2.14 - On the Replication of Fermion Generations: Four Generational
Grand Unification with Eighteen Preons and Nine Independent
Flavor/Color/Generation Degrees of Freedom, and a Preon
Discussion of Mesons and Meson Decay

The existence of multiple fermionic generations has been a
puzzle ever since Pauli first inquired about the muon, 'who ordered
this?' At the time, he was referring to the then recent discovery
of a particle identical with the electron in all respects, except
for the distinguishing fact that this particle was about two-hundred
times as heavy as the electron. Today, we know that this type of rep-
lication takes place not only for leptons, but also for quarks.

We know too, beyond the muon "family" or "generation," \( ^7 \) that there exists
yet a third tau generation; and further, although no additional
\( ^7 \) such generations have to date been observed, \( ^7 \) there exists no known
theoretical principle to preclude the existence of additional generations
beyond the known three. Hence, it is common practice to hold open
the possibility that one or more additional generations beyond the
observed electron, muon and tauon generations might actually also
exist in nature, and to suppose that the contemporary absence of
experimental evidence for any additional generations, if they do
exist, is most probably due to limitations on the energy scales at
which we are able, with present technology, to perform experiments.

Harari has noted of the generation puzzle, that 'the presently
accepted pattern of generations has two independent (his emphasis)
striking features:

1. Within each generation, the pattern of quarks is very sim-
ilar to the pattern of leptons.
2. Each generation is similar to the other generations.\(^{2.2}\)

This first point refers apparently refers to the fact that the number of quark generations, whatever that number may be, seems to be equal to the number of lepton generations (this is also mandated theoretically, to avoid fermion loop anomalies\(^{2.2}\)); and also, to the fact that within a single generation, the number of quark flavors, which is two, up and down, is equal to the number of lepton flavors, electron and neutrino. This similarity is also manifest in the fact, for left-handed chiral projections, that both the quark pair and the lepton pair form weak isospin doublets, which is to say that the two quark flavors within a single generation decay into one another through exactly the same weak interaction process as the two lepton flavors within that generation decay into one another, namely, by the emission of an electroweak $W^\pm$ or $W^0$ vector Boson. This is of course the particular area of similarity which led us initially the discussion of preonic flavor grand unification in Section 2.10. The second point refers apparently to the fact, aside from the very important difference in particle masses, that all other features of the various fermions, in particular their flavor and color quantum numbers, are the same from one generation to the next. Thus for example, all of the electron, muon and tauon have identical electrostatic charge $Q=-1$, while all of their associated neutrinos have $Q=0$. Similarly for the $u, c, t$ quarks with $Q=+2/3$ and the $d, s, b$ quarks with $Q=-1/3$. Naturally, all of the quarks come in three colors; while at high energies, where lepton number becomes a fourth color, the leptons within each generation will color transform as
a fourth quark color, within that same generation.

One of the more successful theoretical approaches to the replication of fermion generations, known as "horizontal symmetry," was first developed by Pati and Salam.\textsuperscript{2,23} In this approach, the four colors of fermion (three quark colors, one lepton color) are said to follow a "vertical" symmetry, while the replication of fermion generations is realized by a horizontal duplication of the as often as is necessary to account for all generations. vertical color symmetry.\textsuperscript{A} For the sake of discussion, we shall say therefore that these two symmetries, color and generation, are "orthogonal" symmetries. Thus for example, it is now common practice to form the color x (flavor/generation) matrix: (Distinguish $l^\pm$ lepton and $l^\pm$ lepton by context)

$$
\begin{pmatrix}
\nu^e_L & e_L & \nu^\mu_L & \mu_L & \nu^\tau_L & \tau_L & \ldots
\\
u^e_R & d_R & c_R & s_R & t_R & b_R & \ldots
\\
u^\mu_R & d_G & c_G & s_G & t_G & b_G & \ldots
\\
u^\tau_R & d_B & c_B & s_B & t_B & b_B & \ldots
\end{pmatrix}_{L,R}
$$

in order to depict the basic flavor/color/generation and chiral combinations of fermion.\textsuperscript{2,24} In the above, the vector formed out of the quarks ($u,d,c,s,t,b$) is thought to form an SU(6) flavor group, but it is just as well known that this is only an approximate, not an exact symmetry. Put into other terms, while we can utilize SU(6) flavor to enumerate particle states, this means that we cannot effectively utilize this group to precisely predict fermion masses.

An important limitation in the above is uncovered by observing that the horizontal rows actually involve a mixture of flavor weak iso-spin and generation symmetries. That is, on the one hand, the decays $u \leftrightarrow d$, $c \leftrightarrow s$, $t \leftrightarrow b$ and $e \leftrightarrow \nu_e$, $\mu \leftrightarrow \nu_\mu$, $\tau \leftrightarrow \nu_\tau$ are simply ordinary weak decays, and need not have anything to do with the
intergenerational symmetry. On the other hand, the decays $u \leftrightarrow c \leftrightarrow t$, $d \leftrightarrow s \leftrightarrow b$ and $e \leftrightarrow \tau \leftrightarrow \tau$, $\nu_\tau \leftrightarrow \nu_\mu \leftrightarrow \nu_e$ directly involve the generation symmetry, though they need not have anything to do with weak beta decay. Of course the composite forms of decay, for example, the decay $u \leftrightarrow s$, involve both weak beta decay, which is part of flavor symmetry, and generation decay. It is this composite form of decay which is observed experimentally as Cabibbo-type mixing. However, from a theoretical standpoint, one can obtain an important simplification by clearly separating the treatment of the weak (flavor) that beta decay from of intergenerational decay.

When considered on these two forms of decay a composite basis, form the basis for properly describing observed Cabibbo mixing. In short, while Cabibbo mixing involves both flavor (weak beta-decay) and generation decay, it is nevertheless advantageous to distinguish generation from flavor as a completely independent symmetry, with its own set of independent generation quantum numbers. Consequently, in the same manner that we earlier used the flavor preon model in Section 10 to "factor out" the very similar forms of weak beta-decay as between the quarks and leptons in a given generation, we should wish to similarly "factor out" the replication of weak beta decay as it takes place within all of the different generations of (2.374). This line of reasoning is further developed by considering the vertical columns in (2.374). Here, we see lepton number regarded as a fourth fermion color. As we know from the prior discussion however, the decay of a quark into a(n anti) lepton involves the emission of an exotic $x^{+1/3}u$ or $x^{-1/3}u$ intermediate vector boson and, due to the same overlaps which reduced from eight to six the number of flavor/color preons and linearly in-

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dependent \(^{\text{diagonalized}}\) generators, this may be thought of alternatively as the decay between an \(A\) and a \(B\) flavor of preon, or, the decay between and \(R,G\) or \(B\), and an \(\overline{L}\) color of preon. (see ie., (2.279)). It is this alternative description which is most readily depicted by forming the preons into the flavor/color six-vector which includes \((A_R,A_G,A_B,B_I,C,D)\). The point here is that hyperweak decay, which may be regarded as a form of flavor decay, and which can also be thought of as ordinary beta decay that takes place simply through a different mode of the same channel process (contrast (2.278), (2.279)), is depicted in (2.374) as a vertical symmetry; while ordinary weak beta decay, which is also a form of flavor decay (and in fact is the only other form of flavor decay which produces a change in flavor quantum numbers), is depicted in (2.374) as a horizontal symmetry. This would lead us to believe that some aspects of flavor are horizontal, while other aspects are vertical; in short, that flavor symmetry is orthogonal to itself. This of course, is not so, as the original flavor preon quadruplet \((A,B,C,D)\) places both the hyperweak and ordinary forms of beta decay into a linear vector which obeys simple flavor \(SU(4) \times U(1)\) at high energies. Again for discussion, this is to say that hyperweak and ordinary beta (except in the sense of the (2.278), (2.279) verticals) flavor decays are not orthogonal \(^{\text{hyperweak}}\) as (2.374) might lead us to believe. Rather, we should say that these flavor decays are "co-linear," which means that it is not necessary to make hyperweak decay vertical, and ordinary beta decay horizontal, in order to obtain a proper description of the two. Consequently, we shall wish to reformulate the matrix (2.374) in such a manner that both the hyperweak and ordinary modes of beta decay can be represented in a co-linear fashion.
It is also worth noting that flavor and color as a combined symmetry are also colinear, insofar as the hyperweak preon transition $A \leftrightarrow B$ may at once be regarded as both a flavor and a color transition.

