

2.14 - On the Replication of Fermion Generations: Four Generational Grand Unification with Eighteen Preons and Nine Independent Flavor/Color/Generation Degrees of Freedom, and a Preonic Discussion of Mesons and Meson Decay

The existence of multiple fermionic generations has been a puzzle ever since Pauli first inquired about the muon, 'who ordered this?' At the time, he was referring to the then recent discovery of a particle identical with the electron in all respects, except for the distinguishing fact that this particle was about two-hundred times as heavy as the electron. Today, we know that this type of replication takes place not only for leptons, but also for quarks. We know too, beyond the muon "family" or "generation,"<sup>that</sup> there exists yet a third tau generation; and further, although no additional such generations have to date been observed,<sup>that</sup> there exists no known theoretical principle to preclude the existence of additional generations beyond the known three. Hence, it is common practice to hold open the possibility that one or more additional generations beyond the observed electron, muon and tauon generations might actually also exist in nature, and to suppose that the contemporary absence of experimental evidence for any additional generations, if they do exist, is most probably due to limitations on the energy scales at which we are able, with present technology, to perform experiments.

Harari has noted of the generation puzzle, that 'the presently accepted pattern of generations has two independent (his emphasis) striking features:

1. Within each generation, the pattern of quarks is very similar to the pattern of leptons.

2. Each generation is similar to the other generations. <sup>-2.21</sup>

This first point refers apparently to the fact that the number of quark generations, whatever that number may be, seems to be equal to the number of lepton generations (this is also mandated theoretically, to avoid fermion loop anomalies<sup>-2.22</sup>); and also, to the fact that within a single generation, the number of quark flavors, which is two, up and down, is equal to the number of lepton flavors, electron and neutrino. This similarity is also manifest in the fact, for left-handed chiral projections, that both the quark pair and the lepton pair form weak isospin doublets, which is to say that the two quark flavors within a single generation decay into one another through exactly the same weak interaction process as the two lepton flavors within that generation decay into one another, namely, by the emission of an electroweak  $W^{+u}$  or  $W^{-u}$  vector Boson. This is of course the particular area of similarity which led us initially to the discussion of preonic flavor grand unification in Section 10. The second point refers apparently to the fact, aside from the very important difference in particle masses, that all other features of the various fermions, in particular their flavor and color quantum numbers, are the same from one generation to the next. Thus for example, all of the electron, muon and tauon have identical electrostatic charge  $Q=-1$ , while all of their associated neutrinos have  $Q=0$ . Similarly for the u,c,t quarks with  $Q=+2/3$  and the d,s,b quarks with  $Q=-1/3$ . Naturally, all of the quarks come in three colors; while at high energies, where lepton number becomes a fourth color, the leptons within each generation will color transform as

a fourth quark color, within ~~the~~ same generation.

One of the more successful theoretical approaches to the replication of fermion generations, known as "horizontal symmetry," was first developed by Pati and Salam.<sup>-2.23</sup> In this approach, the four colors of fermion (three quark colors, one lepton color) are said to follow a "vertical" symmetry, while the replication of fermion generations is realized by a horizontal duplication of the as often as is necessary to account for all generations. vertical color symmetry. For the sake of discussion, we shall say therefore that these two symmetries, color and generation, are "orthogonal" symmetries. Thus for example, it is now common practice to form the color x (flavor/generation) matrix: (Distinguish L = lepton and L = left by context)

$$\begin{pmatrix} \nu_L^e & e_L & \nu_L^u & \mu_L & \nu_L^\tau & \tau_L & \dots \\ u_R & d_R & c_R & s_R & t_R & b_R & \dots \\ u_G & d_G & c_G & s_G & t_G & b_G & \dots \\ u_B & d_B & c_B & s_B & t_B & b_B & \dots \end{pmatrix}_{L,R} \quad (2.374)$$

in order to depict the basic flavor/color/generation and chiral combinations of fermion.<sup>-2.24</sup> In the above, the vector formed out of the quarks (u,d,c,s,t,b) is thought to form an SU(6) flavor group, but it is just as well known that this is only an approximate, not an exact symmetry. Put into other terms, while we can utilize SU(6) flavor to enumerate particle states, this means that we cannot effectively utilize this group to precisely predict fermion masses. An important limitation in the above is uncovered by observing that the horizontal rows actually involve a mixture of flavor weak isospin and generation symmetries. That is, on the one hand, the decays  $u \leftrightarrow d$ ,  $c \leftrightarrow s$ ,  $t \leftrightarrow b$  and  $e \leftrightarrow \nu_e$ ,  $\mu \leftrightarrow \nu_\mu$ ,  $\tau \leftrightarrow \nu_\tau$  are simply ordinary weak <sup>beta-</sup> decays, and need not have anything to do with the

intergenerational symmetry. On the other hand, the decays  $\Lambda^u \leftrightarrow c \leftrightarrow t$ ,  $d \leftrightarrow s \leftrightarrow b$  and  $e \leftrightarrow \mu \leftrightarrow \tau$ ,  $\nu_e \leftrightarrow \nu_\mu \leftrightarrow \nu_\tau$  directly involve the generation symmetry, though they need not have anything to do with weak beta decay. Of course the composite forms of decay, for example, the decay  $u \leftrightarrow s$ , involve both weak beta decay, which is part of flavor symmetry, and generation decay. It is this composite form of decay which is observed experimentally as Cabibbo-type mixing. However, from a theoretical standpoint, one can obtain an important simplification by clearly separating <sup>out</sup> the treatment of the weak (flavor) beta decay from <sup>that</sup> of intergenerational decay. When considered on a composite basis, <sup>these two forms of decay</sup>  $\Lambda$  form the basis for properly describing observed Cabibbo mixing. In short, while Cabibbo mixing involves both flavor (weak beta-decay <sup>in particular</sup>) and generation decay, it is nevertheless advantageous to distinguish generation from flavor as a completely independent symmetry, with its own set of independent generation quantum numbers. Consequently, in the same manner that we earlier used the flavor preon model in Section 10 to "factor out" the very similar forms of weak beta-decay as between the quarks and leptons in a given generation, we should wish to similarly "factor out" the replication of weak beta decay as it takes place within all of the different generations of (2.374). This line of reasoning is further developed by considering the vertical columns in (2.374). Here, we see lepton number regarded as a fourth fermion color. As we know from the prior discussion however, the <sup>hyperweak</sup>  $\Lambda$  decay of a quark into a(n anti) lepton involves the emission of an exotic  $X^{+1/3u}$  or  $X^{-1/3u}$  intermediate vector boson and, due to the same overlaps which reduced from eight to six the number of flavor/color preons and linearly in-

