieved by considering the decomposition of real fermions and bosons into complex preons. In short, in order to fully implement color, it is necessary merely to give to the A flavor of preon, two additional degrees of internal freedom. These are the so-called color degrees of freedom designated in the usual way by the color generators  $\lambda^3, \lambda^8$ , of SU(3), and by the color eigenstates R,G,B. The remaining B,C,D flavors of preon, need not be given this same internal color symmetry, but only because  $Q_u = R + G + B = 0$  for each of these preons. And again, this is only so, because of the quantum gravitational interaction. The fact that a given real particle is a quark, and that this <sup>given</sup> particle has a particular color, is determined by composing this particle, in part, out of one of the preons  $A_R, A_G, A_B$ . The further fact that this given quark of given color is either an "up" quark, or a "down" quark, is determined in remaining part by whether the colored A preon with which that quark is composed is combined, respectively, with either of a  $\bar{D}$  or  $\bar{C}$  isospin preon. Again, the question of whether a particle is isospin-up, or isospin-down, may be determined independently of the question whether that particle is a quark of any color, or a lepton. This "factoring out" of weak isospin, is also a very important insight that one obtains when considering the complex preonic decomposition of real fermions and bosons.

This brings us to the question of lepton number which, as Pati and Salam have strongly suggested, may be regarded at very high energies as but a fourth color of quark. We know, too, as

shown for example in (2.309), that it is the B preon which is the carrier of (anti)lepton number; while the A, C and D preons all have  $L=0$ . <sup>This too, is a consequence of quantum gravitation.</sup> Returning to examine the hyperweak flavor decay diagrams (2.279) and noting that it is the preonic conversion  $A \leftrightarrow B$  which is responsible for the hyperweak mode of beta-decay involving  $F=2$  bosons <sup>-2.17</sup>, one sees that the implementation of lepton number, as a fourth color of quark, is already strongly suggested by preonic flavor grand unification. Simply, this is implemented by regarding the B flavor of preon, as but a fourth color of the  $A_{\Lambda}$  preon. Redesignating the B preon by  $B_{\bar{L}}$ , with  $\bar{L}$  designating in this context the <sup>(anti)</sup> leptonic color (vs. left handedness), one consolidates  $B_{\bar{L}}$ , along with the three colors  $A_R, A_G, A_B$  into the  $SU(4)$  (actually,  $SU(4) \times U(1)$ ) quadruplet  $(B_{\bar{L}}, A_R, A_G, A_B)$ . Above the energies associated with hyperweak decay, the colored  $A_{\Lambda}$  preons decay into  $B_{\bar{L}}$  preons just as readily as they decay among themselves. Below the hyperweak energy, which will be closely related to the masses of the  $X^{*1/3u}, X^0$  hyperweak bosons (see Table 2.10), the symmetry that incorporates  $B_{\bar{L}}$  into the same  $SU(4) \times U(1)$  color multiplet as  $(A_R, A_G, A_B)$  is spontaneously broken down <sup>to the unbroken exact</sup>  $SU(3)$  (actually,  $SU(3) \times U(1)$ ) symmetry of low energy strong color interactions, with lepton number conserved independently. This scale used for symmetry breaking will be the same scale at which  $A \leftrightarrow B$  transitions become (statistically) forbidden by the preonic flavor theory, see, eg., Table 2.10. Hence, one anticipates certain connections between flavor and color. Including color, the Feynman preonic decomposition diagram for hyperweak decay involving, say, red quarks and hyperweak bosons, and involving all four flavors



of real fermion, may be drawn as such: (compare with the M-channel hyperweak diagram (2.270)(c) (Again, L=Lepton, R=Red, not left, right))



Contrasting the above with the diagram for ordinary strong decay, (2.311), we see that the only difference is that the preonic transition  $A_R \rightarrow A_G$  of (2.311) is replaced by the transition  $A_R \rightarrow B_L$ . This is to say that in (2.312), quarks turn into (anti)leptons in the same manner (and above a certain energy, with the same frequency) that quarks of one color turn into quarks of another color via ordinary low energy strong color interactions, as expected.

To explicitly implement preonic color symmetry including lepton number as the fourth color, it is helpful to begin by defining the following preonic color eigenvectors: (cf. (2.262) for flavor)

$$B_L \equiv \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}; \quad A_R \equiv \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}; \quad A_G \equiv \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}; \quad A_B \equiv \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}, \quad (2.313)$$

diagonalized along with the associated  $SU(4) \times U(1)$  color generators (cf. (2.284), (2.264), (2.265) for flavor)

$$\lambda^0 \equiv \frac{1}{2} \frac{1}{\sqrt{2}} T^0 = \begin{pmatrix} \frac{1}{4} & 0 & 0 & 0 \\ 0 & \frac{1}{4} & 0 & 0 \\ 0 & 0 & \frac{1}{4} & 0 \\ 0 & 0 & 0 & \frac{1}{4} \end{pmatrix} \quad (2.314)(a)$$

$$\lambda^3 \equiv \frac{1}{2} T^3 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & -\frac{1}{2} \end{pmatrix} \quad (2.314)(b)$$

$$\lambda^8 \equiv \frac{1}{2} \frac{1}{\sqrt{3}} T^8 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1/3 & 0 & 0 \\ 0 & 0 & -1/6 & 0 \\ 0 & 0 & 0 & -1/6 \end{pmatrix} \quad (2.314)(c)$$

$$\lambda^{15} \equiv \frac{1}{2} \frac{1}{\sqrt{6}} T^{15} = \begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 \\ 0 & -1/12 & 0 & 0 \\ 0 & 0 & -1/12 & 0 \\ 0 & 0 & 0 & -1/12 \end{pmatrix} \quad (2.314)(d)$$

