2.4 - The Fifth-Dimensional Origin of Left and Right Handed Chiral Projections and the Continuity Equation in Five Dimensions: Hermitian Conjugacy, Adjoint Spinors, and the Finite Operators Parity (P) and Axiality (A)

In the earlier discussion, it was suggested that chirality appears naturally, from a geometric standpoint, to be associated with a fifth spacetime dimension. Particularly, this conclusion appears almost unavoidable when one considers the structural properties of the Dirac matrices including γ^5. The five-dimensional metric deduced by considering the structural properties of γ^5, was given in (2.18)(a). One should note that $g^{55}=1$; hence we deal here with a "cylinder" world not unlike that first proposed in the early 1920's by Kaluza and Klein.[-2.3] At this time, we shall examine this five-dimensional possibility a little more fully, and will simultaneously examine the behavior of the four and five dimensional Dirac equations under the various operations of hermitian conjugation, ordinary complex conjugation, and transposition. This will lead us to consider in more detail the closely related C, P and T operators introduced briefly in the earlier discussion.

To begin with, consider the Dirac equation in covariant form, eq. (2.37), though generalized to five dimensions along the lines of the bilinear equation (2.48). With $\hbar$ and c explicitly restored, this is given by: $(U = 0,1,2,3,5)$

$$\left[\gamma^U\left(i d_U - \frac{eQ}{\hbar c}\cdot A_U\right) - \frac{mc^2}{\hbar c}\right]\Psi = 0 \,, \tag{2.88}$$

where the wavefunction Ψ is given by: (eq. (2.31))

$$\Psi = N\cdot u(E^U)\, e^{-iE^V x_V} \tag{2.89}$$

and the Compton wavelength $\lambda\!\!\!^{-}_C$ is given by:

$$mc^2/\hbar c = 1/\lambda\!\!\!^{-}_C \,. \tag{2.90}$$

The vector field A_U comes about directly in response to the requirement

(2.36) that the Dirac equation retain invariance under wavefunction transformations of the form:

$$\psi \longrightarrow \psi' = e^{ia(x^U)} , \tag{2.91}$$

and as a consequence of this requirement for local gauge symmetry, the vector A^U, which one associates with the photon, transforms according to:

$$eQA^U \longrightarrow eQA^{U'} = eQA^U - d^U a , \tag{2.92}$$

eq. (2.38).

Let us start by demanding that the Dirac equation remain invariant under hermitian conjugation, ie., under conjugate transposition. In 5-space, this is to say that:

$$\begin{aligned} &\left[\gamma^U(id_U - \tfrac{eQ}{\hbar c} A_U) - 1/\lambda_C\right]\psi \\ &= \psi_+\left[\gamma_+^U(-id_U - \tfrac{eQ}{\hbar c} A_U) - 1/\lambda_C\right] = 0 . \end{aligned} \tag{2.93}$$

It is helpful to consider that for each of γ^U, the Hermitian conjugate is given by:

$$\gamma_+^0 = \gamma^0;\ \gamma_+^1 = -\gamma^1;\ \gamma_+^2 = -\gamma^2;\ \gamma_+^3 = -\gamma^3;\ \gamma_+^5 = \gamma^5 , \tag{2.94}$$

as is easily shown by explicit analysis of the γ^U matrices (2.8), (2.9) and (2.12). In ordinary four-dimensional spacetime, these results may be summarized by:

$$\gamma_+^u = \gamma^0 \gamma^u \gamma^0 , \tag{2.95}$$

but note however, that:

$$\gamma_+^5 = -\gamma^0\gamma^5\gamma^0 . \tag{2.96}$$

This means that (2.95) only holds in four-spacetime, but not in five dimensional spacetime with chirality. Including the chirality γ^5, equation (2.95) must be generalized to the following:

$$\gamma_+^U = -\gamma^0\gamma^5\gamma^U\gamma^5\gamma^0 . \tag{2.97}$$

First we examine the four-spacetime hermitian conjugacy of (2.93).

