2.9 - Introduction to Isospin Preons in Electroweak Theory: The Preonic Decomposition of Four Real Electroweak Bosons, A^u, W^{+u}, W^{-u}, Z^u, into Two Complex Preons, $\Uparrow$, $\Downarrow$, Later C, D, Denoting "Isospin Up" and "Isospin Down"

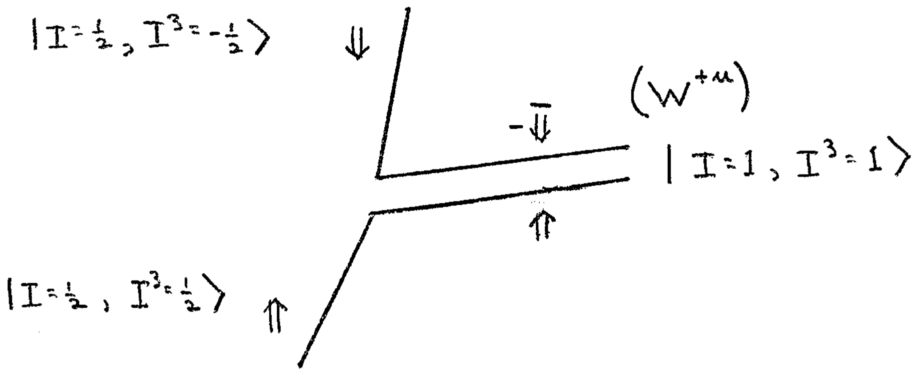
In the preceding section, we took great pains to clearly develop the origin of the spin space classification of the various polarization states of a covariant (real and virtual) vector boson. The particular decomposition of Bose polarizations into Fermi polarizations depicted graphically in Figure 2.1, and developed more explicitly in Eqs. (2.225)-(2.227) and Table 2.2, utilizing the eigensolutions (2.223) for the Pauli matrices (2.217), describes more than simply the polarization decomposition of Bosons into Fermions; it describes and is directly related to the spinor decomposition of spacetime. (Chirality again excepted for the moment.) This is to say, by engaging in an exercise quite similar to that of the previous section, one is readily able to decompose the four real dimensions of spacetime into two complex spinor states; and to describe various transformations in spacetime in terms of their corresponding transformations in the two-complex Argand dimensional spinor space. This is what led Penrose and Rindler to emphasize so heavily, the indispensable importance of two-complex dimensional spinors, as a means for decomposing four-real dimensional spacetime into a more "elemental" sort of representation. —2.6 This will be discussed in greater detail at the outset of Section 3, not part of the current draft.

What concerns us in particular here, is the fact that this same sort of decomposition can easily be applied to the real Bose particles of Weinberg/Salam Electroweak flavor theory, so that the four real particles $(W^{+u}, W^{-u}, A^u, Z^u)$ of electroweak theory may be decomposed into two complex spinor-like particles which, by analogy, one might refer to as "isospin up" and "isospin down", designated $\Uparrow$ and $\Downarrow$. These complex spinor-like flavors of particle are precisely those entities which we shall refer to as "preons" throughout the discussions of grand unification. In the same way that spinors are an essential element in the deeper understanding of spacetime, so too are preons an indispensable element in the deeper description and understanding of the elementary particles. Let us now be more specific.

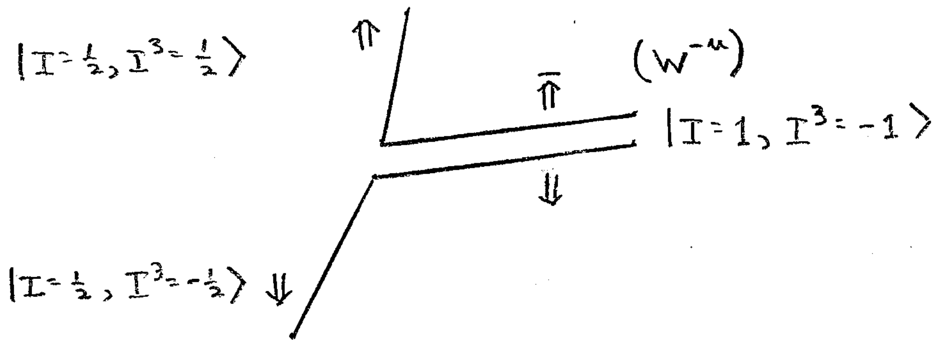
As noted early in the introductory section, the weak isospin model of nuclear beta-decay has long been known. In particular, in order to model weak beta decay, it is conventional to assign to any given particle a particular numerical value for a quantum mechanical degree of freedom known as weak isospin, and designated I^3. This is related to the electromagnetic Coulomb charge Q, and to a third (defined) interaction known as hypercharge, designated Y, by the familiar: (Some authors use $Q \equiv Y/2 + I^3$ as the Y definition.)