<sup>diagonalized</sup>  
 dependent generators, this may be thought of alternatively as the decay between an A and a B flavor of preon, or <sup>as</sup> the decay between and R,G or B, and an L color of preon. (see ie., (2.279)). It is this alternative in description which is most readily depicted by forming the preons into the flavor/color six-vector which includes  $(A_R, A_G, A_B, B_L, C, D)$ . The point here is that hyperweak decay, which may be regarded as a form of flavor decay, and which can also be thought of as ordinary beta decay that takes place simply through a different mode of the same channel process (contrast (2.278), (2.279)), is depicted in (2.374) as a vertical symmetry; while ordinary weak beta decay, which is also a form of flavor decay (and in fact is the only other form of flavor decay <sup>besides hyperweak</sup> which produces a change in flavor quantum numbers), is depicted in (2.374) as a horizontal symmetry. This would lead us to believe that some aspects of flavor are horizontal, while other aspects are vertical; in short, that flavor symmetry is orthogonal to itself. This of course, is not so, as the original flavor preon quadruplet  $(A, B, C, D)$  places both the hyperweak and ordinary forms of beta decay into a linear vector which obeys simple flavor  $SU(4) \times U(1)$  at high energies. Again for discussion, this is to say that hyperweak and ordinary beta flavor decays are not orthogonal <sup>(except in the sense of the (2.278), (2.279) vertices)</sup> as (2.374) might lead us to believe. Rather, we should say that these flavor decays are "co-linear," which means that it is not necessary to make hyperweak decay vertical, and ordinary beta decay horizontal, in order to obtain a proper description of the two. Consequently, we shall wish to reformulate the matrix (2.374) in such a manner that both the hyperweak and ordinary modes of beta decay can be represented in a co-linear fashion.

It is also worth noting that flavor and color as a combined symmetry are also colinear, insofar as the hyperweak preon transition  $A \leftrightarrow B$  may at once be regarded as both a flavor and a color transition.

If one follows the approach suggested above, and regards generation symmetry as a symmetry that is completely distinct from flavor and color symmetries, and therefore as a symmetry that carries its own distinct set of <sup>conserved</sup> quantum numbers separately from both of flavor and color, then two preliminary questions immediately arise. First, which particles, both complex and real, carry the generational charges, and; second, what is the (high energy) gauge group of the generation symmetry? In short, what are the particle eigenstate solutions of generation symmetry, and what are the generation charge eigenvalues for these various eigenstate solutions? One begins to answer this question by noting, as Harari points out, that both the quarks and the leptons, ie., that all fermions, follow a similar pattern of generations. This would tend to suggest that it is fermion number  $F$  itself, which is the key to identifying which particles carry a generation charge, just as quark number  $Q_u$  is the key to identify<sup>ing</sup> which particles carry a (three) color charge. If we label the generation charges with the names of the generations themselves, ie., by  $e, \mu, \tau, \dots$ , and if we also assume for the moment, to be discussed shortly, that there also exists a fourth generation which will be designated as the gamma,  $\gamma$ , generation, then it is possible to expand upon equation (1.1), which has already been utilized to connect flavor and color, so as to write:

$$F = Q_u + L = R + G + B + L = e + \mu + \tau + \gamma . \quad (2.375)$$

Thus for example, the ordinary electron, which has  $F=L=1$  and  $Q_u=0$ ,

also has  $e=1$  and  $\mu=\tau=\gamma=0$ . So too for the electron's neutrino. The c quark on the other hand, as another example, is part of the second (muon) generation, and it has  $F=Q_u=\mu=1$  and  $L=e=\tau=\gamma=0$ , as does the s quark. Any further distinction to be made, for example, as between <sup>two particles in the</sup> the  $(c,s)$  isospin doublet, may be achieved by the further specification of the proper flavor quantum numbers, ie.,  $Q=+2/3$  for c and  $Q=-1/3$  for d, etc. Hence, to fully specify the characteristics of any given particle, it is necessary to examine that particle's quantum numbers, ie., internal degrees of freedom, for all three of the flavor, color and generation symmetry groups. Additionally, insofar as the flavor generators  $G^0, I^3, Y^8, B^{15}$  are concerned, it is also necessary to be concerned with whether it is a left or right handed chiral projection (see Table 2.7) under consideration. <sup>Generation symmetry, by itself, is most readily</sup> Generation symmetry, by itself, is most readily presumed to be chiral symmetric, as is color symmetry, <sup>to be seen below.</sup>

Given that it is Fermion number  $F$  which is apparently associated with the generation charge, it is possible to answer the first question raised in the prior paragraph, namely, which particles carry the generation charge? The answer: those particles which carry fermion number, or which are composed out of particles which carry fermion number. If we again note that the A preon is the carrier of quark number,  $F=Q_u=1$ , that the B preon is the carrier of (anti)lepton number  $F=L_e=-1$ , and that the C and D preons (electroweak isospin up and down) both carry  $F=Q_u=L=0$ , then this answer can be represented in a more concrete way. In particular, this now indicates that it is the A and  $\bar{B}$  preons, each with  $F=1$ , that are to be regarded as the complex preonic carriers of the generation charge. The C and D preons on the other hand, each with  $F=0$ , are completely uninvolved in carrying generation charges. The

set of real particles which carries the generation charge therefore, is simply that set of particles which is composed out of one or more A and/or B preons and/or antipreons. The C and D preons, <sup>which mediate low energy weak  $\beta$ -decay,</sup> are again irrelevant to generation symmetry. This is how one "factors out"  $\beta$ -decay.