With explicit substitution of the four-dimensional (2.95), the hermitian conjugate equation (2.93) is easily written: (u=0,1,2,3)

$$\Psi^{+}\left[\gamma^{0}\,\gamma^{u}\,\gamma^{0}\,\left(id_{u}+\frac{eQ}{\hbar c}A_{u}\right)+1/\lambda_{C}\right]. \tag{2.98}$$

Multiplying from the right by γ^0 and noting that $\gamma^0\gamma^0 = 1$, (this is equivalent with the fact that the metric time component g^{00} has +1 signature, see (2.17)-(2.19)) this is easily rewritten as:

$$\bar{\Psi}\left[\gamma^{u}\,\left(id_{u}+\frac{eQ}{\hbar c}A_{u}\right)+1/\lambda_{C}\right]=0\ , \tag{2.99}$$

where we have defined both a new (adjoint) wavefunction $\bar{\Psi}$ and a new (parity) operation P given by:

$$\bar{\Psi} \equiv \Psi^{+}\gamma^{0} \equiv \Psi^{+}P^{-1}. \tag{2.100}$$

Note that the sign of both of Q and $1/\lambda_C$ in (2.98) has reversed with respect to that in the original Dirac equation (2.88). The sign reversal for Q indicates, whatever the electrostatic charge of the original Ψ, that the adjoint $\bar{\Psi}$ has an electrostatic charge which is precisely the opposite of the original charge. On the other hand, the sign reversal for $1/\lambda_C$ means, if we multiply (2.88) from the left by $\bar{\Psi}$, (2.99) from the right by Ψ and then add, that:

$$id_{u}J^{u} \equiv id_{u}\,(\bar{\Psi}\gamma^{u}\Psi)=0, \tag{2.101}$$

which no longer contains $1/\lambda_C$. Because $id_u\,(\bar{\Psi}\gamma^u\Psi) = 0$, it becomes possible to identify the term $\bar{\Psi}\gamma^u\Psi$ with the particle current, and eq. (2.101) above, with the conservation (continuity) of that current. One should also note, from (2.100), that the definition of $\bar{\Psi}$ includes a parity transformation P. This means, in addition to opposite charge, that $\bar{\Psi}$ and Ψ also have opposite intrinsic parity. In more general terms, $\bar{\Psi}$ is often regarded as the antiparticle wavefunction. All of this of course, is a standard part of any analysis of the Dirac equation.

For analysis in five-spacetime including chirality, the above must be somewhat modified. Particularly, the five-dimensional hermitian conjugate equation (2.93) must now be evaluated using the five-dimensional relationship (2.97). Hence, the five-dimensional analog of (2.98) is

$$\Psi_{+}\left[-\gamma^{0}\gamma^{5}\gamma^{U}\gamma^{5}\gamma^{0}(-id_{U} - \frac{eQ}{\hbar c}A_{U}) - 1/\lambda_{C}\right] \quad . \qquad (2.102)$$

This time, we must multiply from the right by $\gamma^{0}\gamma^{5}$, using both $\gamma^{0}\gamma^{0} = 1$ and $\gamma^{5}\gamma^{5} = 1$, ie., $g^{00}=g^{55}=1$. (2.102) is then reduced to:

$$\overline{\Psi}_{A}\left[\gamma^{U}(id_{U} + \frac{eQ}{\hbar c}A_{U}) - 1/\lambda_{C}\right] = 0 \qquad (2.103)$$

where we retain the definition (2.100) derived by considering hermitian conjugacy in ordinary spacetime, but are forced into a new set of definitions by the existence of the fifth spacetime chirality dimension, particularly, (see (2.97)):(the minus sign here is important)

$$\overline{\Psi}_{A} \equiv \overline{\Psi}(-\gamma^{5}) = \Psi^{\dagger}(-\gamma^{0}\gamma^{5}) \equiv \overline{\Psi}A^{-1} = \Psi^{\dagger}P^{-1}A^{-1} \qquad (2.104)$$

where $A^{-1} \equiv -\gamma^{5}$ denotes a chiral (axial) transformation on $\overline{\Psi}$. Once again, the adjoint wavefunction has electrostatic charge and intrinsic parity opposite to that of the original wavefunction, but here, there are two important differences. First, the adjoint wavefunction required to ensure hermitian conjugacy in five dimensions, is no longer a "vector" wavefunction, as it was in the four dimensional (2.99). Instead, it is an "axial vector" wavefunction, which gives it quite a different character. (The word "vector" in this context has a different meaning than it does in the context of spacetime.) Secondly, while the sign of the mass term $mc^{2}/\hbar c = 1/\lambda_{C}$ (eq. (2.90)) is reversed in the four dimensional (2.98) with the consequence that the four current $J^{u}{}_{;u}=0$ is conserved as in (2.101), the sign of the mass term in the five dimensional (2.103) does <u>not</u> reverse itself with respect to that in the original Dirac equation in five dimensions, (2.88). This affects the fermion mass term in the five dimensional Lagrangian , as is discussed below.