If one follows the approach suggested above, and regards generation symmetry as a symmetry that is completely distinct from flavor and color symmetries, and therefore as a symmetry that carries its own distinct set of conserved quantum numbers separately from both of flavor and color, then two preliminary questions immediately arise. First, which particles, both complex and real, carry the generational charges, and; second, what is the (high energy) gauge group of the generation symmetry? In short, what are the particle eigenstate solutions of generation symmetry, and what are the generation charge eigenvalues for these various eigenstate solutions? One begins to answer this question by noting, as Harari points out, that both the quarks and the leptons, i.e., that all fermions, follow a similar pattern of generations. This would tend to suggest that it is fermion number $F$ itself, which is the key to identifying which particles carry a generation charge, just as quark number $Q_u$ is the key to identify which particles carry a (three) color charge. If we label the generation charges with the names of the generations themselves, i.e., by $e, \mu, \tau, \ldots$, and if we also assume for the moment, to be discussed shortly, that there also exists a fourth generation which will be designated as the gamma, $\gamma$, generation, then it is possible to expand upon equation (1.1), which has already been utilized to connect flavor and color, so as to write:

$$F = Q_u + L = R + G + B + L = e + \mu + \tau + \gamma.$$  \hfill (2.375)

Thus for example, the ordinary electron, which has $F=L=1$ and $Q_u=0$, 2.164
also has $e=1$ and $\mu=\tau=\gamma=0$. So too for the electron's neutrino. The $c$ quark on the other hand, as another example, is part of the second (muon) generation, and it has $F=Q_u=M=1$ and $L=e=\tau=\gamma=0$, as does the $s$ quark. Any further distinction to be made, for example, as between the $(c,s)$ isospin doublet, may be achieved by the further specification of the proper flavor quantum numbers, i.e., $Q=+2/3$ for $c$ and $Q=-1/3$ for $d$, etc. Hence, to fully specify the characteristics of any given particle, it is necessary to examine that particle's quantum numbers, i.e., internal degrees of freedom, for all three of the flavor, color and generation symmetry groups. Additionally, insofar as the flavor generators $G^0, t^3, y^8, b^{15}$ are concerned, it is also necessary to be concerned with whether it is a left or right handed chiral projection (see Table 2, 7) under consideration. Generation symmetry, by itself, is most readily presumed to be chiral symmetric, as is color symmetry, to be seen below.

Given that it is Fermion number $F$ which is apparently associated with the generation charge, it is possible to answer the first question raised in the prior paragraph, namely, which particles carry the generation charge? The answer: those particles which carry fermion number, or which are composed out of particles which carry fermion number. If we again note that the $A$ preon is the carrier of quark number, $F=Q_u=1$, that the $B$ preon is the carrier of (anti)lepton number $F=L_c=-1$, and that the $C$ and $D$ preons (electroweak isospin up and down) both carry $F=Q_u=L=0$, then this answer can be represented in a more concrete way. In particular, this now indicates that it is the $A$ and $B$ preons, each with $F=1$, that are to be regarded as the complex preonic carriers of the generation charge. The $C$ and $D$ preons on the other hand, each with $F=0$, are completely uninvolved in carrying generation charges. The
set of real particles which carries the generation charge therefore, is simply that set of particles which is composed out of one or more A and/or B preons and/or antipreons. The C and D preons are again irrelevant to generation symmetry. This is how one "factors out" β-decay.

As to the second question, what is the gauge group of generation symmetry, we shall hazard an educated guess that the proper gauge group, as in flavor and color symmetry, is SU(4)xU(1) at high energies; which is to say that there are a total of four distinct generations, three known, one hypothesized. The reason for supposing that a fourth generation exists, at this point in time, is largely aesthetic. In particular, because the C and D preons are not involved in generation symmetry (note that these are also not involved in color, and are in fact only involved in electroweak isospin flavor weak beta decay) it is possible to represent the vertical symmetry of (2.374), in terms of a preonic decomposition, by the four-color vector containing \((A_R; A_G; A_B; B_L)\). The replication of fermion generations is then achieved by the fourfold duplication of the vertical color symmetry, to form the \(4\times4\) preonic square matrix:

\[
\begin{pmatrix}
B_L^e & B_L^u & B_L^d & B_L^s \\
A_R^e & A_R^u & A_R^d & A_R^s \\
A_G^e & A_G^u & A_G^d & A_G^s \\
A_B^e & A_B^u & A_B^d & A_B^s
\end{pmatrix}
\]

(2.376)

It is the desire to make this a \(4\times4\) square matrix which is the particular aesthetic ground upon which a fourth generation is introduced. At relatively low energies, where hyperweak \(A \leftrightarrow B\) transitions are highly suppressed, one has only three colors, hence the appropriate choice would be a \(3\times3\) subset of this matrix involving only the colored
A preons, and the $e, \mu, \tau$ generations, consistent with low energy experimental observations. While we have argued for a square matrix on aesthetic grounds, which happens also to work out consistently with low energy observation of exactly three generations, it would probably be fruitful to determine if there is any theoretical basis for actually mandating that a square matrix be utilized. One would suppose that there is, and if so, this would provide an additional basis for concern about (2.374), which is currently 4x6, and which in this context it would therefore be the discovery of the third generation in particular would become 4x8 with a fourth generation. The real problem with (2.374) is the repeated duplication of the isospin degree of freedom, question co-linearly with generation freedom. In fact, since isospin decays involve the preonic transformation $C \leftrightarrow D$, and since $C$ and $D$ each carry $F=0$ and are hence irrelevant to generation decay, it is preferable to factor these preons out of consideration entirely, and to focus merely on the generational properties of the A and B preons. This sort of reduction cannot not in any way be achieved by examining strictly real particles. Rather, it is vital in this situation to directly consider the preonic decomposition of the generation symmetry, as in (2.376), therefore eliminating from consideration the superfluous, and in this context confusing isospin degree of freedom. Ironically, or perhaps not so, this all brings us back to our initial considerations in Section 10, on the fundamental similarities between quarks and leptons within a single generation, as per the earlier quoted remark by Harari, and between the quarkonic and leptonic forms of beta decay via the electroweak $W^\pm$ vector bosons, which are themselves composed strictly out of C and D preons, see (2.297)(b); and the even earlier discussions leading to (2.234)(a) and (b).
It has earlier been noted, from a preonic point of view, that admixtures of the A and B flavors of preon between themselves, and of the C and D flavors of preon between themselves, are responsible for the vector Boson flavors of a particle; while a composition which involves either of the A or B flavors, in combination with either of the C or D flavors, results in the composition, or more precisely, the "flavor polarizations" for the spin half real Fermions. See for example, eqs. (2.280) for flavor only, and (2.318) for flavor and color within a single generation. Now, with the inclusion also of at least three, and perhaps four fermionic generations, the color x generation matrix (meaning that color and generation are orthogonal, i.e., not colinear symmetries) (2.376) indicates, wherever we previously considered only the preon color vector (B_L, A_R, A_G, A_B), that we must now consider the replication of this quadruplet over three or four distinct generations, i.e., we must now consider (B_L, A_R, A_G, A_B) e, μ, τ, γ. In this sense, generation symmetry is much more closely related to the strong color interaction, than it is to the flavor interactions. As flavor and color can be made colinear in the sextuplet (A_R, A_G, A_B, B_L, C, D), the overall symmetry is in fact flavor/color x generation. Whereas we started with two preons, (C, D) in electroweak/color theory, went to four preons in electroweak, strong and quantum gravitational flavor theory, and then to six preons in flavor/color theory, it is now necessary, with generation symmetry as well, to start with (for four generations) a total of eighteen preons; that is, the sixteen in (2.376), and the electroweak (C, D) pair. Logically, this now yields 18 x 18 = 324 real particles, considering

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all conjugacy states. It is also helpful to think of this as

\[(16 + 2) \times (16 + 2) = 256 + 64 + 4 = 324\] particles, given the manner

in which fermion and boson flavors and colors are composed out of

preons, again, eqs. (2.280), (2.318). The 256 corresponds to the

\(16 \times 16\) combinations of A and B preon that can be formed out of the

sixteen distinct flavor/color/generation preons in (2.376). These

are all vector bosons associated with the strong four-color interaction,

and are either gluons, \(G^u\), charged hyperweak bosons, \(X^{+1/3u}\) and

\(X^{-1/3u}\), or neutral hyperweak bosons, \(X^0_u\). This will be discussed

in more detail shortly. The 4 corresponds to the \(2 \times 2\) combinations

of C and D preon which are possible, and is comprised of the usual

four bosons of electroweak theory, i.e., the photon \(A^u\), charged

weak bosons \(W^{+u}, W^{-u}\), and the electroweak neutral bosons \(W_0^u(=\gamma^u)\).

Thus, prior to conjugacy reductions, one starts with \(256 + 4 = 260\)

real flavor/color/generation combinations of real vector boson.