Please note, while we have borrowed the  $T^{\bar{v}}$  matrices used to form the very similar flavor generators  $G^0, I^3, Y^8, B^{15}$ , that the above color generators  $\lambda^0, \lambda^3, \lambda^8, \lambda^{15}$  have a completely distinct significance. In short, the former generators are for flavor while the latter are for color. It is also useful, similarly to the flavor product (2.291), to form the square matrix product of the above color generators. This is used, in particular, to define the colors L,R,G,B. Raising and lowering indices with the Kronecker delta once again, and examining diagonalized matrices only ( $\bar{v} = 0,3,8,15$ ), one may write:

$$\begin{aligned} \frac{1}{2} T^{\bar{v}} T_{\bar{v}} &= \begin{pmatrix} \lambda^0 + 3\lambda^{15} & 0 & 0 & 0 \\ 0 & \lambda^0 + 2\lambda^8 - \lambda^{15} & 0 & 0 \\ 0 & 0 & \lambda^0 + \lambda^3 - \lambda^8 - \lambda^{15} & 0 \\ 0 & 0 & 0 & \lambda^0 - \lambda^3 - \lambda^8 - \lambda^{15} \end{pmatrix} \\ &\equiv \begin{pmatrix} -L & 0 & 0 & 0 \\ 0 & R & 0 & 0 \\ 0 & 0 & G & 0 \\ 0 & 0 & 0 & B \end{pmatrix} \end{aligned} \quad (2.315)$$

In short, we define:

$$L = -\lambda^0 - 3\lambda^{15} \quad (2.316)(a)$$

$$R = \lambda^0 + 2\lambda^8 - \lambda^{15} \quad (2.316)(b)$$

$$G = \lambda^0 + \lambda^3 - \lambda^8 - \lambda^{15} \quad (2.316)(c)$$

$$B = \lambda^0 - \lambda^3 - \lambda^8 - \lambda^{15} \quad (2.316)(d)$$

Finally, and at long last, it is possible to define in more precise terms, the connection between flavor and color symmetries. By virtue of (2.316), along with (2.300), it now becomes possible to write both the quark number  $Q_u = R + G + B$  and the lepton number  $L$  in terms of both the flavor generators (2.284), (2.264), (2.265) and the color <sup>generators</sup> (2.314). This interconnection is established as follows: (using (2.316))

$$Q_u = R + G + B = 3\lambda^0 - 3\lambda^{15} = G^0 + 3B^{15} \quad (2.317)(a)$$

$$L = -\lambda^0 - 3\lambda^{15} = B^{15} - 2Y^8 - G^0 \quad (2.317)(b)$$

In this manner, it is possible to reduce what are originally eight linearly independent generators (ie. quantum degrees of freedom) down to six.

The original eight generators are of course, the four  $SU(4) \times U(1)$  high energy flavor generators  $G^0, I^3, Y^8, B^{15}$ , and the four  $SU(4) \times U(1)$  high energy color generators  $\lambda^0, \lambda^3, \lambda^8, \lambda^{15}$ . At the same time, what are originally eight independent preons, four preons each from flavor  $SU(4) \times U(1)$  and color  $SU(4) \times U(1)$ , are reduced down to six preons, namely  $(A_R, A_G, A_B, B_L, C, D)$ . The important point, that that color itself is not directly attached to a particular flavor of preon, namely the A preon, which by virtue of quantum gravitation has  $Q_u = R+G+B=1$ ; while (anti) lepton number, as a fourth color, is similarly attached directly to a particular flavor of preon, namely the B preon, again from the discussion in the prior two sections. This helps to simplify (reduce) the number of real particle states as well.

In the original electroweak preon theory, where we only had the C and D (isospin up and isospin down) preons to work with, it was possible to create four real bosons via preon composition, see Section 2.9. These are of course, the standard  $A^u, W^{\pm u}, W^0_u$ .

As the  $W^{\pm u}$  are one another's antiparticles, these four electroweak particles actually represent three conjugally independent particles. When we considered flavor absent color, this was increased somewhat. Using (A,B,C,D), one could compose <sup>4x4=</sup>16 real particles, of which 10 are independent, see (2.280) and Table 2.8. For Fermions and Bosons respectively, this breaks down as follows:

Real Particles      Real Particles  
With Antiparticle    Without Antiparticle

Fermions

(u,d)	4		2
( $\nu$ ,e)	4		2

Bosons

Electroweak

A <sup>u</sup>	1		1
W <sup>0u</sup>	1		1
W <sup>±</sup>	2		1

Table 2.11 - Real Particle Totals, Flavor Absent Color and Generation

Gluon/Exotica

G <sup>u</sup>	1		1
X <sup>0u</sup>	1		1
X <sup>±1/3u</sup>	2		1

Total Fermions	8		4
Total Bosons	8		6
Total Real			6
Particles	16		10

For flavor with color, utilizing the six preons (A<sub>R</sub>,A<sub>G</sub>,A<sub>B</sub>,B<sub>L</sub>,C,D), one can now compose <sup>6x6=</sup>36 logical flavor/color combinations of real particle. The breakdown at this point, following the lines of Table 2.11, and including color, now turns out as follows:

	<u>Real Particles</u>	<u>Real Particles</u>
	<u>With Antiparticle</u>	<u>Without Antiparticle</u>

Fermions

(u,d) <sub>R,G,B</sub>	12	6
( $\bar{u}$ ,e) <sub>L</sub>	4	2
S.Total	16	8

Bosons

Electroweak

A <sup>u</sup>	1	1
W <sup>0u</sup>	1	1
W <sup>±u</sup>	2	1
S.Total	4	3

Table 2.12- Real Particle Totals, Flavor With Color, Absent Generation

Gluon/Exotica

G <sup>u</sup> <sub>0u</sub> ( $\overline{RGB}$ )(RGB)	9 = 3 ⊗ 3	6
X <sup>±1/3u</sup> <sub>(RGB)</sub>	1	1
	6	3
S.Total	16	10