To identify the fermion mass term, it is helpful first to define the vector and axial wavefunctions, in terms of left and right handed chiral wavefunctions. The usual definitions here are the following: (with $\Psi_V \equiv \Psi$)

$$\Psi_V \equiv \Psi_R + \Psi_L = \Psi_V \qquad (2.105)(a)$$

$$\Psi_A \equiv \Psi_R - \Psi_L = \gamma^5 \Psi_V \, , \qquad (2.105)(b)$$

along with the inverses:

$$\Psi_R = \tfrac{1}{2}\,(\Psi_V + \Psi_A) = \tfrac{1}{2}\,(1 + \gamma^5)\,\Psi_V \qquad (2.106)(a)$$

$$\Psi_L = \tfrac{1}{2}\,(\Psi_V - \Psi_A) = \tfrac{1}{2}\,(1 - \gamma^5)\,\Psi_V \, . \qquad (2.106)(b)$$

Utilizing the adjoint wavefunction (2.100), $\overline{\Psi} = \Psi^\dagger\gamma^0$, one derives a similar set of relationships for the $\overline{\Psi}$, namely:

$$\overline{\Psi}_V = \Psi^\dagger_V{}^{0} = \overline{\Psi}_R + \overline{\Psi}_L = \overline{\Psi} \qquad (2.107)(a)$$

$$\overline{\Psi}_A = \Psi^\dagger_A{}^{0} = \overline{\Psi}_R - \overline{\Psi}_L = \overline{\Psi}_V\,(-\gamma^5) \qquad (2.107)(b)$$

and inverses:

$$\overline{\Psi}_R = \Psi^\dagger_R{}^{0} = \tfrac{1}{2}(\overline{\Psi}_V + \overline{\Psi}_A) = \overline{\Psi}_V\,\tfrac{1}{2}(1 - \gamma^5) \qquad (2.108)(a)$$

$$\overline{\Psi}_L = \Psi^\dagger_L{}^{0} = \tfrac{1}{2}(\overline{\Psi}_V - \overline{\Psi}_A) = \overline{\Psi}_V\,\tfrac{1}{2}(1 + \gamma^5). \qquad (2.108)(b)$$

Note that (2.107)(b) is used to account for the definition in (2.104), and note too the sign reversal which takes place in the adjoint projections. As noted in (2.87), γ^5 becomes associated with helicity Σ^3 at high energy. Also useful is the electrostatic charge Q associated with each of the chiral projections (2.105)-(2.108) above. This set of relationships is given by:

$$Q_V = \tfrac{1}{2}\,(Q_R + Q_L) \qquad (2.109)(a)$$

$$Q_A = \tfrac{1}{2}\,(Q_R - Q_L) \qquad (2.109)(b)$$

and inverses:

$$Q_R = Q_V + Q_A \qquad (2.110)(a)$$

$$Q_L = Q_V - Q_A \quad . \qquad (2.110)(b)$$

Because the electromagnetic interaction is known to be left-right symmetric, one may summarize the relationships among (2.109)(2.110) according to:

$$Q_V = Q_R = Q_L \qquad (2.111)(a)$$

$$Q_A = 0 \quad . \qquad (2.111)(b)$$

It is useful to note, for later discussions of flavor symmetry, that a similar set of relationships must exist for quark number Q_u and lepton number L, in order to ensure the left-right chiral symmetry of the strong color interaction. As was discussed at length in the introduction, it is this requirement, that Q, Q_u and L possess and intrinsic Lagrangian chiral left-right symmetry, which will be affirmatively utilized as a gauge condition, to break the gauge symmetry of the chosen G.U.T. symmetry group, thus giving rise to the conservation of the electrostatic and "color with lepton number" charges, and to the masslessness of the photon and colored gluons. Specifically, the reader may wish to refer back to Table 1.1, and to eqs. (1.2)(a) and (1.11). Eq. (2.111) above is another way of saying that $Q_{LR} = Q_R = Q_L$. For certain other flavor quantum numbers, such as Y and I^3, this ~~left-right~~ symmetry does not hold true, as in eq. (1.2)(b), and it is in this situation, where left-right symmetry is violated by a particular interaction, that sets of equations similar to (2.109)(2.110) become of particular interest. (See, eg., Table 2.9 infra)