The remaining 64 = 32 + 32 refers to the fermions, which involve,

as in some "menus," one preon from A or B, and one from C or D. Here,

there are 32 real fermions and 32 antifermions, with the 32 real

fermions comprised of the eight flavor/color combinations \(u_R, u_G, u_B,\)

\(d_R, d_G, d_B, e_L, \nu_L\), duplicated over four generations. It is helpful to

begin by considering the 32 conjugally independent fermions alone,

and later, to bring in the various bosons. As (2.376) was developed

with the explicit purpose of explaining the replication of real

fermions in terms of a preonlic decomposition, the examination of

the real fermions may double as a check on the overall theoretical

consistency.

The flavor/color/generation decomposition of the real fermions
is rather graphically depicted by examining the composition of the color × generation matrix \((2.376)\), with similar 4×4 matrices involving the C and D preons in an appropriate manner. Continuing with the flavor/color fermion compositions which have applied all along, as in \((2.280),(2.318)\), but supplementing this with multiple generations, one may depict the preonic decomposition of the 32 real fermions in the form:

\[
\begin{pmatrix}
  \psi^e_L & \psi^u_L & \psi^\tau_L & \psi^\gamma_L \\
  \psi^e_R & \psi^u_R & \psi^\tau_R & \psi^\gamma_R \\
  \psi^e_G & \psi^u_G & \psi^\tau_G & \psi^\gamma_G \\
  \psi^e_B & \psi^u_B & \psi^\tau_B & \psi^\gamma_B \\
\end{pmatrix}_{L=R} \equiv \begin{pmatrix}
  \psi^e_L & \psi^u_L & \psi^\tau_L & \psi^\gamma_L \\
  \psi^e_R & \psi^u_R & \psi^\tau_R & \psi^\gamma_R \\
  \psi^e_G & \psi^u_G & \psi^\tau_G & \psi^\gamma_G \\
  \psi^e_B & \psi^u_B & \psi^\tau_B & \psi^\gamma_B \\
\end{pmatrix}_{L=R} \oplus \begin{pmatrix}
  \bar{c} & \bar{c} & \bar{c} & \bar{c} \\
  \bar{c} & \bar{c} & \bar{c} & \bar{c} \\
  \bar{c} & \bar{c} & \bar{c} & \bar{c} \\
  \bar{c} & \bar{c} & \bar{c} & \bar{c} \\
\end{pmatrix}_{L=R}
\]

\[(2.377)(a)\]

\[
\begin{pmatrix}
  \psi^e_L & \psi^u_L & \psi^\tau_L & \psi^\gamma_L \\
  \psi^e_R & \psi^u_R & \psi^\tau_R & \psi^\gamma_R \\
  \psi^e_G & \psi^u_G & \psi^\tau_G & \psi^\gamma_G \\
  \psi^e_B & \psi^u_B & \psi^\tau_B & \psi^\gamma_B \\
\end{pmatrix}_{L=R} \equiv \begin{pmatrix}
  \psi^e_L & \psi^u_L & \psi^\tau_L & \psi^\gamma_L \\
  \psi^e_R & \psi^u_R & \psi^\tau_R & \psi^\gamma_R \\
  \psi^e_G & \psi^u_G & \psi^\tau_G & \psi^\gamma_G \\
  \psi^e_B & \psi^u_B & \psi^\tau_B & \psi^\gamma_B \\
\end{pmatrix}_{L=R} \oplus \begin{pmatrix}
  \bar{d} & \bar{d} & \bar{d} & \bar{d} \\
  \bar{d} & \bar{d} & \bar{d} & \bar{d} \\
  \bar{d} & \bar{d} & \bar{d} & \bar{d} \\
  \bar{d} & \bar{d} & \bar{d} & \bar{d} \\
\end{pmatrix}_{L=R}
\]

\[(2.377)(b)\]

In terms of the usual notation, and introducing the \(x, y, \gamma, \nu^\gamma\) fermions of the \(\gamma\) generation, the sixteen distinct flavor/generation combinations of fermion (suppressing color) in the above are better known by the symbols:

\[
(u, c, t, x) \equiv (u^e, u^u, u^\tau, u^\gamma) \quad (2.378)(a)
\]

\[
(d, s, b, y) \equiv (d^e, d^u, d^\tau, d^\gamma) \quad (2.378)(b)
\]

\[
(\nu^e, \nu^u, \nu^\tau, \nu^\gamma) \equiv (\nu^e, \nu^u, \nu^\tau, \nu^\gamma) \quad (2.378)(c)
\]

\[
(e, \mu, \tau, \gamma) \equiv (e^e, e^u, e^\tau, e^\gamma) \quad (2.378)(d)
\]

The \(L=R\) and \(L\neq R\) is used to designate chirality. Specifically, because the A and B preons are chiral symmetric \((L=R)\) for all interactions, while the C and D are not \((L\neq R)\), see Table 2.7, the consequence here...
is that the real fermions, which all involve a combination of $A$ or $B$ ($I \neq R$), and $C$ or $D$, are chiral non-symmetric (with respect to $G^0, I^3, Y^8, B^{15}$, see Table (2.9)) due in particular to the non-symmetry of $C$ and $D$, as is to be expected. Consequently, the 32 distinct real flavor/color/generation fermions have a total of $2 \times 32 = 64$ distinct chiral projections.

At this point, we can begin to examine Cabibbo-type mixing for both quarks and leptons. To do this, it is helpful first to recall the six neutral current generators $Q, Z, (1/3)Q_u, X, \lambda^3$ and $\lambda^8$, which may be consolidated from (2.345), (2.368) and (2.352), as such:

\begin{align*}
Q &= y^8 + I^3 \\
Z &= I^3 - Q \sin^2 \theta_W \\
\frac{1}{3}Q_u &= \lambda^0 - \lambda^{15} \\
X &= \lambda^{15} + \frac{1}{3}Q_u \sin^2 \theta_S \\
\lambda^3 &= \lambda^3 \\
\lambda^8 &= \lambda^8,
\end{align*}

recall that $\lambda^3$ and $\lambda^8$ do not mix. Once these six generators are specified, it is possible to deduce therefrom all of $G^0, I^3, Y^8, B^{15}, \lambda^0, \lambda^3, \lambda^8, \lambda^{15}$ and $Q_u, L, F$, as noted earlier in the discussion following (2.328). The associated preon decomposition for the above, from (2.351), (2.373) and (2.352), is given by: ($Z^u$ for left-handed projections)

\begin{align*}
A^u &= \frac{1}{3}(\overline{E}B + \overline{C}C) - \frac{2}{3}\overline{D}D \\
Z^u &= \frac{1}{3}(\overline{C}C - \overline{D}D) - \left(\frac{1}{3}(\overline{E}B + \overline{C}C) - \frac{2}{3}\overline{D}D\right) \sin^2 \theta_W \\
G^u &= \frac{1}{3}(\overline{A}^R\overline{A}^R + \overline{A}^G\overline{A}^G + \overline{A}^B\overline{A}^B) \\
\lambda^0 &= \frac{1}{4}\overline{E}B + \frac{1}{12}(\overline{A}^R\overline{A}^R + \overline{A}^G\overline{A}^G + \overline{A}^B\overline{A}^B) + \frac{1}{3}(\overline{A}^R\overline{A}^R + \overline{A}^G\overline{A}^G + \overline{A}^B\overline{A}^B) \sin^2 \theta_S \\
G^3u &= \frac{1}{2}(\overline{A}^G\overline{A}^G - \overline{A}^B\overline{A}^B) \\
G^8u &= \frac{1}{3}\overline{A}^R\overline{A}^R - \frac{1}{6}(\overline{A}^G\overline{A}^G + \overline{A}^B\overline{A}^B).
\end{align*}

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For right-handed $Z^u_A$, the $\frac{1}{3}(\bar{C}C-DD)$ term may be eliminated. All other neutral currents are left-right chiral symmetric. Given the above, it helps now to recast Table 2.7 into a more developed form including flavor and color for a single generation. Using (2.379) and Tables 2.7, 2.13, this is as follows: (Showing only conjugally independent states)