$$Q \equiv Y + I^3 \quad . \qquad \text{E.}\ (2.229)$$

As noted in the introduction, Q is the only one of these degrees of freedom which respects the left-right chiral symmetry of the fifth dimension. These assignments of weak isospin number are done completely by analogy with the assignments of spin number outlined in the prior section. Specifically, designating a casimir isospin I which plays the same role as the casimir spin S, one models the beta-decay vertex in exactly the same manner as the spin vertices E.(2.228)(a) and (b). This is to say that beta-decay, in both its quarkonic and leptonic manifestations, may be described by the flavor (as opposed to spin) vertices:



E. (2.230)(a)



E. (2.230)(b)

In order to place these diagrams on a firm mathematical footing, one begins in exactly the same manner as in (2.217), namely, with the Pauli matrices, which we now write as:

$$t^{\underline{u}} = \tfrac{1}{2}\left(\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}, \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}, \begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}, \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\right), \tag{2.231}$$

where $t^{\underline{u}}$ is used to indicate that we are no longer using these matrices to describe spacetime spin; rather we are using them to describe flavor isospin. The underlining of indices is used to designate a flavor index, as distinguished from a spacetime index. Corresponding with (2.218) one forms a preonic isospin doublet, and adjoint doublet, given by:

$$\bar{\chi} = (\bar{\Uparrow}\ \bar{\Downarrow}) \quad ; \quad \chi = \begin{pmatrix}\Uparrow\\ \Downarrow\end{pmatrix} \quad . \tag{2.232}$$

From here, the analysis is identical to that performed in the last section in almost all respects. One first obtains eigensolutions for the SU(2)xU(1) flavor matrices (2.231) and assigns these solutions to the doublets (2.232) in exactly the same manner as was done for spin. One is led directly to a table describing certain flavor degrees of freedom, which is identical to Table 2.2 in a number of important respects, but with a few important differences as well. Specifically, one is led to the following table of particle flavor:

		I	I^3	Y	Q	Z
	$\Uparrow$ $\Downarrow$	$\frac{1}{2}$ $\frac{1}{2}$	$\frac{1}{2}$ $-\frac{1}{2}$	$\frac{1}{2}$ $\frac{1}{2}$	1 0	$\frac{1}{2}-\mathrm{Sin}^2\theta_W$ $-\frac{1}{2}$
	$-\bar{\Downarrow}$ $\bar{\Uparrow}$	$\frac{1}{2}$ $\frac{1}{2}$	$\frac{1}{2}$ $-\frac{1}{2}$	$-\frac{1}{2}$ $-\frac{1}{2}$	0 -1	$\frac{1}{2}$ $-\frac{1}{2}+\mathrm{Sin}^2\theta_W$
$(W^{+\mu})$ $(W^{-\mu})$	$-\bar{\Downarrow}\Uparrow$ $\bar{\Uparrow}\Downarrow$	1 1	1 -1	0 0	1 -1	$-\mathrm{Cos}^2\theta_W$ $+\mathrm{Cos}^2\theta_W$
(A^μ)	$\bar{\Uparrow}\Uparrow$ $-\bar{\Downarrow}\Downarrow$	1 1	0 0	0 0	0 0	0 0
(Z^μ)	$\frac{1}{2}\bar{\Uparrow}\Uparrow(\mathrm{Cos}^2\theta_w-\mathrm{Sin}^2\theta_w)$ $-\frac{1}{2}\bar{\Downarrow}\Downarrow$	$\mathrm{Cos}^2\theta_W$	0	0	0	0

Table 2.3 - Isospin Flavor Degrees of Freedom for Real Vector Bosons, and Complex Preonic Flavor Decomposition

where: (compare (2.225))

$$I(I+1) \equiv \underline{t}^{1\,2} + \underline{t}^{2\,2} + \underline{t}^{3\,2} \quad (2.233)(a)$$

$$I^3 \equiv \underline{t}^3 \quad (2.233)(b)$$

$$Y \equiv \underline{t}^0 \quad (2.233)(c)$$

$$Q \equiv \underline{t}^0 + \underline{t}^3 = Y + I^3 , \quad (2.233)(d)$$

$$Z \equiv I^3\mathrm{Cos}^2\theta_W - Y\mathrm{Sin}^2\theta_W = I^3 - Q\ \mathrm{Sin}^2\theta_W \quad (2.233)(e)$$

and where the preonic flavor composition of the Bose particles, is given by

$$W^{+u} \equiv B^u\ (-\bar{\Downarrow}\Uparrow) = |I=1,\ I^3=1,\ Y=0,\ Q=1,\ Z=\mathrm{Cos}^2\theta_W\rangle \quad (2.234)(a)$$