As to the second question, what is the gauge group of generation symmetry, we shall hazard an educated guess that the proper gauge group, as in flavor and color symmetry, is  $SU(4) \times U(1)$  at high energies; which is to say that there are a total of four distinct generations, three known, one hypothesized. The reason for supposing that a fourth generation exists, at this point in time, is largely aesthetic. In particular, because the C and D preons are not involved in generation symmetry (note that these are also not involved in color, and are in fact only involved in electroweak isospin flavor weak beta decay) it is possible to represent the vertical symmetry of (2.374), in terms of a preonic decomposition, by the four-color vector containing  $(A_R, A_G, A_B, B_L)$ . The replication of fermion generations is then achieved by the fourfold <sub>horizontal</sub> duplication of the vertical color symmetry, to form the  $4 \times 4$  preonic square matrix:

$$\begin{pmatrix} B_L^e & B_L^u & B_L^d & B_L^s \\ A_R^e & A_R^u & A_R^d & A_R^s \\ A_G^e & A_G^u & A_G^d & A_G^s \\ A_B^e & A_B^u & A_B^d & A_B^s \end{pmatrix} \quad (2.376)$$

It is the desire to make this a square matrix which is the particular aesthetic ground upon which a fourth generation is introduced. At relatively low energies, where hyperweak  $A \leftrightarrow B$  transitions are highly suppressed, one has only three colors, hence the appropriate choice would be a  $3 \times 3$  subset of this matrix involving only the colored



A preons, and the  $e, \mu, \tau$  generations, consistent with low energy experimental observations. While we have argued for a square matrix on aesthetic grounds, which happens also to work out consistently with low energy observations of exactly three generations, it would probably be fruitful to determine if there is any theoretical basis for actually mandating that a square matrix be utilized. One would suppose that there is, and if so, this would provide an additional basis for concern about (2.374), which is currently  $4 \times 6$ , and which would become  $4 \times 8$  with a fourth generation. <sup>In this context, it would therefore be the discovery of the third generation in particular</sup> The real problem with <sup>which would be used to call (2.374) into question</sup> (2.374) <sup>however,</sup> is the repeated duplication of the isospin degree of freedom, <sup>weak</sup> co-linearly with generation freedom. In fact, since <sup>weak</sup> isospin decays involve the preonic transformation  $C \leftrightarrow D$ , and since C and D each carry  $F=0$  and are hence irrelevant to generation decay, it is preferable to factor these preons out of consideration entirely, and to focus merely on the generational properties of the A and B preons. This sort of reduction cannot not in any way be achieved by examining strictly real particles. Rather, it is vital in this situation to directly consider the preonic decomposition of the generation symmetry, as in (2.376), therefore eliminating from consideration the superfluous, and in this context confusing isospin degree of freedom. Ironically, or perhaps not so, this all brings us back to our initial considerations in Section 10, on the fundamental similarities between quarks and leptons within a single generation, as per the earlier quoted remark by Harari, and between the quarkonic and leptonic forms of beta decay via the electroweak  $W^{\pm u}$  vector bosons, which are themselves composed strictly out of C and D preons, see (2.297)(b); and <sup>to</sup> the even earlier discussions leading to (2.234)(a) and (b).

It has earlier been noted, from a preonic point of view, that admixtures of the A and B flavors of preon between themselves, and of the C and D flavors of preon between themselves, are res-  
possible for the vector Boson flavors of  $\wedge$  <sup>real</sup> particle; while a composition which involves either of the A or B flavors, in combination with either of the C or D flavors, results in the composition, or more precisely, the "flavor polarizations" for the spin half real Fermions. See for example, eqs. (2.280) for flavor only, and (2.318) for flavor and color within a single generation. . Now, with the inclusion also of at least three, and perhaps four fermionic generations, the color x generation matrix (meaning that color and generation are orthogonal, ie., not colinear symmetries) (2.376) indicates, wherever we previously considered only the preon color vector  $(B_L, A_R, A_G, A_B)$ , that we must now consider the replication of this quadruplet over three or four distinct generations, ie., we must now consider  $(B_L, A_R, A_G, A_B)_{e, \mu, \tau, \chi}$ . In this sense, generation symmetry is much more closely related to the strong color interaction, than it is to the flavor interactions. As flavor and color can be made colinear in the sextuplet  $(A_R, A_G, A_B, B_L, C, D)$ , the overall symmetry is in fact flavor/color x generation. Whereas we started with two preons,  $(C, D)$  in electroweak theory, went to four preons  $\wedge$  <sup>(A, B, C, D)</sup> in electroweak, strong and quantum gravitational flavor theory, and then to six preons  $\wedge$  <sup>(A\_R, A\_G, A\_B, B\_L, C, D)</sup> in flavor/color theory, it is now necessary, with generation symmetry as well, to start with (for four generations) a total of eighteen preons; that is, the sixteen in (2.376), and the electroweak (C, D) pair. Logically, this now yields  $18 \times 18 = 324$  real particles, considering

all conjugacy states. It is also helpful to think of this as  $(16 + 2) \times (16 + 2) = 256 + 64 + 4 = 324$  particles, given the manner in which fermion and boson flavors and colors are composed out of preons, again, eqs. (2.280), (2.318). The 256 corresponds to the  $16 \times 16$  <sub>(meson)</sub> combinations of A and B preon that can be formed out of the sixteen distinct flavor/color/generation preons in (2.376). These are all vector bosons associated with the strong-four-color interaction, and are either gluons,  $G^u$ , charged hyperweak bosons,  $X^{+1/3u}$  and  $X^{-1/3u}$ , or neutral hyperweak bosons,  $X_0^u$ . This will be discussed in more detail shortly. The 4 corresponds to the  $2 \times 2$  <sub>(meson)</sub> combinations of C and D preon which are possible, and is comprised of the usual four bosons of electroweak theory, ie., the photon  $A^u$ , charged weak bosons  $W^{+u}, W^{-u}$ , and the electroweak neutral bosons  $W_0^u (=Z^u)$ . Thus, prior to conjugacy reductions, one starts with  $256 + 4 = 260$  real flavor/color/generation combinations of real vector boson. The remaining  $64 = 32 + 32$  refers to the fermions, which involve, as in some "menus," one preon from A or B, and one from C or D. Here, there are 32 real fermions and 32 antifermions, with the 32 real fermions comprised of the eight flavor/color combinations  $u_R, u_G, u_B, d_R, d_G, d_B, e_L, \nu_L$ , duplicated over four generations. It is helpful to begin by considering the 32 conjugally independent fermions alone, and later, to bring in the various bosons. As (2.376) was developed with the explicit purpose of explaining the replication of real fermions in terms of a preonic decomposition, the examination of the real fermions may double as a check on the overall theoretical consistency.