<table>
<thead>
<tr>
<th>Q</th>
<th>$Z_L$</th>
<th>$Z_R$</th>
<th>$\frac{1}{3}q_u$</th>
<th>X</th>
<th>$\lambda^3$</th>
<th>$\lambda^8$</th>
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<tr>
<td>A</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1/3</td>
<td>-1/12+(1/3)$\sin^2\theta_S$</td>
</tr>
<tr>
<td>A</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1/3</td>
<td>-1/12+(1/3)$\sin^2\theta_S$</td>
<td>-1/6</td>
</tr>
<tr>
<td>B</td>
<td>$-1/3$</td>
<td>$-(1/3)\sin^2\theta_W$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>B</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1/3</td>
<td>$1/6$</td>
</tr>
<tr>
<td>C</td>
<td>$-1/3$</td>
<td>$-(1/3)\sin^2\theta_W$</td>
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<td>0</td>
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<td>0</td>
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<td>C</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>D</td>
<td>$-1/3$</td>
<td>$-(1/3)\sin^2\theta_W$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
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<td>0</td>
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</tr>
<tr>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$G^u_{RB}$</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$G^u_{BG}$</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$X^u_{RL}$</td>
<td>$1/3$</td>
<td>$(1/3)\sin^2\theta_W$</td>
<td>1/3</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$X^u_{RL}$</td>
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<td>0</td>
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<td>1/3</td>
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</tr>
<tr>
<td>$X^u_{BL}$</td>
<td>$1/3$</td>
<td>$(1/3)\sin^2\theta_W$</td>
<td>1/3</td>
<td>0</td>
<td>0</td>
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</tr>
<tr>
<td>$X^u_{BL}$</td>
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<td>0</td>
<td>0</td>
<td>1/3</td>
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</tr>
<tr>
<td>$\nu^u_L$</td>
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<td>$\frac{1}{2}+(1/3)\sin^2\theta_W$</td>
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<td>$\frac{1}{2}$</td>
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</tr>
<tr>
<td>$\nu^u_L$</td>
<td>0</td>
<td>$\frac{1}{2}+(1/3)\sin^2\theta_W$</td>
<td>0</td>
<td>$\frac{1}{2}$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
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<td>$-(2/3)\sin^2\theta_W$</td>
<td>1/3</td>
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</tr>
<tr>
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<tr>
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<td>$-(2/3)\sin^2\theta_W$</td>
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</tr>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>1/3</td>
<td>0</td>
</tr>
<tr>
<td>$d^u_R$</td>
<td>$-1/3$</td>
<td>$-(1/3)\sin^2\theta_W$</td>
<td>1/3</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$d^u_R$</td>
<td>$-1/3$</td>
<td>$-(1/3)\sin^2\theta_W$</td>
<td>1/3</td>
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<td>0</td>
<td>1/3</td>
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</tr>
<tr>
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<td>-$\sin^2\theta_W$</td>
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</tr>
<tr>
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</tr>
<tr>
<td>$Z^u$</td>
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<td>0</td>
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<td>0</td>
</tr>
<tr>
<td>$X^u$</td>
<td>0</td>
<td>0</td>
<td>0</td>
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<td>0</td>
</tr>
<tr>
<td>$G^u$</td>
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</tr>
</tbody>
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Table 2.14 - Complete Flavor/Color Classification of All Conjugally Independent Complex Preons and Real Fermions and Bosons, Including Neutral Currents and Right-Handed Chiral Projections for Non-Symmetric $Z$ Neutral Current. (21 Real States)
Note in the above that \( U_L, \ell_L \) designates "lepton" not "left." 

With this, we are ready to discuss Cabibbo mixing for quarks and leptons.

To illustrate the important features of Cabibbo mixing, one can select any relevant quark and lepton pair, as the analysis is the same for any other relevant pairs. For example, for quarks, we may choose as representative the decay mode \( u \leftrightarrow s \), while for leptons we may select \( \nu_e \leftrightarrow u \). In a simple two-generational approach, this mixing is controlled for each of quarks and leptons by a single (one each) Cabibbo mixing angle \( \Theta_C \), which appears to differ for quarks and leptons. For quarks, this angle at accessible energies is measured to be about \( \Theta_C(Q_u) \approx 13^0 \), while for leptons, there is in fact no observable mixing, \( \Theta_C(L) \approx 0^0 \). Chirality aside, the current terms describing the quarkonic and leptonic forms of mixing, respectively, are: (Note, \( (Q_u) \) and \( (L) \) in this context are merely labels for the angles.)

\[
\begin{align*}
J_{\text{Quark}}^u &= (u \ c) \ \chi^u \begin{pmatrix} \cos \Theta_C(Q_u) & \sin \Theta_C(Q_u) \\ -\sin \Theta_C(Q_u) & \cos \Theta_C(Q_u) \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix} \\
&\text{(2.381)(a)}
\end{align*}
\]

and

\[
\begin{align*}
J_{\text{Lepton}}^u &= (\nu \ e \ \nu_\mu) \ \chi^u \begin{pmatrix} \cos \Theta_C(L) & \sin \Theta_C(L) \\ -\sin \Theta_C(L) & \cos \Theta_C(L) \end{pmatrix} \begin{pmatrix} e \\ \mu \end{pmatrix} \\
&\text{(2.381)(b)}
\end{align*}
\]

We then examine the preonic decomposition of any particular generation changing transition, for example, \( u \leftrightarrow s \), \( \nu_e \leftrightarrow \mu \), for quarks and leptons respectively. To simplify matters, we assume also that the quarks involved in (2.381)(a) do not exchange any color during the entire decay process, i.e., that what begins as a red quark remains as a red quark, and does not strongly interact with any other quark. In short, if any strong gluons are to be exchanged during this process, these are to be limited to the SU(3) singlet gluon of (2.380)(c),
\( G_0^u = (1/3) (\tilde{A}_R^A A_R^A + \tilde{A}_G^A A_G^A + \tilde{A}_B^A A_B^A) \), and none of the remaining eight color carrying gluons of \( 3 \otimes 3 = 8 + 1 \) for \( SU(3) \otimes U(1) \). Later, we shall both relax and elaborate upon this restriction. For now, the simple net effect is that this allows us to disregard, i.e., suppress color, when considering the Cabibbo mixing of quarks at low energies. Consequently for quarks, the preon decomposition of the \( u \leftrightarrow s \) decay diagram may be drawn as follows:

\[
\begin{align*}
\equiv \text{\( \mu \rightarrow s + G_{\mu e}^{\mu} + W^{\pm \mu} \)} \\
\equiv \text{\( \mu \rightarrow s + X_{\mu e}^{\mu} + W^{\pm \mu} \)} \\
\text{(2.382)(a)}
\end{align*}
\]

while for leptons, the \( \nu_e \leftrightarrow \mu \) diagram may be drawn as follows:

\[
\begin{align*}
\equiv \text{\( e \rightarrow \nu^{\mu} + X_{\mu e}^{\mu} + W^{-\mu} \)} \\
\text{(2.382)(b)}
\end{align*}
\]

Noting from (2.376) that it is now important to distinguish the \( A \) and \( B \) flavors of preon by their generation, and imposing the requirement that generation number must be conserved through any vertex, it becomes necessary that these decays be modelled not by a three, but rather by a four particle (and preon) vertex. If it were not necessary to distinguish \( A \) and \( B \) by their generation then, assuming that no color is exchanged, it would be possible simply to connect the \( A \) preons in (2.382)(a), and the \( B \) preons in (2.382)(b),
resulting in the usual three particle vertex, along single worldlines, as indicated by the "dotted" connection line. This of course, simply brings us back to ordinary single generational beta-decay, as in the diagrams (2.278). This brings us to the critical issue.

If we recall again the forms (2.380)(c) and (d) for the strong interaction neutral current gluon singlet $G^u_0$, and the neutral hyperweak vector bosons $X^u_0$, namely, with color suppressed,

$$G^u_0 = \bar{A}A$$

$$X^u_0 = \frac{1}{2}(\bar{E}B - \bar{A}A) + \bar{A}A \sin^2 \Theta_S,$$

see (2.325), and if we compare this with the diagrams (2.382), one may arrive at some very interesting conclusions. Focusing on the fourth particle introduced into the decay diagrams, namely the $\bar{A}A^e$ in (2.382)(a) and the $\bar{E}B^e$ in (2.382)(b), it now appears that the $G^u_0$ and $X^u_0$ above should actually be regarded among other things to be bi-generational, in precisely the same manner that they are already regarded to be bi-colored. This is somewhat apparent when thinking about meson combinations of the preons in (2.376), but it helps to bring this point out in the context of Cabibbo mixing. Further, and this is the real key, it appears as though the former interaction (2.382)(a) for quarks, which involves a bi-generational (and if color is exchanged, bi-colored) combination $\bar{A}A$, can be mediated by either of the $G^u_0$ or the $X^u_0$, since each contains the $\bar{A}A$ combination, as shown above in (2.383). For leptons however, the latter interaction (2.382)(b) involves a bi-generational $\bar{E}B$ combination, and can therefore be mediated only by $X^u_0$. At presently accessible energies, which are way below anything approaching the super-massive $X^u_0$, the only

2.175
particle in (2.383) which can be produced is the $G^u_0$ (and the gluons $G^u$ generally if color is exchanged). Consequently, the only form of Cabibbo mixing which can be observed is the quarkonic, (2.382)(a), and not the leptonic, (2.382)(b). At hyperweak energies however, which are at least $\approx 10^{15}$ GeV., when the $X^u_0$ boson might be more readily produced, it would then be possible to observe lepton mixing as well as quark mixing. In other words, the Cabibbo mixing of quarks is mediated by both massless Gluons and very massive hyperweak bosons, while that of leptons is mediated only by the hyperweak bosons. At very high energies, where leptons are but a fourth quark color, and where the hyperweak bosons are consequently mere gluons which at least in part, carry the fourth color, the distinction between these bosons, and between quarks and leptons disappears; with the consequence that both quarks and leptons will engage in Cabibbo mixing with equal frequencies. At low energies however, where hyperweak boson production is virtually impossible, it is still perfectly possible to produce bi-generational gluons; however, these couple only to quarks, and not to leptons. Consequently, Cabibbo mixing will be observed for quarks, but not for leptons. One notes too, because of the lepton mixing which can take place once energies on the $X^u$ scale are reached, that the neutrino is expected to have some very small, though finite rest mass; and also, and as a consequence of this, to possess both left and right handed chiral projections.