With the various chiral projections identified, one may return to the original Dirac equation in five dimensions, (2.88), and multiply this equation from the left by $\pm\overline{\Psi}_V$; while one also multiplies the adjoint equation in five dimensions, (2.103), from the right by $\pm\Psi_A$. As between (2.88) and (2.103), this yields four combinations of + and - sign. Adding (2.88) and (2.103) using each of the four sign combinations, and making use particularly of (2.106) and (2.108), one arrives at four distinct component equations:

$$\bar{\Psi}_R\ (\gamma^U i d_U - \frac{mc^2}{\hbar c})\ \Psi_R = 0 \qquad (2.112)(a)$$

$$\bar{\Psi}_L\ (\gamma^U i d_U - \frac{mc^2}{\hbar c})\ \Psi_L = 0 \qquad (2.112)(b)$$

$$\bar{\Psi}_R\ (\gamma^U i d_U - \frac{mc^2}{\hbar c})\ \Psi_L = 0 \qquad (2.112)(c)$$

$$\bar{\Psi}_L\ (\gamma^U i d_U - \frac{mc^2}{\hbar c})\ \Psi_R = 0\ . \qquad (2.112)(d)$$

The above can be simplified to some degree. First, utilizing the commutivity properties of the γ^U matrices in conjunction with (2.106) and (2.108), it is possible to show that the following hold identically:

$$\bar{\Psi}_R\ \gamma^5\ \Psi_R = 0 \qquad (2.113)(a)$$

$$\bar{\Psi}_L\ \gamma^5\ \Psi_L = 0 \qquad (2.113)(b)$$

$$\bar{\Psi}_R\ \gamma^u\ \Psi_L = 0 \qquad (2.113)(c)$$

$$\bar{\Psi}_L\ \gamma^u\ \Psi_R = 0\ . \qquad (2.113)(d)$$

As a result, (2.112) may be simplified to:

$$\bar{\Psi}_R\ (\gamma^u i d_u - \frac{mc^2}{\hbar c})\ \Psi_R = 0 \qquad (2.114)(a)$$

$$\bar{\Psi}_L\ (\gamma^u i d_u - \frac{mc^2}{\hbar c})\ \Psi_L = 0 \qquad (2.114)(b)$$

$$\bar{\Psi}_R\ (\gamma^5 i d_5 - \frac{mc^2}{\hbar c})\ \Psi_L = 0 \qquad (2.114)(c)$$

$$\bar{\Psi}_L\ (\gamma^5 i d_5 - \frac{mc^2}{\hbar c})\ \Psi_R = 0\ . \qquad (2.114)(d)$$

Similarly, one can form the identities:

$$\bar{\Psi}_R \Psi_R = -\bar{\Psi}_R \gamma^5 \Psi_R = 0 \qquad (2.115)(a)$$

$$\bar{\Psi}_L \Psi_L = \bar{\Psi}_L \gamma^5 \Psi_L = 0 \qquad (2.115)(b)$$

$$\bar{\Psi}_R \Psi_L = -\bar{\Psi}_R \gamma^5 \Psi_L \qquad (2.115)(c)$$

$$\bar{\Psi}_L \Psi_R = \bar{\Psi}_L \gamma^5 \Psi_R\ . \qquad (2.115)(d)$$

With these, (2.114) is further reduced to:

$$\overline{\Psi}_R\gamma^u id_u\Psi_R = \frac{mc^2}{\hbar c}\overline{\Psi}_R\Psi_R = -\frac{mc^2}{\hbar c}\Psi_R\gamma^5\Psi_R = 0 \qquad (2.116)(a)$$
$$\overline{\Psi}_L\gamma^u id_u\Psi_L = \frac{mc^2}{\hbar c}\overline{\Psi}_L\Psi_L = \frac{mc^2}{\hbar c}\Psi_L\gamma^5\Psi_L = 0 \qquad (2.116)(b)$$
$$\overline{\Psi}_R\gamma^5 id_5\Psi_L = \frac{mc^2}{\hbar c}\overline{\Psi}_R\Psi_L = -\frac{mc^2}{\hbar c}\Psi_R\gamma^5\Psi_L \qquad (2.116)(c)$$
$$\overline{\Psi}_L\gamma^5 id_5\Psi_R = \frac{mc^2}{\hbar c}\overline{\Psi}_L\Psi_R = \frac{mc^2}{\hbar c}\Psi_L\gamma^5\Psi_R\ . \qquad (2.116)(d)$$