The flavor/color/generation decomposition of the real fermions

is rather graphically depicted by examining the composition of the color x generation matrix (2.376), with similar 4x4 matrices involving the C and D preons in an appropriate manner. Continuing with the flavor/color fermion compositions which have applied all along, as in (2.280), (2.318), but supplementing this with multiple generations, one may depict the preonic decomposition of the 32 real fermions in the form:

$$\begin{pmatrix} \nu_L^e & \nu_L^\mu & \nu_L^\tau & \nu_L^\gamma \\ d_R^e & d_R^\mu & d_R^\tau & d_R^\gamma \\ d_G^e & d_G^\mu & d_G^\tau & d_G^\gamma \\ d_B^e & d_B^\mu & d_B^\tau & d_B^\gamma \end{pmatrix}_{L \neq R} \equiv \begin{pmatrix} \bar{B}_L^e & \bar{B}_L^\mu & \bar{B}_L^\tau & \bar{B}_L^\gamma \\ A_R^e & A_R^\mu & A_R^\tau & A_R^\gamma \\ A_G^e & A_G^\mu & A_G^\tau & A_G^\gamma \\ A_B^e & A_B^\mu & A_B^\tau & A_B^\gamma \end{pmatrix}_{L=R} \oplus \begin{pmatrix} C & C & C & C \\ \bar{C} & \bar{C} & \bar{C} & \bar{C} \\ \bar{C} & \bar{C} & \bar{C} & \bar{C} \\ \bar{C} & \bar{C} & \bar{C} & \bar{C} \end{pmatrix}_{L \neq R} \quad (2.377)(a)$$

$$\begin{pmatrix} e_L^e & e_L^\mu & e_L^\tau & e_L^\gamma \\ u_R^e & u_R^\mu & u_R^\tau & u_R^\gamma \\ u_G^e & u_G^\mu & u_G^\tau & u_G^\gamma \\ u_B^e & u_B^\mu & u_B^\tau & u_B^\gamma \end{pmatrix}_{L \neq R} \equiv \begin{pmatrix} \bar{B}_L^e & \bar{B}_L^\mu & \bar{B}_L^\tau & \bar{B}_L^\gamma \\ A_R^e & A_R^\mu & A_R^\tau & A_R^\gamma \\ A_G^e & A_G^\mu & A_G^\tau & A_G^\gamma \\ A_B^e & A_B^\mu & A_B^\tau & A_B^\gamma \end{pmatrix}_{L=R} \oplus \begin{pmatrix} D & D & D & D \\ \bar{D} & \bar{D} & \bar{D} & \bar{D} \\ \bar{D} & \bar{D} & \bar{D} & \bar{D} \\ \bar{D} & \bar{D} & \bar{D} & \bar{D} \end{pmatrix}_{L \neq R} . \quad (2.377)(b)$$

In terms of the usual notation, and introducing the  $x, y, \nu, \nu^c$  fermions of the  $\gamma$  generation, the sixteen distinct flavor/generation combinations of fermion (suppressing color) in the above are better known by the symbols:

$$(u, c, t, x) \equiv (u^e, u^\mu, u^\tau, u^\gamma) \quad (2.378)(a)$$

$$(d, s, b, y) \equiv (d^e, d^\mu, d^\tau, d^\gamma) \quad (2.378)(b)$$

$$(\nu^c, \nu^{\mu c}, \nu^{\tau c}, \nu^{\gamma c}) \equiv (\nu^e, \nu^\mu, \nu^\tau, \nu^\gamma) \quad (2.378)(c)$$

$$(e, \mu, \tau, \gamma) \equiv (e^e, e^\mu, e^\tau, e^\gamma) . \quad (2.378)(d)$$

The L=R and L $\neq$ R is used to designate chirality. Specifically, because the A and B preons are chiral symmetric (L=R) for all interactions, while the C and D are not (L $\neq$ R), see Table 2.7<sup>2.9</sup>, the consequence here

is that the real fermions, which all involve a combination of A or B and C or D, are chiral non-symmetric, <sup>(L≠R)</sup>, (with respect to  $G^0, I^3, Y^8, B^{15}$ , see Tables <sup>2.7 and</sup> 2.9) due in particular to the <sup>chiral</sup> non-symmetry of <sup>the electroweak</sup> C and D, as is to be expected. Consequently, the 32 distinct real flavor/color/generation fermions have a total of  $2 \times 32 = 64$  distinct chiral projections.

At this point, we can begin to examine Cabibbo-type mixing for both quarks and leptons. To do this, it is helpful first to recall the six neutral current generators Q, Z,  $(1/3)Q_u$ , X,  $\lambda^3$  and  $\lambda^8$ , which may be consolidated from (2.345), (2.368) and (2.352), as such:

$$Q = Y^8 + I^3 \quad (2.379)(a)$$

$$Z = I^3 - Q \sin^2 \theta_W \quad (2.379)(b)$$

$$\frac{1}{3}Q_u = \lambda^0 - \lambda^{15} \quad (2.379)(c)$$

$$X = \lambda^{15} + \frac{1}{3}Q_u \sin^2 \theta_S \quad (2.379)(d)$$

$$" \lambda^3 = \lambda^3 " \quad (2.379)(e)$$

$$" \lambda^8 = \lambda^8 " \quad (2.379)(f)$$

recall that  $\lambda^3$  and  $\lambda^8$  do not mix. Once these six generators are specified, it is possible to deduce therefrom all of  $G^0, I^3, Y^8, B^{15}, \lambda^0, \lambda^3, \lambda^8, \lambda^{15}$  and  $Q_u, L, F$ , as noted earlier in the discussion following (2.328). The associated preon decomposition for the above, from

(2.351), (2.373) and (2.352), is given by: ( $Z^u$  for left-handed projections)

$$A^u = \frac{1}{3}(\bar{B}B + \bar{C}C) - \frac{2}{3}\bar{D}D \quad (2.380)(a)$$

$$Z^u = \frac{1}{2}(\bar{C}C - \bar{D}D) - \left( \frac{1}{3}(\bar{B}B + \bar{C}C) - \frac{2}{3}\bar{D}D \right) \sin^2 \theta_W \quad (2.380)(b)$$

$$G_0^u = \frac{1}{3}(\bar{A}_R A_R + \bar{A}_G A_G + \bar{A}_B A_B) \quad (2.380)(c)$$

$$X_0^u = \frac{1}{4}\bar{B}_L B_L - \frac{1}{12}(\bar{A}_R A_R + \bar{A}_G A_G + \bar{A}_B A_B) + \frac{1}{3}(\bar{A}_R A_R + \bar{A}_G A_G + \bar{A}_B A_B) \sin^2 \theta_S \quad (2.380)(d)$$

$$G^3 u = \frac{1}{2}(\bar{A}_G A_G - \bar{A}_B A_B) \quad (2.380)(e)$$

$$G^8 u = \frac{1}{3}\bar{A}_R A_R - \frac{1}{6}(\bar{A}_G A_G + \bar{A}_B A_B) \quad (2.380)(f)$$