An alternative way of viewing this conclusion is to note, from Table 2.14, that the generator $(1/3)Q_u$, which in the context of flavor/color and generation symmetry is to be associated with Cabibbo mixing mediated by bi-colored, bi-generational gluons, is equal
to 1/3 for all quarks, but is equal to 0 for the leptons. Consequently, only the quarks experience the "Strong" form of Cabibbo mixing, mediated by ordinary gluons. On the other hand, the hyperweak X generator is non-zero for both quarks and leptons. Hence, when the hyperweak gluons (bi-generational) are used to mix generations, one will observe both quark and lepton mixing, i.e., both quarks and leptons will experience the "hyperweak" form of Cabibbo mixing, mediated by high energy hyperweak bosons. Of course, since hyperweak production requires enormous energies, this form of mixing cannot be observed for all practical purposes, with today's experimental capabilities. This may be contrasted with the behavior of the neutrino in ordinary electroweak interactions. Noting particularly from Table 2.14 that the neutrino is the only fermion for which the electromagnetic charge generator we deduce the well known result that $Q = 0$, this particle does not interact electromagnetically. Nevertheless, the neutral current $Z_L$ generator is non-zero for all fermions; hence all fermions, including the left-handed neutrino, can interact via the neutral current $Z$ interaction, and of course, via the charged $W^\pm$ boson interaction. The exception here is the right handed neutrino, for which $Z_R = 0$ also. Hence this neutrino component will not couple through any of $Z^u (= W^0u)$ and $W^\pm u$ either. In fact, if one examines just the neutrino in Table 2.14, and recalling that the $X$ generator is chiral symmetric, it becomes apparent that the only interaction through which the right-handed neutrino projection will couple, is that associated with the $X$ generator ($X = -\frac{1}{3}$). Thus, to actually detect the right-handed neutrino projection, thereby establishing a non-zero rest mass, it would be necessary to produce one of the hyperweak $X^u_0$, $X^+1/3u$ bosons, which again, is virtually impossible with current technological capabilities. This contrast may
be summarized by stating that left-handed neutrinos in electroweak theory serve a role similar to that of leptons in strong/hyperweak theory, insofar as each interacts only through the massive, and not through the massless vector bosons. In strong/hyperweak interactions, the various vector bosons carry the additional responsibility of mediating intergenerational decays; and the difficulty in producing the massive hyperweak bosons is reflected in the difficulty that one has observing leptonic Cabibbo mixing, and the neutrino rest mass, and the right-handed neutrino chiral components. In a somewhat more tabular form, this contrast between electroweak and strong/hyperweak theory is illustrated by examining the various flavors of boson in each theory, along with the various flavors and chiral projections of fermion which may interact through these bosons, as such:

<table>
<thead>
<tr>
<th>Electroweak Theory</th>
<th>Boson Mediators</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fermions</td>
<td>W0^u, W^*u</td>
</tr>
<tr>
<td>( u_L ), e_L, R, d_L, R</td>
<td>W0^u, W^*u, A^u</td>
</tr>
</tbody>
</table>

**Strong/Hyperweak Theory**

<table>
<thead>
<tr>
<th>Fermions</th>
<th>Boson Mediators</th>
</tr>
</thead>
<tbody>
<tr>
<td>( u_L, R, e_L, R )</td>
<td>X0^u, X^±1/3u</td>
</tr>
<tr>
<td>( u_L, R, d_L, R )</td>
<td>X0^u, X^±1/3u, G^u</td>
</tr>
</tbody>
</table>

Table 2.15 - Fermion Flavors and Chiral Projections, and Associated Flavors of Mediating Vector Boson, for Electroweak and Strong/Hyperweak Interaction Theory.

It is the absence of the gluons, \( G^u \), as mediators of strong/hyperweak due to the fact that \( G^u \) for \( u_L, e_L \) leptonic interactions involving \( u_L \), which is concurrently responsible for the absence of observed Cabibbo lepton mixing at accessible energies, and for the apparent masslessness of the neutrino and the consequent absence of right handed chiral neutrino projections. Again, this is but another way of viewing our earlier results reg-
arding the Cabibbo mixing of quarks and leptons.

At this point, we are now prepared to examine directly the various vector bosons which can be composed out of the 18 preon states, namely the 16 in (2.376) and the (C,D). Recalling that it is possible to compose \( 324 = (16 + 2) \times (16 + 2) = 256 + 64 + 4 \) real particles out of these 18 preons, and that the 64 represents fermions and anti-fermions, already discussed, we turn now to examine the (flavor/color/generation) \( 256 + 4 = 260 \) logical combinations of real vector boson. The 4, which describes the \( 2 \times 2 = 3 + 1 \) preon decomposition of the four electroweak flavor particles \( A^u, W_0^u, W^+_u \) into the C and D preons has already been examined in depth, see, eg., Figure 2.2 and discussion leading thereto. Consequently, what is now of particular interest to us here, are the \( 16 \times 16 = 256 \) logical real vector bosons which can be composed by the meson combination of the sixteen color/generation combinations of A and B flavored preon shown in (2.376). Working at ultra-high (gravitational) energies, prior to any form of spontaneous symmetry breaking, it is helpful to separate color from generation. Thus, from (2.376), one may form sixteen logical color combinations, namely the twelve mixed color states \( LR, LG, LB, \overline{RL}, \overline{RG}, \overline{RB}, GL, GR, GB, \overline{EL}, \overline{ER}, \overline{EG} \) and the four pure states \( \overline{LL}, \overline{RR}, \overline{GG}, \overline{BB} \) which, as we know, are used in the composition of the hyperweak/strong neutral current vector bosons, \( G_0^u, X_0^u, G^{3u}, G^B_u \), see (2.380)(c)-(f). When conjugacy is considered, these sixteen combinations may be reduced to the ten conjugally independent \( LR, LG, LB, \overline{GR}, \overline{BG}, \overline{RB} \) and \( \overline{LL}, \overline{RR}, \overline{GG}, \overline{BB}, \) for example. That six of the twelve mixed states may be reduced through conjugacy. Note by the way, as we are dealing here strictly with A and B preons, that all vector bosons currently under discussion are chiral symmetric. For generation, assuming four gen-

2.179
number of erations, the various logical meson generation combinations which can be formed from (2.376) totals sixteen as well, namely the twelve mixed generation states \( \bar{\nu}_r, \bar{\nu}_m, \bar{\nu}_e, \bar{\tau}_r, \bar{\tau}_m, \bar{\tau}_e, \bar{\mu}_r, \bar{\mu}_m, \bar{\mu}_e, \bar{e}_r, \bar{e}_m, \bar{e}_e \), and the four neutral states \( \bar{\nu}_r, \bar{\tau}_r, \bar{\mu}_r, \bar{e}_r \). Here too, six of the twelve mixed states can be reduced out once conjugacy is considered, hence we again reduce down to ten conjugally independent combinations, for example, \( \bar{\nu}_r, \bar{\nu}_m, \bar{\nu}_e, \bar{\tau}_r, \bar{\tau}_e, \bar{\mu}_e, \bar{e}_e \), and \( \bar{\nu}_e, \bar{\tau}_r, \bar{\mu}_r, \bar{e}_r \). Thus, with conjugacy considered, one has ten independent color and ten independent generation combinations available for the strong/hyperweak interaction gluons. As noted in the earlier discussion following (2.383), the gluons are responsible for mediating both color and generational decays, which is to say that the gluons are bi-colored and bi-generational. Thus, considering color and generation together, at hyperweak energies where \( SU(4) \times U(1) \) applies unbroken for both color and generation, there are a total of \( 10 \times 10 = 100 \) conjugally independent color/generation combinations of strong/hyperweak gluon. Considering four colors and four generations, following the type of approach used in Tables 2.11, 2.12, this may also be thought of as \( 100 = \frac{1}{3}(4)(4+1) \times \frac{1}{3}(4)(4+1) \) conjugally independent gluons and hyperweak bosons, as opposed to the \( 256 = (4 \times 4) \times (4 \times 4) \) such gluons and hyperweak bosons which were considered prior to reducing out the conjugate gluon states. With the three conjugally independent electroweak \( A^u, Z^u, W^u \) bosons and the 32 conjugally independent fermions, this results in a total of \( 135 = 100 + 32 + 3 \) conjugally independent real fermions and vector bosons (32 fermions, 103 bosons), down from the original total of \( 324 = 256 + 64 + 4 \) (64 fermions, 260 bosons) prior to the reduction of conjugate states. 