Total Fermions	16	8
Total Bosons	20	13
Total Real Particles	<u>36</u>	<u>21</u>

This may all be summarized by noting, for N preons, that the number of real particles including antiparticles is simply N<sup>2</sup>; while the number of real particles excluding antiparticles is given by N(N+1)/2. Including color, it is again helpful to highlight the definition and composition of the real fermions and bosons in a form similar to that given in (2.280), though now for six rather than four preons. Leaving the explicit form of the neutral currents aside for the moment (these will involve various linear combinations of the states shown, including two neutral gluons, one singlet neutral gluon, and the X<sup>0u</sup>, W<sup>0u</sup> (=Z<sup>u</sup>), A<sup>u</sup>), this may be written as follows:

$$\begin{pmatrix} \bar{A} \\ \bar{A} \\ \bar{A} \\ \bar{B} \\ \bar{B} \\ \bar{C} \\ \bar{C} \\ \bar{D} \end{pmatrix} \begin{pmatrix} A_R & A_G & A_B & B_{\bar{L}} & C & D \end{pmatrix} = \begin{pmatrix} \bar{A} \\ \bar{A} \\ \bar{A} \\ \bar{B} \\ \bar{B} \\ \bar{C} \\ \bar{C} \\ \bar{D} \end{pmatrix} \begin{pmatrix} A_R & A_G & A_B & B_{\bar{L}} \end{pmatrix} \begin{pmatrix} \bar{A} \\ \bar{A} \\ \bar{A} \\ \bar{B} \\ \bar{B} \\ \bar{C} \\ \bar{C} \\ \bar{D} \end{pmatrix} \begin{pmatrix} C & D \end{pmatrix}$$

$$= \begin{pmatrix} \bar{A} \bar{R} A_R & \bar{A} \bar{R} A_G & \bar{A} \bar{R} A_B & \bar{A} \bar{R} B_{\bar{L}} & \bar{A} \bar{R} C & \bar{A} \bar{R} D \\ \bar{A} \bar{G} A_R & \bar{A} \bar{G} A_G & \bar{A} \bar{G} A_B & \bar{A} \bar{G} B_{\bar{L}} & \bar{A} \bar{G} C & \bar{A} \bar{G} D \\ \bar{A} \bar{B} A_R & \bar{A} \bar{B} A_G & \bar{A} \bar{B} A_B & \bar{A} \bar{B} B_{\bar{L}} & \bar{A} \bar{B} C & \bar{A} \bar{B} D \\ \bar{B} \bar{L} A_R & \bar{B} \bar{L} A_G & \bar{B} \bar{L} A_B & \bar{B} \bar{L} B_{\bar{L}} & \bar{B} \bar{L} C & \bar{B} \bar{L} D \\ \bar{C} A_R & \bar{C} A_G & \bar{C} A_B & \bar{C} B_{\bar{L}} & \bar{C} C & \bar{C} D \\ \bar{D} A_R & \bar{D} A_G & \bar{D} A_B & \bar{D} B_{\bar{L}} & \bar{D} C & \bar{D} D \end{pmatrix} \tag{2.318}$$

$$= \begin{pmatrix} \bar{A} \bar{R} A_R & G_{\bar{R}G}^u & G_{\bar{R}B}^u & X_{\bar{R}3}^{+1u} & \bar{d}_{\bar{R}} & \bar{u}_{\bar{R}} \\ G_{\bar{G}R}^u & \bar{A} \bar{G} A_G & G_{\bar{G}B}^u & X_{\bar{G}3}^{+1u} & \bar{d}_{\bar{G}} & \bar{u}_{\bar{G}} \\ G_{\bar{B}R}^u & G_{\bar{B}G}^u & \bar{A} \bar{B} A_B & X_{\bar{B}3}^{+1u} & \bar{d}_{\bar{B}} & \bar{u}_{\bar{B}} \\ X_{\bar{R}3}^{-1u} & X_{\bar{G}3}^{-1u} & X_{\bar{B}3}^{-1u} & \bar{B} \bar{L} B_{\bar{L}} & \bar{\nu}_{\bar{L}} & e_{\bar{L}} \\ d_{\bar{R}} & d_{\bar{G}} & d_{\bar{B}} & \bar{e}_{\bar{L}} & \bar{C} C & W^{-u} \\ u_{\bar{R}} & u_{\bar{G}} & u_{\bar{B}} & \bar{e}_{\bar{L}} & W^{+u} & \bar{D} D \end{pmatrix} = \begin{pmatrix} \text{Strong/} & & & & & \\ \text{Hyperweak} & & & & & \\ \text{Bosons} & & & & & \\ \text{Fermions} & & & & & \\ \text{Fermions} & & & & & \\ \text{Electron/} & & & & & \\ \text{Weak} & & & & & \\ \text{Bosons} & & & & & \end{pmatrix}$$

Given the above, one sees from a slightly different perspective why it is that six preons can be composed into thirty-six real particles, with twenty-one conjugally independent particles. Clearly, the reduction of twenty-one independent real flavor/color combinations of Bosons within a single generation and Fermion down to only six independent complex preons represents a very significant simplification in elementary particle classification. Note again, as before, that the A and B preons combined between themselves, and the C and D preons combined between themselves, both serve to produce real Bose particles. On the other hand, the real Fermi particles are produced by mixtures involving either of the A or B preons, in combination with either of the C or D preons. The fact that

→ For these:

$$\lambda^3 = \frac{1}{2}G - \frac{1}{2}B \quad (2.320)(c)$$

$$\lambda^8 = \frac{1}{3}R - \frac{1}{6}G - \frac{1}{6}B \quad (2.320)(d)$$

real fermions and bosons belong to the same flavor/color multiplets in the above, is one of the particularly strong features of the preonic approach.