Since  $I^3 = 0$ ,

For right-handed  $Z^u$ , the  $\frac{1}{2}(\overline{CC-DD})$  term may be eliminated. All other neutral currents are left-right chiral symmetric. Given the above, it helps now to recast Table 2.7 into a more developed form including flavor and color for a single generation. Using (2.379) and Tables 2.7, 2.13, this is as follows: (Showing only conjugally independent states)

	Q	Z <sub>L</sub>	Z <sub>R</sub>	$\frac{1}{3}Q_u$	X	$\lambda^3$	$\lambda^8$
A <sub>R</sub>	0	0	0	1/3	$-1/12 + (1/3)\sin^2\theta_S$	0	1/3
A <sub>G</sub>	0	0	0	1/3	$-1/12 + (1/3)\sin^2\theta_S$	$\frac{1}{2}$	-1/6
A <sub>B</sub>	0	0	0	1/3	$-1/12 + (1/3)\sin^2\theta_S$	$-\frac{1}{2}$	-1/6
B <sub>L</sub>	1/3	$-(1/3)\sin^2\theta_W$	$-(1/3)\sin^2\theta_W$	0	$\frac{1}{4}$	0	0
C <sub>L</sub>	1/3	$-\frac{1}{2} - (1/3)\sin^2\theta_W$	$-(1/3)\sin^2\theta_W$	0	0	0	0
D	-2/3	$-\frac{1}{2} + (2/3)\sin^2\theta_W$	$(2/3)\sin^2\theta_W$	0	0	0	0
G <sub>U</sub>	0	0	0	0	0	$-\frac{1}{2}$	$\frac{1}{2}$
G <sub>GR</sub>	0	0	0	0	0	$-\frac{1}{2}$	$-\frac{1}{2}$
G <sub>RB</sub>	0	0	0	0	0	1	0
G <sub>BG</sub>	0	0	0	0	0	0	0
X <sub>RL</sub> <sup>u</sup>	-1/3	$(1/3)\sin^2\theta_W$	$(1/3)\sin^2\theta_W$	1/3	$-(1/3)\cos^2\theta_S$	0	1/3
X <sub>GL</sub> <sup>u</sup>	-1/3	$(1/3)\sin^2\theta_W$	$(1/3)\sin^2\theta_W$	1/3	$-(1/3)\cos^2\theta_S$	$\frac{1}{2}$	-1/6
X <sub>BL</sub> <sup>u</sup>	-1/3	$(1/3)\sin^2\theta_W$	$(1/3)\sin^2\theta_W$	1/3	$-(1/3)\cos^2\theta_S$	$-\frac{1}{2}$	-1/6
$\nu_L$	0	$-\frac{1}{2} + \frac{1}{2}\sin^2\theta_W$	0	0	$-\frac{1}{4}$	0	0
e <sub>L</sub>	-1	$-\frac{1}{2} + \sin^2\theta_W$	$\sin^2\theta_W$	0	$-\frac{1}{4}$	0	0
u <sub>R</sub>	2/3	$\frac{1}{3} - (2/3)\sin^2\theta_W$	$-(2/3)\sin^2\theta_W$	1/3	$-1/12 + (1/3)\sin^2\theta_S$	0	1/3
u <sub>G</sub>	2/3	$\frac{1}{3} - (2/3)\sin^2\theta_W$	$-(2/3)\sin^2\theta_W$	1/3	$-1/12 + (1/3)\sin^2\theta_S$	$\frac{1}{2}$	-1/6
u <sub>B</sub>	2/3	$\frac{1}{3} - (2/3)\sin^2\theta_W$	$-(2/3)\sin^2\theta_W$	1/3	$-1/12 + (1/3)\sin^2\theta_S$	$-\frac{1}{2}$	-1/6
d <sub>R</sub>	-1/3	$-\frac{1}{3} + (1/3)\sin^2\theta_W$	$(1/3)\sin^2\theta_W$	1/3	$-1/12 + (1/3)\sin^2\theta_S$	0	1/3
d <sub>G</sub>	-1/3	$-\frac{1}{3} + (1/3)\sin^2\theta_W$	$(1/3)\sin^2\theta_W$	1/3	$-1/12 + (1/3)\sin^2\theta_S$	$\frac{1}{2}$	-1/6
d <sub>B</sub>	-1/3	$-\frac{1}{3} + (1/3)\sin^2\theta_W$	$(1/3)\sin^2\theta_W$	1/3	$-1/12 + (1/3)\sin^2\theta_S$	$-\frac{1}{2}$	-1/6
W <sup>+</sup>	1	$\cos^2\theta_W$	$-\sin^2\theta_W$	0	0	0	0
A <sub>U</sub>	0	0	0	0	0	0	0
Z <sub>U</sub>	0	0	0	0	0	0	0
G <sub>U</sub>	0	0	0	0	0	0	0
G <sub>U</sub>	0	0	0	0	0	0	0
G <sub>U</sub>	0	0	0	0	0	0	0
G <sub>U</sub>	0	0	0	0	0	0	0
G <sub>U</sub>	0	0	0	0	0	0	0
G <sub>U</sub>	0	0	0	0	0	0	0

Table 2.14 - Complete Flavor/Color Classification of All Conjugally Independent Complex Preons and Real Fermions and Bosons, Including Neutral Currents and Right-Handed Chiral Projections for Non-Symmetric Z Neutral Current. (21 Real States)

Note in the above, that  $\nu_L, e_L$  designates "lepton", not "left."

With this, we are ready to discuss Cabibbo mixing for quarks and leptons.