2.180
This is all illustrated most clearly by an examination of the strong interaction gluons. When we began at the outset in section 11 to discuss grand unified flavor symmetry absent color and generation, and once quantum gravitation was considered in section 12, it was noted that the A flavor preon was the exclusive flavor carrier of quark number, with $Q_u = 1$, see (2.308)(a). At the same time, we had but a single colorless gluon $G^u = \bar{\Lambda}A$, eq. (2.299)(b) mediation of strong which was to be associated with the colorless flavor interactions. In section 13 it was shown, to account for color, that the A preon, with $Q_u = r + g + b = 1$, in fact is best regarded so as to exist in three color varieties, $A_R, A_G, A_B$. By combining preons and antipreons in the usual manner, one of course arrives at $3 \otimes 3 = 8 \oplus 1$ distinct bi-colored gluons, with the usual eight color combinations $\bar{A}_R A_G, \bar{A}_G A_R, \bar{A}_R A_B, \bar{A}_B A_R, \bar{A}_G A_B, \bar{A}_B A_G, \frac{1}{2} (\bar{A}_R A_G - \bar{A}_G A_R), \frac{1}{2} (\bar{A}_R A_B - \bar{A}_B A_R), \frac{1}{3} (\bar{A}_R A_G + \bar{A}_B A_B)$, and the singlet gluon $\frac{1}{3} (\bar{A}_R A_G + \bar{A}_B A_B)$ generated during the spontaneous symmetry breakdown of strong/hyperweak high energy $SU(4)xU(1)$ color as a fourth color with lepton number, to low energy strong $SU(3)xU(1)$ color, with of color lepton number conserved independently, see., i.e. (2.352), (2.373)(a). Of these, $\frac{1}{2}(3)(3+1) = 6$ gluons are conjugally independent. Now, accounting further for generation, and confining ourselves at this juncture to the three observed $e, \mu, \gamma$ generations, one has a total of nine color/generation combinations for the A flavor of preon which, from (2.376) may be put into the matrix form:

\[
\begin{pmatrix}
A^e_R & A^\mu_R & A^\gamma_R \\
A^e_G & A^\mu_G & A^\gamma_G \\
A^e_B & A^\mu_B & A^\gamma_B
\end{pmatrix}
\]  

(2.384)

2.181
Now, accounting for the three low-energy generations, we find a total of $9 \times 9 = (3 \times 3) \times (3 \times 3) = (8 + 1) \times (8 + 1) = 81$ distinct color/generation combinations of gluons. Of these, $\frac{1}{2}(3)(3+1) \times \frac{1}{2}(3)(3+1) = 36$ are conjugally independent. That is, out of the 256 total strong/hyperweak gluons and bosons exactly $\frac{81}{4}$ of these particles are the gluons which mediate interactions involving the first three fermionic generations; while out of the total of 100 conjugally independent strong/hyperweak gluons and bosons of four generations, precisely 36 of these mediate strong interactions of the first three generations. This 36 is the product of, for instance, the six color combinations $\bar{G}G$, $\bar{G}B$, $B\bar{B}$, $\frac{1}{3}(\bar{G}G-\bar{B}B)$, $\frac{1}{3}(\bar{R}G+\bar{G}B)$, and the six generation combinations $\bar{e}u$, $\bar{e}u$, $\bar{e}e$, $\bar{e}e$, $\bar{e}e$, and neutral current combinations of $\bar{e}e$, $\bar{e}u$, $\bar{e}e$. The crucial point to be emphasized here is that the gluons, once generation is considered, now assume double duty, as the carriers of both the strong color and the generation interaction charges. Now, we know most certainly, insofar as color is concerned, that these gluons are massless, since the SU(3) x U(1) color symmetry of these gluons goes unbroken at low energies. However, insofar as generation is concerned, it appears that the gluons may in fact carry a non-zero rest mass, due to the spontaneous breakdown at lower energies of the generation gauge group SU(4) x U(1). In fact, one of the most significant advances of the past generation of physicists involves the use of massive vector gauge particles as the mediators of certain interactions. Thus for instance it is often said, the weak interaction is "weak" not because of its coupling, but because the (W$^\pm$) bosons which mediate this interaction are fairly massive, and cannot therefore be produced without supplying a certain amount.
of energy. The scale of energy required is of course set by the Fermi coupling $G_F$ which has an associated energy of $\sqrt{s} \approx 246$ GeV., though this magnitude is in some sense a mere detail peculiar to weak interactions. The important point is that this mass scale is not equal to zero, i.e., $\sqrt{s} \neq 0$, and this applies independently of the actual numeric magnitude of $G_F$ and its associated energy. If $\sqrt{s}$ were larger for example, this means simply that even more energy would be needed to produce weak interaction phenomena such as beta-decay, and so at low energies, that the amplitudes for observing these decays would be even further reduced. The $X^u$ hyperweak bosons, discussed previously, provide an ultra-high energy example of this approach. Particularly, the energy required to produce these bosons is probably greater than $\approx 10^{15}$ GeV., and perhaps as large as the gravitational $1.22 \times 10^{19}$ GeV. Hence at low energies, the interactions involving these bosons are essentially unobservable. This brings us back to the gluons. Because the gluons are now regarded as mediators of both color and generation interactions, and because certain generation transitions occur with higher amplitudes than do certain others, it seems plausible that the ratios of various amplitudes will depend directly upon the relative masses of the various gluons mediating intergenerational decay. For example, transitions of the form $e \leftrightarrow e$ occur with greater frequency than do $e \leftrightarrow \mu$ transitions; while the $e \leftrightarrow \gamma$ transition is even less frequent than the other two. This suggests that the gluons with $A^e A^e$ are least massive, that $A^\mu A^e$ are somewhat more massive, and that the $A^\mu A^e$ gluons are the most massive, as among the mediators of these three transitions. Thus, the reason that the transition $e \leftrightarrow \mu$ occurs far less frequently than $e \leftrightarrow e$ is tied
directly to the fact, during the spontaneous breakdown of higher energy generation symmetry, (irrespective of how this breakdown may in fact take place) that the $\Delta^{\mu}\Delta^\nu$ gluons somehow acquire a mass that is much larger than those involving $\Delta^\mu\Delta^\nu$. Of course, the amplitudes for various intergenerational transitions are in turn established by the Cabibbo mixing angles; hence it appears as though there should be a way to relate the different masses acquired by the various bi-generational gluons directly to the Cabibbo mixing angles. Consequently, as is the case with weak and hyperweak interactions, the amplitude ratios for the occurrence of various intergenerational transitions are set during spontaneous generational symmetry breakdown, by the massive vector bosons (in this case, Gluons) which are generated during the symmetry breakdown. As such, Cabibbo mixing and related generational questions may be qualitatively and quantitatively analyzed on the basis of well established techniques involving the use of massive gauge particles. We emphasize again that the gluons do not acquire any mass by virtue of color symmetry. Insofar as color is concerned, gluons are still massless as always. The mass which we propose that certain gluons will actually acquire, is due to the fact that the gluons are also involved in the generation symmetry. The quantitative details regarding masses of specific gluons must of course be determined by the manner in which the generation symmetry is broken, and this will not be discussed here. Nevertheless, the qualitative proposal, that gluons do in fact acquire varying masses due to generation symmetry, leads to a number of avenues by which the results developed here may in fact be subjected to experimental testing within the scope of present technology.

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One area where it may be possible to directly observe the bi-
gen-erational properties of gluons is in the observation of hadronic,
non-leptonic meson decay. It is helpful, for example, to examine
for any particular meson, all of the possible intermediate vector
bosons and combinations thereof, into which that meson might possibly
decay. Then, by inverting this analysis, one derives an inverse
classification of all of the mesons into which each of the various
intermediate vector bosons may further decay. By combining the
allowable meson $\rightarrow$ boson decays with the further allowable boson
$\rightarrow$ meson decays, one then arrives at an exhaustive listing of all
possible meson $\rightarrow$ meson decays that might be possible in nature.