Finally, with the help (2.113) and (2.115), one may write the following identities:

$$\overline{\Psi}\Psi = \overline{\Psi}_L\Psi_R + \overline{\Psi}_R\Psi_L = \overline{\Psi}_L\gamma^5\Psi_R - \overline{\Psi}_R\gamma^5\Psi_L \qquad (2.117)(a)$$
$$\overline{\Psi}\gamma^u\Psi = \overline{\Psi}_L\gamma^u\Psi_L + \overline{\Psi}_R\gamma^u\Psi_R \qquad (2.117)(b)$$
$$\overline{\Psi}\gamma^5\Psi = \overline{\Psi}_L\gamma^5\Psi_R + \overline{\Psi}_R\gamma^5\Psi_L = \overline{\Psi}_L\Psi_R - \overline{\Psi}_R\Psi_L\ . \qquad (2.117)(c)$$

Further, since $\Psi = N\cdot ue^{-iE^UX_U}$ (see (2.89)), and, using eq. (2.100), $\overline{\Psi} = \Psi^{\dagger}\gamma^0$ along with a similar $\overline{u} = u^{\dagger}\gamma^0$ for spinors, since $= N^{\dagger}\cdot ue^{iE^UX_U}$, one may write the antisymmetric relationship:

$$\overline{\Psi}\gamma^U\ i(d_U\Psi) = -i(d_U\overline{\Psi})\gamma^U\Psi = \overline{\Psi}\gamma^U E_U\Psi. \qquad (2.118)$$

Thus, using (2.117) to consolidate (2.116), and using (2.118) to write a similar set of equations involving the adjoint derivative $d_U\overline{\Psi}$, one may write:

$$\overline{\Psi}\gamma^u(id_u\Psi) = 0 \quad ; \quad (id_u\overline{\Psi})\gamma^u\Psi = 0 \qquad (2.119)(a)$$
$$\overline{\Psi}\gamma^5(id_5\Psi) = \frac{mc^2}{\hbar c}\overline{\Psi}\Psi \quad ; \quad (id_5\overline{\Psi})\gamma^5\Psi = -\frac{mc^2}{\hbar c}\overline{\Psi}\Psi \qquad (2.119)(b)$$

hence:

$$\overline{\Psi}\gamma^U(id_U\Psi) = \frac{mc^2}{\hbar c}\overline{\Psi}\Psi \quad ; \quad (id_U\overline{\Psi})\gamma^U\Psi = -\frac{mc^2}{\hbar c}\overline{\Psi}\Psi \qquad (2.119)(c)$$

The fact that the fermion mass term does not vanish from either of (2.119)(b) or (c) above stems directly from the absence of a sign reversal for this same term in the adjoint five dimensional equation (2.103). As will be discussed below the above suggests that <u>it may be possible to obtain a strictly geometric interpretation of particle rest mass</u> in the Q.E.D. Lagrangian. Particularly, combining the two parts of (2.119)(c), one arrives as should be expected, at a five dimensional continuity equation:

$$J^U{}_{;U} \equiv (\overline{\Psi}\gamma^U\Psi)_{;U} = \overline{\Psi}_{;U}\gamma^U\Psi + \overline{\Psi}\gamma^U\Psi_{;U} = 0\ , \qquad (2.120)(a)$$

along with the individual spacetime and chirality component equations:

$$J^u{}_{;u} = (\bar{\Psi}\gamma^u\Psi)_{;u} = \bar{\Psi}_{;u}\gamma^u\Psi + \bar{\Psi}\gamma^u\Psi_{;u} = 0 \qquad (2.120)(b)$$

$$J^5{}_{;5} = (\bar{\Psi}\gamma^5\Psi)_{;5} = \bar{\Psi}_{;5}\gamma^5\Psi + \bar{\Psi}\gamma^5\Psi_{;5} = 0 \qquad (2.120)(c)$$

using (2.119)(a) and (b). Eq. (b) above is of course just the usual continuity equation, ie., equation of current conservation, in ordinary spacetime, (2.101), as is to be expected.