To illustrate the important features of Cabibbo mixing, one can select any relevant quark and lepton pair, as the analysis is the same for any other relevant pairs. For example, for quarks, we may choose as representative the decay mode  $u \leftrightarrow s$ , while for leptons we may select  $\nu_e \leftrightarrow \mu$ . In a simple two-generational approach, this mixing is controlled for each of quarks and leptons by a single (one each) Cabibbo mixing angle  $\Theta_C$ , which appears to differ for quarks and leptons. For quarks, this angle at accessible energies is measured to be about  $\Theta_C(Q_u) \approx 13^\circ$ , while for leptons, there is in fact no observable mixing,  $\Theta_C(L) \approx 0^\circ$ . Chirality aside, the current terms describing the quarkonic and leptonic forms of mixing, respectively, are: (Note,  $(Q_u)$  and  $(L)$  in this context are merely labels for the angles.)

$$J_{\text{Quark}}^u = (u \ c) \gamma^u \begin{pmatrix} \cos \Theta_C(Q_u) & \sin \Theta_C(Q_u) \\ -\sin \Theta_C(Q_u) & \cos \Theta_C(Q_u) \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix} \quad (2.381)(a)$$

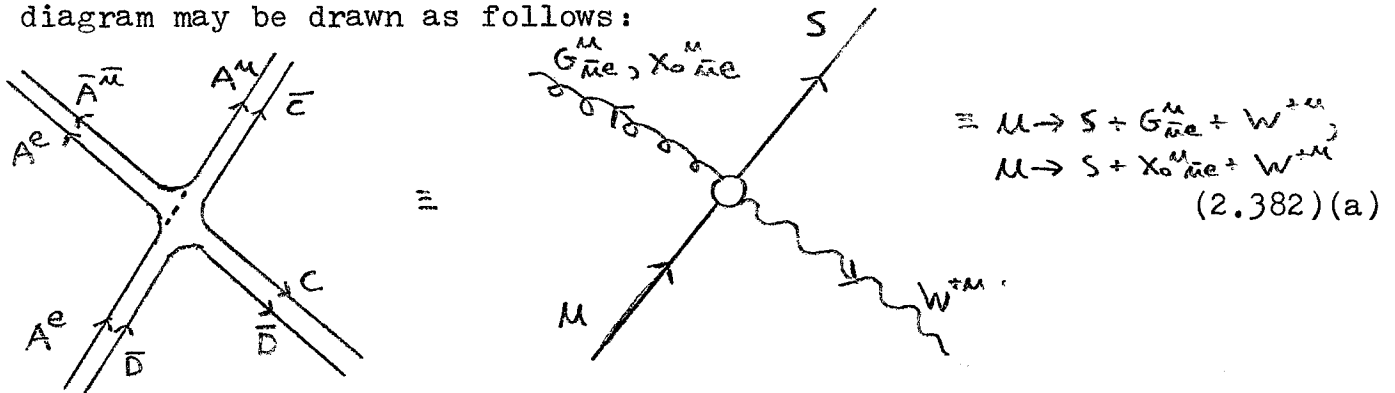
and

$$J_{\text{Lepton}}^u = (\nu^e \ \nu^\mu) \gamma^u \begin{pmatrix} \cos \Theta_C(L) & \sin \Theta_C(L) \\ -\sin \Theta_C(L) & \cos \Theta_C(L) \end{pmatrix} \begin{pmatrix} e \\ \mu \end{pmatrix}. \quad (2.381)(b)$$

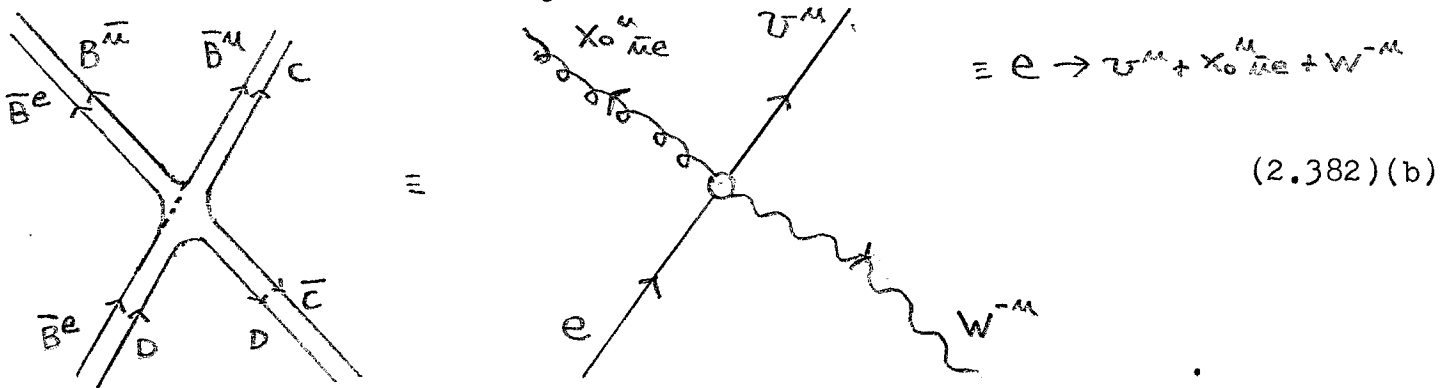
We then examine the preonic decomposition of any particular generation changing transition, for example,  $u \leftrightarrow s, \nu_e \leftrightarrow \mu$  for quarks and leptons respectively. To simplify matters, we assume also that the quarks involved in (2.381)(a) do not exchange any color during the entire decay process, ie., that what begins <sup>for example</sup> as a red quark remains as a red quark, and does not strongly interact with any other quark. In short, if any strong gluons are to be exchanged during this process, these are to be limited to the SU(3) singlet gluon of (2.380)(c),

$G_0^u = (1/3) (\bar{A}_R A_R + \bar{A}_G A_G + \bar{A}_B A_B)$ , and none of the remaining eight color carrying gluons of  $3 \otimes 3 = 8 \oplus 1$  for  $SU(3) \times U(1)$ . Later, we shall both relax and elaborate upon this restriction. For now, the simple net effect is that this allows us to disregard, ie., suppress color, when considering the Cabibbo mixing of quarks at low energies.