More complicated decays can then be developed by the composite
utilization of this analysis. We begin this analysis by noting that
there are precisely three distinct flavors of strong/hyperweak vector
boson, suppressing color and generation. Similarly, there are precisely
three distinct flavors of electroweak boson. These bosons, and their
similarities, are summarized in Tables 2.8, 2.15. From $(2.380)$\textsuperscript{(a)-(d)}
and, for example, $(2.280)$, the conjugally independent strong/hyperweak
in preonic decomposition, intermediate vector boson flavors are the following: (Recall\textsuperscript{2.325})

\begin{align*}
g^u &= \AA \quad \text{(2.385)(a)} \\
x^0 \AA &= \frac{1}{2} (\bar{BB} - \AA) + \AA \sin^2 \Theta \, S \quad \text{(2.385)(b)} \\
x^{-1} \AA &= \bar{BA} \, , \quad \text{(2.385)(c)}
\end{align*}

while the similar decompositions for the electroweak bosons are the
following:

\begin{align*}
A^u &= \frac{1}{3} (\bar{BB} + \bar{CC}) - \frac{2}{3} \bar{DD} \quad \text{(2.386)(a)} \\
w^0 \AA &= \frac{1}{2} (\bar{CC} - \bar{DD}) - \left( \frac{1}{3} (\bar{BB} + \bar{CC}) - \frac{2}{3} \bar{DD} \right) \sin^2 \Theta \, W \quad \text{(2.386)(b)} \\
w^{+u} &= \bar{DC} \, . \quad \text{(2.386)(c)}
\end{align*}
Of course, for the strong/hyperweak gluons (2.385), there is also color and generation to be considered. For the electroweak bosons (2.386), there is of course no color or generation to contend with, i.e., these are all of the bosons. Noting that \( A^u \) and \( W^0u \) both contain a \( \overline{B}B \) term, this indicates that when \( \overline{B}B \) appears in either of \( A^u \) or \( W^0u \), that both \( B \)'s must be in the same generation. When the \( \overline{B}B \) term appears in \( X^{0u} \) we know however, from prior discussion, that any combination of generations is possible. Now let's talk about mesons. (hadronic)

We note first of all that each so-called meson is in fact a combination of a quark/antiquark, and further, that each quark and each antiquark is a combination of a preon/antipreon. Thus, what one conventionally refers to as a meson in fact contains a total of four preonic components, two preon, and two antipreon. Next we return, for example, to the vertex diagram (2.382)(a) for \( u \leftrightarrow s \) particle decay. We note that there are actually two distinct intermediate vector bosons that must be involved in this decay. One of these is either of the strong/hyperweak \( G^u_{\mu e}, X^{0u}_{\mu e} \) (although at low energies, production of the \( X^0 \) bosons will not take place). These bosons mediate the decay from the \( e \), to the \( \mu \) generation. The other boson involved is the electroweak \( W^+u \), which mediates the weak isospin beta decay from the upper member of a quark doublet, to the lower member. The \( G^u \) and \( X^{0u} \) can of course mediate strong color exchange as well, but for the current discussion, it pays to suppress the consideration of color exchange, and to focus on generation and isospin exchange. The above establishes a general approach to meson decay. In general, this tells us that meson decay will take place at a four particle vertex, similar to (2.382), rather than the three particle vertex
which is ordinarily anticipated. This also tells us that the decay vertex will involve two vector bosons, one strong/hyperweak, the other electroweak, rather than the single vector boson ordinarily anticipated. Of course, there are certain situations where these vertices may in fact be simplified. For instance, if (2.382)(a) had been used to describe the $u \leftrightarrow d$, rather than the $u \leftrightarrow s$ transition, then the gluon involved would be $G_{ee}^u$, and it would be possible to directly connect the worldlines of the two preons, as shown by the broken lines in (2.382). Thus, within a single generation, it is possible to reduce down to the usual three particle vertices. (Though four particle vertices are of course still permitted here, but are not mandatory.) The same applies any time it is possible to connect preon worldlines directly, hence the simplification of the four particle vertices should be considered in appropriate situations.

Now we consider specific mesons. To simplify discussion, and without loss of generality, it is possible to work with just the first two generations of quark, namely, the $u,d,c,s$ quarks, and their various $\bar{q}q$ mesons. At first sight, this gives us 16 distinct mesons to consider, namely, the twelve mixed mesons $\bar{u}c, \bar{c}u, \bar{s}d, \bar{d}s, \bar{u}d, \bar{d}u, \bar{u}s, \bar{s}u, \bar{d}c, \bar{c}d, \bar{s}c, \bar{c}s$, and the four neutral mesons $\bar{u}u, \bar{d}d, \bar{c}c, \bar{s}s$. As among the four neutral mesons, one anticipates some degree of mixing, which will ultimately be determined by the manner in which the generational symmetry is broken, and the neutral meson components are therefore mixed. Of course, only six of the twelve mixed mesons are conjugally independent, hence we need really only consider ten mesons, for example, the six mixed $\bar{u}c, \bar{s}d, \bar{d}u, \bar{u}s, \bar{d}c, \bar{c}s$, and the four neutral $\bar{u}u, \bar{d}d, \bar{c}c, \bar{s}s$. All of these mesons have of
course been studied rather extensively by experiment, and each
has a well settled labelling convention. For each of these mesons,
it is possible to obtain a preonic decomposition. Then, by rearrange-
ment of the preons into the constituents of the various vector bosons
in (2.385), (2.386), and by allowing for simplification of certain
vertices from four down to three particles, as discussed above, it
is possible to list for each distinct meson, all of the various inter-
mediate vector bosons or combinations thereof into which that meson
might possibly decay. For the six mixed mesons, this is given as such:

\[
D^0 = \bar{u}c = \bar{A}^e \bar{D}A^u \rightarrow \bar{A}^e A^u \bar{D}D = G^u_{e\mu} + A^u, G^u_{e\mu} + W^0 u, G^u_{e\mu},
\]
\[
K^0 = \bar{s}d = \bar{A}^\mu \bar{C} \bar{C}A^e \rightarrow \bar{A}^\mu C^e \bar{C}C = G^u_{\mu e} + A^u, G^u_{\mu e} + W^0 u, G^u_{\mu e},
\]
\[
\eta^+ = \bar{d}u = \bar{A}^e \bar{C} \bar{D}A^e \rightarrow \bar{A}^e A^u \bar{D}C = G^u_{e\mu} + W^0 u, G^u_{e\mu} + W^0 u, W^+ u
\]
\[
K^+ = \bar{s}u = \bar{A}^\mu \bar{C} \bar{D}A^u \rightarrow \bar{A}^\mu A^u \bar{D}C = G^u_{\mu e} + W^0 u, G^u_{\mu e} + W^0 u, W^+ u
\]
\[
D^+ = \bar{d}c = \bar{A}^e \bar{C} \bar{D}A^u \rightarrow \bar{A}^e A^u \bar{D}C = G^u_{e\mu} + W^0 u, G^u_{e\mu} + W^0 u, W^+ u
\]
\[
P^+ = \bar{s}c = \bar{A}^\mu \bar{C} \bar{D}A^u \rightarrow \bar{A}^\mu A^u \bar{D}C = G^u_{\mu e} + W^0 u, G^u_{\mu e} + W^0 u, W^+ u
\]

For the four neutral mesons, this becomes:

\[
\eta^0(u)\bar{u}u = \bar{A}^e \bar{D} \bar{D}A^e \rightarrow \bar{A}^e A^u \bar{D}D = G^u_{e\mu} + A^u, G^u_{e\mu} + W^0 u, G^u_{e\mu}, A^u, W^0 u,
\]
\[
\eta^0(d)\bar{d}d = \bar{A}^e \bar{C} \bar{C}A^e \rightarrow \bar{A}^e A^u \bar{C}C = G^u_{e\mu} + A^u, G^u_{e\mu} + W^0 u, G^u_{e\mu}, A^u, W^0 u,
\]
\[
\psi^0 = \bar{c}c = \bar{A}^\mu \bar{D} \bar{D}A^u \rightarrow \bar{A}^\mu A^u \bar{D}D = G^u_{\mu e} + A^u, G^u_{\mu e} + W^0 u, G^u_{\mu e}, A^u, W^0 u,
\]
\[
\phi^0 = \bar{s}s = \bar{A} \bar{C} \bar{C}A^e \rightarrow \bar{A} C^u \bar{C}C = G^u_{\mu e} + A^u, G^u_{\mu e} + W^0 u, G^u_{\mu e}, A^u, W^0 u,
\]