Finally, it is helpful to compile all of the conjugally independent preons and real fermions and bosons into a tabular form, along the lines of Table 2.7, but showing the color quantum numbers for all particles, rather than flavor<sup>alone</sup>. A few points should be noted here. First, because of the connection (2.317) between flavor and color, it is possible to rewrite eqs. (2.316) and certain linear combinations thereof in the form:

$$R = \lambda^0 + 2\lambda^8 - \lambda^{15} = \frac{1}{3}Q_u + 2\lambda^8 \quad (2.319)(a)$$

$$G = \lambda^0 + \lambda^3 - \lambda^8 - \lambda^{15} = \frac{1}{3}Q_u + \lambda^3 - \lambda^8 \quad (2.319)(b)$$

$$B = 0 - 3 - 8 - 15 = \frac{1}{3}Q_u - \lambda^3 - \lambda^8 \quad (2.319)(c)$$

$$L = -\lambda^0 - 3\lambda^{15} = B^{15} - 2Y^8 - G^0 \quad (2.319)(d)$$

$$Q_u = R + G + B = 3\lambda^0 - 3\lambda^{15} = G^0 + 3B^{15} \quad (2.319)(e)$$

$$F = Q_u + L = 2\lambda^0 - 6\lambda^{15} = 4B^{15} - 2Y^8 \quad (2.319)(f)$$

With the above, it is possible to further write the  $\lambda^0$  and  $\lambda^{15}$  generators completely in terms of the flavor generators  $G^0$ ,  $Y^8$ ,  $B^{15}$ , as such:

$$\lambda^0 = \frac{1}{4}(Q_u - L) = \frac{1}{2}(G^0 + Y^8 + B^{15}) \quad (2.320)(a)$$

$$\lambda^{15} = -\frac{1}{4}L - \frac{1}{12}Q_u = \frac{1}{2}\left(\frac{1}{3}G^0 + Y^8 - B^{15}\right) \quad (2.320)(b)$$

Hence, it is only  $\lambda^3$  and  $\lambda^8$ , which are the usual color generators,

which are linearly independent of the flavor generators. (Again, six, rather than eight degrees of freedom.) Also, recalling the earlier discussion, for example, Tables 2.7 and 2.9, we know that quark number  $Q_u$  and Lepton number  $L$  are left-right chiral symmetric; indeed, these quantum numbers are designed to be so. In light of the above, and given the known chiral symmetry of the strong interaction, i.e.,

that: (contrast (2.302))

$$(L,R,G,B)_{L,R} = (L,R,G,B)_L = (L,R,G,B)_R \quad (2.321)(a)$$

$$(L,R,G,B)_A = 0 \quad , \quad (2.321)(b)$$

it is readily apparant that all of the color generators  $\lambda^0, \lambda^3, \lambda^8, \lambda^{15}$  will also obey the chiral symmetry, ie., that:

$$\lambda_{L,R}^6 = \lambda_L^6 = \lambda_R^6 \quad (2.322)(a)$$

$$\lambda_A^6 = 0 \quad , \quad (2.322)(b)$$

as expected. Hence, one arrives at the <sup>(conjugally independent)</sup> color table (C,D are colorless)

(See (2.319) For L,R,G,S,Q<sub>u</sub>,F.)

	$\lambda^0$	$\lambda^3$	$\lambda^8$	$\lambda^{15}$	L	R	G	B	Q <sub>u</sub>	F
$\overline{B}_L$	$\frac{1}{4}$	0	0	$\frac{1}{4}$	-1	0	0	0	0	-1
$\overline{A}_R$	$\frac{1}{4}$	0	1/3	-1/12	0	1	0	0	1	1
$\overline{A}_G$	$\frac{1}{4}$	$-\frac{1}{6}$	-1/6	-1/12	0	0	1	0	1	1
$\overline{A}_B$	$\frac{1}{4}$	$-\frac{1}{6}$	-1/6	-1/12	0	0	0	1	1	1
$\overline{C}$	0	0	0	0	0	0	0	0	0	0
$\overline{D}$	0	0	0	0	0	0	0	0	0	0
$G_{GR}^u$	0	$-\frac{1}{2}$	$\frac{1}{2}$	0	0	1	-1	0	0	0
$G_{BG}^u$	0	1	0	0	0	0	1	-1	0	0
$G_{RB}^u$	0	$-\frac{1}{2}$	$-\frac{1}{2}$	0	0	-1	0	1	0	0
$X_{RL}^{-1/3u}$	0	0	1/3	-1/3	1	1	0	0	1	2
$X_{GL}^{-1/3u}$	0	$\frac{1}{2}$	-1/6	-1/3	1	0	1	0	1	2
$X_{BL}^{-1/3u}$	0	$-\frac{1}{2}$	-1/6	-1/3	1	0	0	1	1	2
$(\nu, e)_L$	$-\frac{1}{4}$	0	0	$-\frac{1}{4}$	-1	0	0	0	0	1
$(u, d)_R$	$\frac{1}{4}$	0	1/3	-1/12	0	1	0	0	1	1
$(u, d)_G$	$\frac{1}{4}$	$\frac{1}{2}$	-1/6	-1/12	0	0	1	0	1	1
$(u, d)_B$	$\frac{1}{4}$	$-\frac{1}{2}$	-1/6	-1/12	0	0	0	1	1	1
$W^+u$	0	0	0	0	0	0	0	0	0	0
$\overline{A}_R A_R$	0	0	0	0	0	0	0	0	0	0
$\overline{A}_G A_G$	0	0	0	0	0	0	0	0	0	0
$\overline{A}_B A_B$	0	0	0	0	0	0	0	0	0	0
$\overline{B}_L B_L$	0	0	0	0	0	0	0	0	0	0
$\overline{C} C$	0	0	0	0	0	0	0	0	0	0
$\overline{D} D$	0	0	0	0	0	0	0	0	0	0

Table 2.13 - Color Quantum Numbers for Complex Preons and Real Fermions and Bosons