Consequently for quarks, the preon decomposition of the  $u \leftrightarrow s$  decay diagram may be drawn as follows:



while for leptons, the  $\nu_e \leftrightarrow \mu$  diagram may be drawn as follows:



Noting from (2.376) that it is now important to distinguish the A and B flavors of preon by their generation, and imposing the requirement that generation number must be conserved through any vertex, it becomes necessary that these decays be modelled not by a three, but rather by a four particle (and preon) vertex. If it were not necessary to distinguish A and B by their generation then, assuming that no color is exchanged, it would be possible simply to connect the A preons in (2.382)(a), and the B preons in (2.382)(b),



resulting in the usual three particle vertex, along single worldlines, as indicated by the "dotted" connection line. This of course, simply brings us back to ordinary single generational beta-decay, as in the diagrams (2.278). This, <sup>in turn,</sup> brings us to the critical issue.

If we recall again the forms (2.380)(c) and (d) for the strong interaction neutral current gluon singlet  $G_0^u$ , and the neutral hyperweak vector bosons  $X_0^u$ , namely, with color suppressed,

$$G_0^u = \bar{A}A \quad (2.383)(a)$$

$$X_0^u = \frac{1}{2}(\bar{B}B - \bar{A}A) + \bar{A}A \sin^2 \theta_S, \quad (2.383)(b)$$

see (2.325), and if we compare this with the diagrams (2.382), one may arrive at some very interesting conclusions. Focusing on the fourth particle introduced into the decay diagrams, namely the  $\bar{A}\bar{A}^e$  in (2.382)(a) and the  $\bar{B}^e\bar{B}^{\bar{A}}$  in (2.382)(b), it now appears that the  $G_0^u$  and  $X_0^u$  above should actually be regarded among other things to be bi-generational, in precisely the same manner that they are already regarded to be bi-colored. <sup>(When four color interactions are considered.)</sup> This is somewhat apparant when thinking about meson combinations of the preons in (2.376), but it helps to bring this point out in the context of Cabibbo mixing. Further, and this is the real key, it appears as though the former interaction (2.382)(a) for quarks, which involves a bi-generational (and if color is exchanged, bi-colored) combination  $\bar{A}A$ , can be mediated by either of the  $G_0^u$  or the  $X_0^u$ , since each contains the  $\bar{A}A$  combination, as shown above in (2.383). For leptons however, the latter interaction (2.382)(b) involves a bi-generational  $\bar{B}B$  combination, and can therefore be mediated only by  $X_0^u$ . At presently accessible energies, which are way below anything approaching the super-massive  $X_0^u$ , the only

particle in (2.383) which can be produced is the  $G_0^u$  (and the gluons  $G^u$  generally if color is exchanged). Consequently, the only form of Cabibbo mixing which can be observed is the quarkonic, (2.382)(a), and not the leptonic, (2.382)(b). At hyperweak energies however, which are at least  $\approx 10^{15}$  GeV., when the  $X_0^u$  boson might be more readily produced, it would then be possible to observe lepton mixing as well as quark mixing. In other words, the Cabibbo mixing of quarks is mediated by both massless Gluons and very massive hyperweak bosons, while that of leptons is mediated only by the hyperweak bosons. At very high energies, where leptons are but a fourth quark color, and where the hyperweak bosons are consequently mere gluons which at least in part, carry the fourth color, the distinction between these bosons, and between quarks and leptons disappears; with the consequence that both quarks and leptons will engage in Cabibbo mixing with equal frequencies. At low energies however, where hyperweak boson production is virtually impossible, it is still perfectly possible to produce bi-generational gluons; however, these couple only to quarks, and not to leptons. Consequently, Cabibbo mixing will be observed for quarks, but not for leptons. One notes too, because of the lepton mixing which can take place once energies on the  $X^u$  scale are reached, that the neutrino is expected to have some very small, though finite rest mass; and also, and as a consequence of this, to possess both left and right handed chiral projections.

An alternative way of viewing this conclusion is to note, from Table 2.14, that the generator  $(1/3)Q_u$ , which in the context of flavor/color and generation symmetry is to be associated with Cabibbo mixing mediated by bi-colored, bi-generational gluons, is equal

to  $1/3$  for all quarks, but is equal to 0 for the leptons. Consequently, only the quarks experience the "strong" form of Cabibbo mixing, mediated by ordinary gluons. On the other hand, the hyperweak X generator is non-zero for both quarks and leptons. Hence, when the hyperweak gluons (bi-generational) are used to mix generations, one will observe both quark and lepton mixing, ie., both quarks and leptons will experience "hyperweak" form of Cabibbo mixing, mediated by high energy hyperweak bosons. Of course, since hyperweak production requires enormous energies, this form of mixing cannot be observed for all practical purposes, with today's experimental capabilities. This may be contrasted with the behavior of the neutrino in ordinary electroweak interactions. Noting particularly from Table 2.14 that the neutrino is the only fermion for which the electromagnetic charge generator  $Q = 0$ , we deduce the well known result that this particle does not interact electromagnetically. Nevertheless, the neutral current  $Z_L$  generator is non-zero for all fermions; hence all fermions, including the left-handed neutrino, can interact via the neutral current Z interaction, and of course, via the charged  $W^{\pm u}$  boson interaction. The exception here is the right handed neutrino, for which  $Z_R=0$  also. Hence this neutrino component will not couple through any of  $Z^u(=W^{0u})$  and  $W^{\pm u}$  either. In fact, if one examines just the neutrino in Table 2.14, and recalling that the X generator is chiral symmetric, it becomes apparant that the only interaction through which the right-handed neutrino projection will couple, is that associated with the X generator ( $X=-\frac{1}{2}$ ). Thus, to actually detect the right-handed neutrino projection, thereby establishing a non-zero rest mass, it would be necessary to produce one of the hyperweak  $X_0^u, X^{\pm 1/3u}$  bosons, which again, is virtually impossible with current technological capabilities. This contrast may

be summarized by stating that left-handed neutrinos in electroweak theory serve a role similar to that of leptons in strong/hyperweak theory, insofar as each interacts only through the massive, and not through the massless vector bosons. In strong/hyperweak interactions, the various vector bosons carry the additional responsibility of mediating intergenerational decays; and the difficulty in producing the massive hyperweak bosons <sub>at current energies</sub> is reflected in the difficulty that one has observing leptonic Cabibbo mixing, and the neutrino rest mass, and the right-handed neutrino chiral components. In a somewhat more tabular form, this contrast between electroweak and strong/hyperweak theory is illustrated by examining the various flavors of boson in each theory, along with the various flavors and chiral projections of fermion which may interact through these bosons, as such:

Electroweak Theory

<u>Fermions</u>	<u>Boson Mediators</u>
$\nu_L$	$W_0^u, W^{\pm u}$
$e_{L,R}, u_{L,R}, d_{L,R}$	$W_0^u, W^{\pm u}, A^u$

Strong/Hyperweak Theory

<u>Fermions</u>	<u>Boson Mediators</u>
$\nu_{L,R}, e_{L,R}$	$X_0^u, X^{\pm 1/3u}$
$u_{L,R}, d_{L,R}$	$X_0^u, X^{\pm 1/3u}, G^u$

Table 2.15 - Fermion Flavors and Chiral Projections, and Associated Flavors of Mediating Vector Bosons for Electroweak and Strong/Hyperweak Interaction Theory.