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Next, it is possible to invert the above, so as to list for each individual vector boson, all of the further combinations of vector boson and/or meson into which that boson might conceivably decay, again, within the first two quark generations. These are as follows:

\[ G_{\bar{u}u}^e, x_{\bar{u}u}^0 \rightarrow D^0, D^0 + W^0 + D^0, \]
\[ \rightarrow K^0, K^0 + W^0 + K^0, \]
\[ K^0 + W^0, D^0 + W^0, \]
\[ (2.388)(a) \]

\[ G_{\bar{u}u}^e, x_{\bar{u}u}^0 \rightarrow \pi^+ + \pi^-, \]
\[ \pi^0(u) + A^u, \pi^0(u) + W^0, \pi^0(u), \]
\[ \pi^0(d) + A^u, \pi^0(d) + W^0, \pi^0(d) \]
\[ (2.388)(b) \]

\[ G_{\bar{u}u}^e, x_{\bar{u}u}^0 \rightarrow F^+ + W^0, \]
\[ \psi^0 + A^u, \psi^0 + W^0, \psi^0, \]
\[ \phi^0 + A^u, \phi^0 + W^0, \phi^0 \]
\[ (2.388)(c) \]

\[ A^u, W^0 \rightarrow D^0 + G_{\bar{u}u}^e, D^0 + x_{\bar{u}u}^0, K^0 + G_{\bar{u}u}^e, K^0 + x_{\bar{u}u}^0, \]
\[ \pi^0(u) + G_{\bar{u}u}^e, \pi^0(u) + x_{\bar{u}u}^0, \pi^0(u), \]
\[ \pi^0(d) + G_{\bar{u}u}^e, \pi^0(d) + x_{\bar{u}u}^0, \pi^0(d), \]
\[ (2.388)(d) \]

\[ A^u, W^0 \rightarrow F^+ + G_{\bar{u}u}^e, F^+ + x_{\bar{u}u}^0, \phi^0 + G_{\bar{u}u}^e, \phi^0 + x_{\bar{u}u}^0, \phi^0 \]
\[ (2.388)(e) \]

It should also be noted, examining the Q and $\bar{Z}$ charges of $W^+u$ and $X^{+1/3}u$ in Table 2.14, that there is an additional decay mode for $A^u, W^0u$ into the charged $W^0$ and $X^{+1/3}u$ intermediate bosons, i.e.,

\[ A^u, W^0u \rightarrow W^+u + W^0, X^{+1/3}u + X^{-1/3}u \]
\[ (2.389) \]

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By then proceeding to combine the above in various combinations, it is possible to arrive at all of the permitted decay modes for those mesons involving the first two generations of quarks. This sort of approach is readily extended to encompass the third and fourth quark generations also, leading in total to a rather large number of hadronic decay possibilities. It is illustrative, for example, to examine some of the permissible decays for the D and K mesons. Particularly, we know that the $X^0_u$ hyperweak bosons can only be produced at high energies, hence we shall examine only those modes involving the strong bi-generational gluons $G^u$. Additionally, since the photon $A^u$ and electroweak neutral boson $W^0u$ are neutral current particles that can be reduced out of most decay processes of which they are a part, let us examine only those processes that do not involve $A^u$ and $Z^u$. In short, we wish to list all of the allowed decays for the D and K mesons which take place strictly by way of the strong gluons $G^u$ and the electroweak $W^u$. For $D^+$, combining (2.387)(e) with (2.388)(a)and (e), considering only the decays involving $G^u, W^u$, examining only those situations where a $W^u$ pair decays into a meson pair in which each meson is identical aside perhaps from charge, and avoiding the introduction of a second "layer" of vector bosons via the K and D decays in (2.388)(e), one arrives at: (notation simplified by removing some "+" signs)

$$
\begin{align*}
D^+ & \longrightarrow G^u + W^u \\
& \longrightarrow D_{W^u}^0, K_{W^u}^0, K_{W^u}^{-}W^u, D_{W^u}^{+}W^u \\
& \longrightarrow D_{\pi^+}^0, K_{\pi^+}^0, K_{\pi^+}^{-}\pi^+, D_{\pi^-\pi^+}^0, \\
& \quad D_{F^+}^0, K_{F^+}^0, K_{F^+}^{-}F^+, D_{F^-F^+}^0.
\end{align*}
$$

(2.390)(a)

Obviously, by being less restrictive, one can arrive at an almost

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endless variety of $D^+$ decays. Nevertheless, all such decays ultimately must be constructed out of some combination of (2.387)-(2.389). For $D^0$, using (2.387)(a) and (2.388)(a) and (e), and imposing a similar set of restrictions, one arrives at the primary decay modes:

$$D^0 \rightarrow G^u_{e \mu},$$

$$\rightarrow D^0, \overline{K}^0, K^-W^+u, D^+W^-u$$  \hspace{1cm} (2.390)(b)

$$\rightarrow D^0, \overline{K}^0, K^-\pi^+, D^+\pi^-, K^-F^+, D^+F^-.$$  \hspace{1cm} (2.390)(c)

For $K^+$, a similar approach yields: (use (2.387)(d) and (2.388)(a),(e))

$$K^+ \rightarrow G^u_{\mu e} + W^+u$$

$$\rightarrow \overline{D^0}W^+u, K^0W^+u, K^+W^-uW^+u, D^-W^+uW^+u$$

$$\rightarrow \overline{D^0}\pi^+, K^0\pi^+, K^+\pi^-\pi^+, D^-\pi^+\pi^+, \overline{D^0}F^+, K^0F^+, K^+F^-F^+, D^-F^+F^+,$$  \hspace{1cm} (2.390)(c)

while for $K^0$, using (2.387)(b) and (2.388)(a),(e), one gets:

$$K^0 \rightarrow G^u_{\mu e}$$

$$\rightarrow \overline{D^0}, \overline{K}^0, K^-W^-u, D^-W^+u$$  \hspace{1cm} (2.390)(d)

$$\rightarrow \overline{D^0}, \overline{K}^0, K^-\pi^-, D^-\pi^+, K^+F^-, D^-F^+.$$

Thus it is apparent how simple it is, using a preonic decomposition such as that of (2.387),(2.388), to arrive at a complete classification of all the permitted decay modes for the various mesons. Obviously, the observation of those decay modes, and only those decay modes which can be composed from (2.387),(2.388), is one way in which the theory presented here can be qualitatively confirmed by experiment. Much of the above already has experimental support.\textsuperscript{2, 28} Another very interesting possibility relates to observation of the massive bi-generational gluons, as discussed above. In particular, all of the above processes in which a change in generation takes place must be mediated by a
bi-generational gluon. By the arguments presented earlier, these gluons should have a certain non-zero mass which is brought about, not by color, but by the symmetry breaking of the generation group. The probability that any particular intergenerational decay will in fact occur, as discussed earlier, should be very closely related to the mass of the particular gluon mediating that particular decay. The more massive the gluon, the less likely is the associated decay. Given this, one notes that in all of the decays (2.390), and in similar sorts of decays for other mesons, that these gluons are among the intermediate products. For example, in the decay of $K^{+}\rightarrow\pi^{+}u$, (2.390)(c), (see also the related (2.382)(a)) it is expected that $G_{\mu e}^{u}$ and $W^{+u}$ will exist for at least some period of time during the overall process. Assuming that the $G_{\mu e}^{u}$ has a long enough lifetime, it seems as though this particle should be observable along with $W^{+u}$, using techniques similar to those which were in fact used to observe the $W^{+u}$; and that this could provide some confirmation that the gluons are indeed bi-generational, and massive. A similar approach might also be attempted for the gluons that mediate interactions with the $\gamma$ generation, which one would expect to be even more massive, based upon what is currently known about Cabibbo mixing with the third generation. While all of this may seem somewhat involved, it is worth pointing out that all of the above revolves around one basic, and very simple conservation principle. In the same way that it is necessary to conserve flavor and color at any given particle vertex, the real hypothesis here is that it is also necessary to conserve generation symmetry. The problem, is that there is no readily agreed upon mechanism
known to date, which allows one to effectively and simply enforce this principle of generation conservation. For flavor and color, there of course exist intermediate vector bosons which carry just the right amount of flavor and color, so as to ensure conservation of these symmetries across any given vertex. For generation however, the analysis does not yet appear to have advanced to the stage where there is a definitively identifiable boson, or set of bosons, which carry just the necessary generation charge to ensure generation conservation, in addition to flavor and color conservation, at any given vertex. The hypothesis here is that it is in fact unnecessary to introduce any new flavors of boson in order to conserve the generational symmetry. Instead, one works with the existing flavors, particularly the strong interaction gluons and hyperweak bosons of grand unification. By making these bosons bi-generational, as well as bi-colored, (four colors at high energies) one can enforce the generational symmetry using the gluons and hyperweak bosons which are already at hand in existing theory. Of course, by allocating to the gluons the additional function of generation conservation, one changes the character of the gluons from what is normally anticipated. Now, it appears as though any of the gluons which mediate generation mixing would have to acquire some mass, due not to their color, but to their dual role as carriers of the generation charge. This in turn, is a hypothesis which might perhaps be experimentally confirmed, during the detailed analysis of the intermediate Bose products, of intergenerational meson decay.