Thus, we arrive explicitly at the color quantum numbers for the six preons  $A_R, A_G, A_B, B_L, C, D$ ; along with the twenty-one conjugally independent real colors/flavors of particle. Fifteen of these are non-neutral under either of flavor or color, or both, see (2.318). The remaining six states of real particle  $\bar{A}_R \bar{A}_R, \bar{A}_G \bar{A}_G, \bar{A}_B \bar{A}_B, \bar{B}_L \bar{B}_L, \bar{C} \bar{C}, \bar{D} \bar{D}$ , for which all flavor and color quantum numbers are equal to zero (see Tables 2.7, 2.13) are used to construct the six independent neutral current vector Bosons. Among these must be the  $A^u, W^{0u}$  of electroweak theory, along with the hyperweak  $X^{0u}$  and appropriate neutral current gluons. The explicit derivation of these six neutral current Bosons is of interest in its own right. Additionally, it is an important prerequisite to a proper understanding of generation symmetry, which will be the primary topic of the section following. Hence, before we embark upon a more detailed discussion of generation symmetry, it will be helpful to have available the explicit forms for these six neutral bosons. It should also be reemphasized that to this point, we have tacitly assumed the existence of but a single fermionic generation (family), and have attempted merely to specify all of the flavor/color combinations of real particle that may exist in nature, within a single fermionic generation. In final form, this is given in Tables 2.7 and 2.13 for flavor and flavor/color. We know of course that nature actually contains at least three, and possibly more generations. To date, while this symmetry can be characterized somewhat successfully, there has been little real success in understanding the theoretical roots of this symmetry, with the partial exception of Pati and Salam. <sup>-2.18</sup>

To begin consideration of neutral currents, it is appropriate to note an important difference between preonic G.U.T. with and without color, which shows why it was not possible until this point to begin a proper discussion of Without color, one utilizes only the four flavor neutral current

states (A,B,C,D). In matrix form, taking quark number  $Q_u$  from the flavor Table 2.7, it is possible to define the preon<sup>flavor</sup> quadruplet such that: (see (2.300)(a))

$$\bar{\chi}_{(4)} \equiv \begin{pmatrix} \bar{A} \\ \bar{B} \\ \bar{C} \\ \bar{D} \end{pmatrix} ; \quad \chi_{(4)} \equiv (A, B, C, D) \quad (2.323)(a)$$

where:

$$Q_u = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} = G^0 + 3B^{15} \quad (2.323)(b)$$

Consequently, the term:

$$\bar{\chi}_{(4)} Q_u \chi_{(4)} = \bar{A}A \quad (2.323)(c)$$

On the other hand, with color, one utilizes the six preons ( $A_R, A_G, A_B, B_L, C, D$ ). In matrix form, quark number may now be taken from Table 2.13, and may be defined simultaneously with the preon flavor/color sextuplet such that: (A factor of 1/3 is introduced <sup>in (2.324)(c) below</sup> for consistency)

$$\bar{\chi}_{(6)} \equiv \begin{pmatrix} \bar{A}_R \\ \bar{A}_G \\ \bar{A}_B \\ \bar{B}_L \\ \bar{C} \\ \bar{D} \end{pmatrix} ; \quad \chi_{(6)} \equiv (A_R, A_G, A_B, B_L, C, D) \quad (2.324)(a)$$

where:

$$Q_u = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} = R+G+B = 3\lambda^0 - 3\lambda^{15} \quad (2.324)(b)$$

Consequently, the term:

$$\bar{\chi}_{(6)} (1/3) Q_u \chi_{(6)} = (1/3) (\bar{A}_R A_R + \bar{A}_G A_G + \bar{A}_B A_B) \quad (2.324)(c)$$

That is, when going from preonic G.U.T. without color to preonic G.U.T. with color, one may replace all occurrences of the term  $\bar{A}A$  with:

$$\bar{A}A \rightarrow (1/3) (\bar{A}_R A_R + \bar{A}_G A_G + \bar{A}_B A_B) \quad (2.325)$$

Noting that low energy strong interactions obey the  $SU(3) \times U(1)$  sym-

metry group, with the diagonalized  $\lambda^0$  (U(1)) and  $\lambda^3, \lambda^8$  (SU(3)) generators: (contrast (2.314) for the high energy SU(4)xU(1))

$$\lambda^0 = \frac{1}{2} \sqrt{\frac{2}{3}} T^0 = \begin{pmatrix} 1/3 & 0 & 0 \\ 0 & 1/3 & 0 \\ 0 & 0 & 1/3 \end{pmatrix} \quad (2.326)(a)$$

$$\lambda^3 = \frac{1}{2} T^3 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & -\frac{1}{2} \end{pmatrix} \quad (2.326)(b)$$

$$\lambda^8 = \frac{1}{2} \frac{1}{\sqrt{3}} T^8 = \begin{pmatrix} 1/3 & 0 & 0 \\ 0 & -1/6 & 0 \\ 0 & 0 & -1/6 \end{pmatrix}, \quad (2.326)(c)$$

with the 3x3  $T^a$  generators normalized to the usual  $\text{Tr}(T^2)=2$  for all  $T^a$ , one notes that the combination of colored A preons shown in (2.324)(c) may similarly be written, using (2.326)(a) above, as: (note,  $\lambda^0$  is for U(1) of SU(3), not SU(4).)

$$\bar{\chi}_{(3)} \lambda^0 \chi_{(3)} = \bar{\chi}_{(3)} (1/3) Q \chi_{(3)} = (1/3) (\bar{A}_R A_R + \bar{A}_G A_G + \bar{A}_B A_B) \quad (2.327)(a)$$

with the color triplet  $\chi_{(3)}$  defined here such that:

$$\bar{\chi}_{(3)} \equiv \begin{pmatrix} \bar{A}_R \\ \bar{A}_G \\ \bar{A}_B \end{pmatrix}; \quad \chi_{(3)} \equiv (A_R, A_G, A_B) \quad (2.327)(b)$$

This too is important to keep in mind, since we shall wish to break the high energy SU(4)xU(1) color symmetry down to the low energy SU(3)xU(1) color symmetry in such a way as to leave the SU(3)xU(1) subgroup unbroken. The usual low energy gluons are of course described by a  $3 \otimes 3 = 8 \oplus 1$  decomposition, with the U(1) singlet  $\Lambda$  given by the term  $\bar{A}A \rightarrow \frac{1}{3} (\bar{A}_R A_R + \bar{A}_G A_G + \bar{A}_B A_B)$ . Consequently, this singlet must be one of the gluons which remains intact, after the spontaneous breaking of SU(4)xU(1) down to SU(3)xU(1). With this requirement, and with the usual approach to symmetry breaking used in electroweak theory, one now has available all of the pieces necessary to fully specify the neutral currents for both flavor and color symmetry.