Of particular interest also at this point, is the Q.E.D. Lagrangian for the Dirac wave (and adjunct wave) equation. In particular, with $\phi = \Psi, \bar{\Psi}$, the four-dimensional components of (2.88), and the adjoint equation (2.99) respectively, may be rewritten as a Lagrangian density field equation,

$$\left(\frac{d\mathcal{L}}{d\phi^{;u}}\right)^{;u} - \left(\frac{d\mathcal{L}}{d\phi}\right) = 0 \quad , \qquad (2.120).(1)(a)$$

where we have set:

$$\mathcal{L} = \bar{\Psi}\gamma^u(id_u - \frac{eQ}{\hbar c}A_u) - \frac{mc^2}{\hbar c}\bar{\Psi}\Psi - \tfrac{1}{4}F^{uv}F_{uv} \,, \qquad (2.120).(1)(b)$$

and where, in abelian field theory,

$$F^{uv} = A^{v;u} - A^{u;v} = A^{v,u} - A^{u,v} \,. \qquad (2.120).(1)(c)$$

All of this is of course standard Q.E.D. Looking at chirality however, we notice that the Lagrangian mass terms in (2.119)(b) admit of some rather interesting possibilities. Particularly, substituting $(mc^2/\hbar c)\bar{\Psi}\Psi = \bar{\Psi}\gamma^5(id_5\Psi)$ into (1)(b) above, noting from (2.120)(a) that $J^U = \bar{\Psi}\gamma^U\Psi$, from (2.107)(b),(2.104) and (2.105) that $\bar{\Psi}_A = \bar{\Psi}(-\gamma^5)$, $\Psi_A = \gamma^5\Psi$, and by explicit examination of the γ^U matrices (2.8),(2.12)(a), that:

$$-\gamma^5\gamma^u\gamma^5 = \gamma^u \qquad (2.120).(2)(a)$$

$$-\gamma^5\gamma^5\gamma^5 = -\gamma^5 \quad , \qquad (2.120).(2)(b)$$

it becomes possible to assimilate the Fermion mass term in (2.120).(1)(b) above directly into the five dimensional Lagrangian, in the form:

$$\mathcal{L}_{(5)} = \overline{\Psi}\gamma^u(id_u\Psi) - \overline{\Psi}\gamma^5(id_5\Psi) - \frac{eQ}{\hbar c} A_u(\overline{\Psi}\gamma^u\Psi) - \tfrac{1}{4}F^{uv}F_{uv}$$

$$= \overline{\Psi}_A\gamma^U(id_U\Psi_A) - \frac{eQ}{\hbar c} A_u J^u - \tfrac{1}{4}F^{uv}F_{uv} \qquad (2.120).(3)$$

$$\Longrightarrow \overline{\Psi}_A\gamma^U(id_U\Psi_A) - \frac{eQ}{\hbar c} A_U J^U - \tfrac{1}{4}F^{UV}F_{UV} \quad ,$$

where in the final line of the above, we have essentially generalized the Maxwell equations of the Lagrangian to five dimensions, ie.,

$$J^u = F^{vu}{}_{;v} \quad \Longrightarrow \quad J^U = F^{VU}{}_{;U}. \qquad (2.120).(4)$$

hat is particularly interesting about the five dimensional Lagrangian (2.120).(3) above, is that the Fermion mass term $m\overline{\Psi}\Psi$ need no longer appear explicitly in the Lagrangian. That is, in a five dimensional spacetime geometry with chirality, utilizing axially projected wave-functions, the Lagrangian mass term is already included implicitly in the geometric term $\overline{\Psi}_A\gamma^U(id_U\Psi_A)$, by virtue of (2.119). This suggests that the fermion rest mass of a particle, which in four-dimensions is essentially introduced from outside, into the spacetime geometry, may be regarded in five dimensions to itself be part and parcel of the five-dimensional geometry per se. That is, it appears as though it may no longer be necessary, in five dimensions, to introduce matter into geometry, and to determine the effects of matter upon geometry, as though it were a foreign object. Rather, matter is itself given a direct interpretation as part of the five dimensional geometry of spacetime with chirlaity. This should have a significant impact upon the way in which one formally deals with the fermion rest mass, and rest mass in general.