It is the absence of the gluons,  $G^u$ , as mediators of strong/hyperweak leptonic interactions involving  $\nu_L e_A$ , <sup>due to the fact that  $G^u = 0$  for  $\nu_L e$ ,</sup> which is concurrently responsible for the absence of observed Cabibbo lepton mixing at accessible energies, and for the apparant masslessness of the neutrino and the consequent absence of right handed chiral neutrino projections. Again, this is but another way of viewing our earlier results reg-

arding the Cabibbo mixing of quarks and leptons.

At this point, we are now prepared to examine directly the various vector bosons which can be composed out of the 18 preon states, namely the 16 in (2.376) and the (C,D). Recalling that it is possible to compose  $324 = (16 + 2) \times (16 + 2) = 256 + 64 + 4$  real particles out of these 18 preons, and that the 64 represents fermions and anti-fermions, already discussed, we turn now to examine the 256 + 4 = 260 logical combinations of real <sup>(flavor/color/generation)</sup> vector boson. The 4, which describes the  $2 \times 2 = 3 + 1$  preon decomposition of the four electroweak flavor particles  $A^u, W_0^u, W^{\pm u}$  into the C and D preons has already been examined in depth, see, eg., Figure 2.2 and discussion leading thereto. Consequently, what is now of particular interest to us here, are the  $16 \times 16 = 256$  logical real vector bosons which can be composed by the meson combination of the sixteen color/generation combinations of A and B flavored preon shown in (2.376). Working at ultra-high (gravitational) energies, prior to any form of spontaneous symmetry breaking, it is helpful to separate color from generation. Thus, from (2.376), one may form sixteen logical color combinations, namely the twelve mixed color states LR, LG, LB,  $\bar{R}\bar{L}$ ,  $\bar{R}\bar{G}$ ,  $\bar{R}\bar{B}$ ,  $\bar{G}\bar{L}$ ,  $\bar{G}\bar{R}$ ,  $\bar{G}\bar{B}$ ,  $\bar{B}\bar{L}$ ,  $\bar{B}\bar{R}$ ,  $\bar{B}\bar{G}$  and the four pure states  $L\bar{L}$ ,  $R\bar{R}$ ,  $G\bar{G}$ ,  $B\bar{B}$  which, as we know, are used in the composition of the <sup>four</sup> hyperweak/strong neutral current vector bosons,  $G_0^u, X_0^u, G^{3u}, G^{8u}$ , see (2.380)(c)-(f). When conjugacy is considered, these sixteen combinations may be reduced to the ten conjugally independent LR, LG, LB,  $\bar{G}\bar{R}$ ,  $\bar{B}\bar{G}$ ,  $\bar{R}\bar{B}$  and  $L\bar{L}$ ,  $R\bar{R}$ ,  $\bar{G}\bar{G}$ ,  $\bar{B}\bar{B}$ , for example. That <sup>is</sup> six of the twelve mixed states may be reduced through conjugacy. Note by the way, as we are dealing here strictly with A and B preons, that all vector bosons currently under discussion are chiral symmetric. For generation, assuming four gen-

number of  
 erations, the <sup>^</sup>various logical meson generation combinations which  
 can be formed from (2.376) totals sixteen as well, namely the twelve  
 mixed generation states  $\bar{\delta}\tau, \bar{\delta}\mu, \bar{\delta}e, \bar{\tau}\delta, \bar{\tau}\mu, \bar{\tau}e, \bar{u}\delta, \bar{u}\tau, \bar{u}e, \bar{e}\delta, \bar{e}\tau, \bar{e}\mu$ , and the  
 four neutral states  $\bar{\tau}\tau, \bar{\mu}\mu, \bar{e}e$ . Here too, six of the twelve mixed  
 states can be reduced out once conjugacy is considered, hence we  
 again reduce down to ten conjugally independent combinations, for  
 example,  $\bar{\delta}\tau, \bar{\delta}\mu, \bar{\tau}e, \bar{\tau}\mu, \bar{\tau}e, \bar{u}e$ , and  $\bar{\tau}\tau, \bar{\mu}\mu, \bar{e}e$ . Thus, with conjugacy  
 considered, one has ten independent color and ten independent gener-  
 eration combinations available for the strong/hyperweak interaction  
 gluons. As noted in the earlier discussion following (2.383), the  
 gluons <sup>and hyperweak bosons</sup> <sup>^</sup>are responsible for mediating both color and generational  
 decays, which is to say that the gluons <sup>and hyperweak bosons</sup> <sup>^</sup>are bi-colored and bi-genera-  
tional. Thus, considering color and generation together, at hyperweak  
 energies where  $SU(4) \times U(1)$  applies unbroken for both color and gener-  
 eration, there are a total of  $10 \times 10 = 100$  conjugally independent  
 color/generation combinations of strong/hyperweak gluon. Considering  
 four colors and four generations, following the type of approach used  
 in Tables 2.11, 2.12, this may also be thought of as  $100 = \frac{1}{2}(4)(4+1) \times$   
 $\frac{1}{2}(4)(4+1)$  conjugally independent gluons and hyperweak bosons, as  
 opposed to the  $256 = (4 \times 4) \times (4 \times 4)$  such gluons and hyperweak  
 bosons which were considered prior to reducing out the conjugate  
 gluon states. With the three conjugally independent electroweak  
 $A^u, Z^u, W^u$  bosons and the 32 conjugally independent fermions, this  
 results in a total of  $135 = 100 + 32 + 3$  conjugally independent  
 real fermions and vector bosons (32 fermions, 103 bosons), down  
 from the original total of  $324 = 256 + 64 + 4$  (64 fermions, 260  
 bosons) prior to the reduction of conjugate states.