To begin with, we note that we have thus far run across eight

interaction generators, four from SU(4)XU(1) flavor, and four from SU(4)XU(1) color. For flavor, these are the  $G^0, I^3, Y^8, B^{15}$  of, for example, Table 2.7. For color, these are the  $\lambda^0, \lambda^3, \lambda^8, \lambda^{15}$  of, for example, Table 2.13. Seemingly, these are all linearly independent; though on more detailed consideration, we find that this is not so. In particular, as shown in eqs. (2.317), the commonality of the linear combinations  $Q_u$  and L as between both flavor and color,

$$Q_u = 3\lambda^0 - 3\lambda^{15} = G^0 + 3B^{15} \quad (2.328)(a)$$

$$L = -\lambda^0 - 3\lambda^{15} = B^{15} - 2Y^8 - G^0 \quad (2.328)(b)$$

reduces from eight to six the number of linearly independent generators as among all of  $G^0, I^3, Y^8, B^{15}$  and  $\lambda^0, \lambda^3, \lambda^8, \lambda^{15}$ . One may choose from these, for instance, the electroweak  $I^3, Y^8$ , and all four of the color  $\lambda^0, \lambda^3, \lambda^8, \lambda^{15}$ . Via (2.328), one may further use the  $\lambda$  matrices to specify both of  $Q_u$  and L. In turn, given  $Q_u$  and L along with  $Y^8$  and  $I^3$ , <sup>hence  $Q = Y^8 + \frac{1}{2}I^3$ ,</sup> it is a simple matter to appropriately invert (2.300), thereby deducing the remaining  $G^0$  and  $B^{15}$  generators. (ie., see (2.294)<sup>(a,d)</sup>). We shall use this fact to reduce what may at first appear to be eight neutral current preon compositions, namely the four flavor (See Table 2.7 and (2.325), (2.298)):

$$\bar{\chi}_{(s)} G^0 \chi_{(s)} = \frac{1}{12} (\bar{A}_R A_R + \bar{A}_G A_G + \bar{A}_B A_B) + \frac{1}{4} (\bar{B}_L B_L + \bar{C} C + \bar{D} D) \quad (2.329)(a)$$

$$\bar{\chi}_{(s)} I^3 \chi_{(s)} = \frac{1}{2} (\bar{C} C - \bar{D} D) \quad (2.329)(b)$$

$$\bar{\chi}_{(s)} Y^8 \chi_{(s)} = \frac{1}{3} \bar{B}_L B_L - \frac{1}{6} (\bar{C} C + \bar{D} D) \quad (2.329)(c)$$

$$\bar{\chi}_{(s)} B^{15} \chi_{(s)} = \frac{1}{12} (\bar{A}_R A_R + \bar{A}_G A_G + \bar{A}_B A_B - \bar{B}_L B_L - \bar{C} C - \bar{D} D) \quad (2.329)(d)$$

and the four color (See Table 2.13 and (2.314)): (Also (2.324)(a))

$$\bar{\chi}_{(s)} \lambda^0 \chi_{(s)} = \frac{1}{4} (\bar{A}_R A_R + \bar{A}_G A_G + \bar{A}_B A_B + \bar{B}_L B_L) \quad (2.330)(a)$$

$$\bar{\chi}_{(s)} \lambda^3 \chi_{(s)} = \frac{1}{2} (\bar{A}_G A_G - \bar{A}_B A_B) \quad (2.330)(b)$$

$$\bar{\chi}_{(s)} \lambda^8 \chi_{(s)} = \frac{1}{3} \bar{A}_R A_R - \frac{1}{6} (\bar{A}_G A_G + \bar{A}_B A_B) \quad (2.330)(c)$$

$$\bar{\chi}_{(s)} \lambda^{15} \chi_{(s)} = \frac{1}{4} \bar{B}_L B_L - \frac{1}{12} (\bar{A}_R A_R + \bar{A}_G A_G + \bar{A}_B A_B) \quad (2.330)(d)$$

down to six truly independent neutral current terms. Two of these will be the electroweak  $A^u$  and  $Z^u \equiv W^{0u}$ . Two more will be Gluons  $G^u$  involving (2.330)(b) and (c), as these are associated also with the SU(3) low energy color group, see (2.326)(b) and (c). The fifth of these neutral current vector Bosons must be the SU(3)xU(1) singlet (2.327)(a) and (2.324)<sup>(c)</sup> as discussed above. In the process of breaking the high energy color SU(4)xU(1) group down to the low energy color SU(3)xU(1), the sixth and final neutral current boson, which will be generated as a by-product of this symmetry breaking, is the massive hyperweak  $X^{0u}$ . Now let us see how this is done.

First, we turn to the neutral electroweak vector Bosons  $A^u, Z^u$ . These are to be defined identically to the way that they normally are in the Weinberg-Salam theory. Nevertheless, it is instructive to show how these are in fact specified, as this will provide a basis for further discussion of strong gluons and the hyperweak  $X^{0u}$ . A particularly simple approach to the spontaneous generation of the electroweak neutral currents  $A^u, Z^u$  involves consideration of the Lagrangian amplitude terms of the form  $\mathcal{L}_m = g \cdot J^u \cdot B^u$ , see for example, eqs. (1.6)-(1.10) in the introduction. <sup>(see also (2.120), (2.121))</sup> We also wish to consider, in particular, the flavor generators  $I^3$  and  $Y^8$ , along with their associated respective currents  $J_W^{3u}$  and  $J_Y^{8u}$ , running charges  $g_W, g_Y$  and vector bosons  $W^{3u}, B^{8u}$ . We shall wish to consider Weinberg/Glashow type transformations which "mix" these various generators, charges, currents and Bosons, of the form:

$$\begin{pmatrix} g_{(em)} J_{(em)}^u \\ g_{(Z)} J_{(Z)}^u \end{pmatrix} \equiv \begin{pmatrix} g'_Y J_Y^{8u} \\ g'_W J_W^{3u} \end{pmatrix} = \begin{pmatrix} \cos \theta_W & \sin \theta_W \\ -\sin \theta_W & \cos \theta_W \end{pmatrix} \begin{pmatrix} g_Y J_Y^{8u} \\ g_W J_W^{3u} \end{pmatrix} \quad (2.331)(a)$$

for charges and currents, and of the form:

$$\begin{pmatrix} A^u \\ Z^u \end{pmatrix} \equiv \begin{pmatrix} B^{8u} \\ W^{3u} \end{pmatrix} = \begin{pmatrix} \cos \theta_W & \sin \theta_W \\ -\sin \theta_W & \cos \theta_W \end{pmatrix} \begin{pmatrix} B^{8u} \\ W^{3u} \end{pmatrix}, \quad (2.331)(b)$$

where  $\theta_W$  is the so-called Weinberg/Glashow angle, also known as the electroweak mixing angle. The reason (2.331) above are of interest is because these mixing relationships leave invariant, the Lagrangian amplitude  $\mathcal{L}_m = g \cdot J^u B_u$ . Specifically, (Again, see (2.120)-(1)(b))

$$\begin{aligned} \mathcal{L}_m &= g_{(em)} J_{(em)}^u A_u + g(Z) J(Z)^u Z_u \\ &= g_Y J_Y^{8u} B_{8u} + g_W J_W^{3u} W_{3u} \\ &= g_Y J_Y^{8u} B_{8u} + g_W J_W^{3u} W_{3u} = \mathcal{L}_m. \end{aligned} \quad (2.332)$$

This is so because the mixing matrix  $U$  in (2.331) is unitary, i.e., because  $U^\dagger U = 1$ . Next, we utilize the well established relationship  $Q = Y^8 + I^3$  (see (2.300)(c)) to note, if  $\Psi$  designates the Dirac wavefunction of a particular particle under consideration, that:

$$J_{(em)}^u \equiv \bar{\Psi} \gamma^u Q \Psi = J_Y^{8u} + J_W^{3u} = \bar{\Psi} \gamma^u Y^8 \Psi + \bar{\Psi} \gamma^u I^3 \Psi. \quad (2.333)$$

Similarly,

$$J(Z)^u \equiv \bar{\Psi} \gamma^u Z \Psi. \quad (2.334)$$

Because each of the flavor generators in the above flavor transform in exactly the same manner as their associated currents, it is helpful to factor out the  $\bar{\Psi} \gamma^u \Psi$  term from the various currents  $J^u$ , and to work directly with the generators. To go back to the currents merely requires the later reintroduction of the  $\bar{\Psi} \gamma^u \Psi$  term. Thus, we return now to (2.331)(a), setting  $g_Q = g_{(em)}$  ( $\equiv$  running fine structure, i.e., Coulomb charge) and  $g_Z = g(Z)$ , and rewrite this relationship in terms of the associated generators, as:

$$\begin{pmatrix} g_{Q^Q} \\ g_{Z^Z} \end{pmatrix} = \begin{pmatrix} \cos \theta_W & \sin \theta_W \\ -\sin \theta_W & \cos \theta_W \end{pmatrix} \begin{pmatrix} g_Y Y^8 \\ g_W I^3 \end{pmatrix}. \quad (2.335)$$

At low energies, the electroweak  $SU(2)_W \times U(1)_Y$  subgroup must break

down to the electromagnetic  $U(1)_{em} \equiv U(1)_Q$  subgroup; which is to say that eq. (2.335) above must be solved simultaneously with the now familiar:

$$Q = Y^8 + I^3 \quad (2.336)$$

which determines the electrostatic  $Q$  generator of electromagnetism. With the above, so long as  $g_Y, g_W, Y^8$  and  $I^3$  are initially specified, it is possible to uniquely determine the remaining unknowns  $g_Q, g_Z, Z$ , along with the mixing angle  $\Theta_W$ , in terms of the given  $g_Y, g_W, Y^8, I^3$ , with  $Q$  already known from (2.336). Particularly, it is helpful to extract the equation for  $g_Q Q$  from (2.335), and to contrast this directly with (2.336), ie.,

$$\begin{cases} g_Q Q = \cos \Theta_W g_Y Y^8 + \sin \Theta_W g_W I^3 & (2.337)(a) \\ Q = Y^8 + I^3 & (2.337)(b) \end{cases}$$

By simple algebra, a comparison of the above reveals that:

$$\frac{g_Y}{g_Q} \cos \Theta_W = 1 \quad ; \quad \frac{g_W}{g_Q} \sin \Theta_W = 1 \quad , \quad (2.338)$$

which is easily rewritten in the more familiar form:

$$g_Q = g_Y \cos \Theta_W = g_W \sin \Theta_W \quad (2.339)$$

Similarly from (2.338), using the identity  $\sin^2 \Theta_W + \cos^2 \Theta_W = 1$ , it is possible to deduce:

$$\frac{1}{g_Q^2} = \frac{1}{g_Y^2} + \frac{1}{g_W^2} = \frac{g_W^2 + g_Y^2}{g_W^2 g_Y^2} \quad (2.340)$$

Also from (2.338), one finds that:

$$\tan \Theta_W \equiv \frac{\sin \Theta_W}{\cos \Theta_W} = \frac{g_Y}{g_W} \quad (2.341)$$

Consequently,

$$\sin \Theta_W = \frac{g_Y}{\sqrt{g_Y^2 + g_W^2}} \quad ; \quad \cos \Theta_W = \frac{g_W}{\sqrt{g_Y^2 + g_W^2}} \quad (2.342)$$

From here, using the above to set  $g_Y = \sin \theta_W \sqrt{g_Y^2 + g_W^2}$  and  $g_W = \cos \theta_W \sqrt{g_Y^2 + g_W^2}$ , and extracting the remaining equation for  $g_Z$  from (2.335), it is possible to write:

$$g_Z Z = -\sin^2 \theta_W \sqrt{g_Y^2 + g_W^2} \cdot Y^8 + \cos^2 \theta_W \sqrt{g_Y^2 + g_W^2} \cdot I^3 \quad (2.343)$$

The above is readily separated into two equations, one for  $Z$  and one for  $g_Z$ . In particular, one may write:

$$Z = -\sin^2 \theta_W Y^8 + \cos^2 \theta_W I^3 \quad (2.344)(a)$$

$$g_Z = \sqrt{g_Y^2 + g_W^2} \quad , \quad (2.344)(b)$$

which determines the remaining unknowns  $Z$ ,  $g_Z$ . Reintroducing the  $\bar{\Psi} \gamma_u \Psi$  factor in the generator equations (2.336), (2.344)(a) then allows us to summarize the neutral currents:

$$J_{(em)}^u = J_Y^{8u} + J_W^{3u} \quad (2.345)(a)$$

$$\begin{aligned} J_{(Z)}^u &= -\sin^2 \theta_W J_Y^{8u} + \cos^2 \theta_W J_W^{3u} \\ &= J_W^{3u} - J_{(em)}^u \sin^2 \theta_W \\ &= -J_Y^{8u} + J_{(em)}^u \cos^2 \theta_W \quad . \end{aligned} \quad (2.345)(b)$$

Further, the couplings may be summarized, from (2.340) and (2.344)(b) by:

$$\frac{1}{g_Q^2} = \frac{1}{g_Y^2} + \frac{1}{g_W^2} = \frac{g_W^2 + g_Y^2}{g_Y^2 \cdot g_W^2} = \frac{g_Z^2}{g_Y^2 \cdot g_W^2} \quad (2.346)(a)$$

$$g_Z^2 = g_Y^2 + g_W^2 \quad . \quad (2.346)(b)$$

Given  $g_Z$ , above along with (2.342), the unitary mixing matrix in (2.331) may also be summarized by:

$$U = \begin{pmatrix} \cos \theta_W & \sin \theta_W \\ -\sin \theta_W & \cos \theta_W \end{pmatrix} = \begin{pmatrix} g_W/g_Z & g_Y/g_Z \\ -g_Y/g_Z & g_W/g_Z \end{pmatrix} \quad (2.347)$$

From (2.331)(b), the above now allows us to also write:

$$g_Z A^u = g_W B^{8u} + g_Y W^{3u} \quad (2.348)(a)$$

$$g_Z Z^u = -g_Y B^{8u} + g_W W^{3u} \quad . \quad (2.348)(b)$$

The Boson masses are then linearly related to the  $g$  charges by



(experimental)

a constant of proportionality determined by the Fermi coupling

constant  $G_F$ . Specifically,  $-2.20$  ( $k=c=1$ ) ( $v_F \equiv$  Fermi vacuum expectation)

$$\frac{1}{8} g_W^2 = \frac{1}{\sqrt{2}} G_F \cdot M_W^2 = \frac{1}{2} M_W^2 / v_F^2 \quad (2.349)(a)$$

$$\frac{1}{8} g_Z^2 = \frac{1}{\sqrt{2}} G_F \cdot M_Z^2 = \frac{1}{2} M_Z^2 / v_F^2 \quad (2.349)(b)$$

while the presence of  $g_Z$  in front of both  $A^u$  and  $Z^u$  in (2.348) indicates that the photon  $A^u$  is itself massless, ie.,

$$M_A = 0 \quad (2.350)$$

This is just what one would expect. Finally, we shall wish to examine

the preonic decomposition of the electroweak bosons  $A^u, Z^u$ . Utilizing

(2.345), and setting  $J_{(em)}^u = \bar{\chi}_{(e)} \gamma^u Q \chi_{(e)}$ ,  $J_W^{3u} = \bar{\chi}_{(e)} \gamma^u I^3 \chi_{(e)}$ , with  $\chi_{(e)}$  equal

to the preonic flavor sextuplet  $(A_R, A_G, A_B, B_L, C, D)$ , and extracting

the appropriate values of  $Q$  and  $I^3$  from Table 2.7, one arrives at

the preonic decomposition: ( $Z^u$  is for left-handed chiral projections only -  $\frac{1}{2}(CC-DD)$  is eliminated for right handed states)

$$A^u = \bar{\chi}_{(e)} Q \chi_{(e)} = \frac{1}{3} (\bar{B}_L B_{\bar{L}} + \bar{C} C) - \frac{2}{3} \bar{D} D \quad (2.351)(a)$$

$$Z^u = \bar{\chi}_{(e)} (I^3 - Q \sin^2 \theta_W) \chi_{(e)} \\ = \frac{1}{2} (\bar{C} C - \bar{D} D) - \left[ \frac{1}{3} (\bar{B}_L B_{\bar{L}} + \bar{C} C) - \frac{2}{3} \bar{D} D \right] \sin^2 \theta_W \quad (2.351)(b)$$

This yields two of the six neutral currents involving  $\bar{A}_R A_R, \bar{A}_G A_G, \bar{A}_B A_B,$

$\bar{B}_L B_{\bar{L}}, \bar{C} C, \bar{D} D$ , see the discussion following (2.330). Now we turn to

the four remaining neutral current Bosons involving the strong

(massless) interaction gluons  $G^u$ , and the hyperweak (massive) vector Boson  $X^{0u}$ .

In electroweak theory, as illustrated by the prior discussion, one spontaneously breaks the  $SU(2) \times U(1)$  flavor symmetry of the electroweak

interaction so as to ensure the emergence of the low energy  $U(1)$

electromagnetic flavor interaction. In the process, one is led to

to the electroweak neutral current preon decomposition (shown in

(2.351) above. In strong/hyperweak interaction theory, we shall

follow a similar approach. Instead of flavor  $SU(2) \times U(1)$  however,