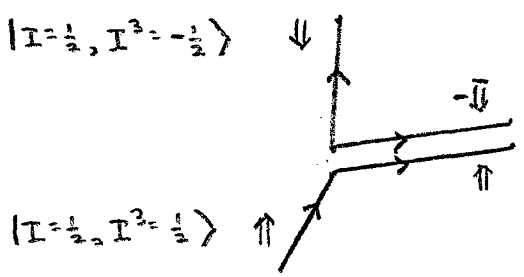
$$W^{-u} \equiv B^u\ (\bar{\Uparrow}\Downarrow) = |I=1,\ I^3=-1,\ Y=0,\ Q=-1, Z=-\mathrm{Cos}^2\theta_W\rangle \quad (2.234)(b)$$

$$A^u \equiv B^u\ (\bar{\Uparrow}\Uparrow) = |I=1,\ I^3=0,\ Y=0,\ Q=0,\ Z=0\rangle \quad (2.234)(c)$$

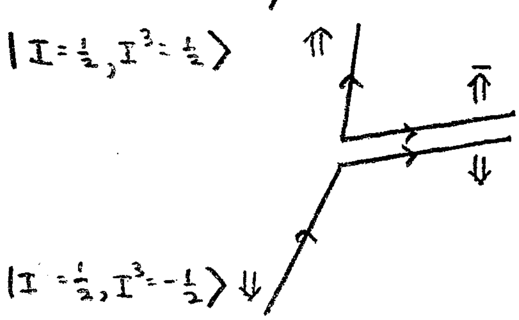
$$Z^u \equiv B^u\ (\tfrac{1}{2}(\bar{\Uparrow}\Uparrow-\bar{\Downarrow}\Downarrow)-\bar{\Uparrow}\Uparrow\mathrm{Sin}^2\theta_w) = B^u\ (\tfrac{1}{2}[\bar{\Uparrow}\Uparrow(\mathrm{Cos}^2\theta_w-\mathrm{Sin}^2\theta_w)-\bar{\Downarrow}\Downarrow]) = |I=\mathrm{Cos}^2\theta_W,\ I^3=0,\ Y=0,\ Q=0,\ Z=0\rangle\ ; \quad (2.234)(d)$$

see (2.227) for comparison.

If we now use the ordered pair (I, I^3) as the primary label for the various flavors of Boson, we find that here, W^{+u}, A^u and W^{-u} form a triplet with respective isospin labels (1,1), (1,0), (1,-1), just as in the case of spin. It is the inclusion of the photon in the same isospin triplet with the $W^{\pm u}$ that in many senses is the central element in electroweak unification. The singlet state however, differs slightly from that for spin symmetry; for isospin, this singlet state has $(\mathrm{Cos}^2\Theta_W, 0)$, where Θ_W, again, is the Weinberg/Glashow electroweak mixing angle. For spin, the singlet state is (0,0). By making the particular set of choices depicted in Table 2.3 and eqs. (2.233),(2.234), it is possible, for the (1,1) and (1,-1) states to arrive identically at the beta decay flavor vertex diagrams:



$$W^{+u} = |I=1, I^3=1, Y=0, Q=1, Z=\mathrm{Cos}^2\Theta_W\rangle \quad (2.235)(a)$$



$$W^{-u} = |I=1, I^3=-1, Y=0, Q=-1, Z=-\mathrm{Cos}^2\Theta_W\rangle \quad (2.235)(b)$$

which are just the diagrams (2.230) that we set out to mathematically represent at the outset, and which are the isospin flavor analogs of the spin vertex diagrams (2.228)(a) and (b). The significant difference between spin and isospin shows up in the "neutral current" Bosons A^u and Z^u. If we set $t^{\underline{u}} = \frac{1}{2}T^{\underline{u}}$ using (2.231), then a particular squared flavor matrix that is very similar to the earlier matrices (2.220) and (2.230) is given by the following:

$$\frac{1}{2}T^{\underline{u}}T_{\underline{u}} = \begin{pmatrix} t_{\underline{0}} + t_{\underline{3}} & t_{\underline{1}} - it_{\underline{2}} \\ t_{\underline{1}} + it_{\underline{2}} & t_{\underline{0}} - t_{\underline{3}} \end{pmatrix} = \begin{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \\ \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \end{pmatrix}, \tag{2.236}$$

noting, when the Pauli matrices are used to describe isospin rather than spin symmetry, that indices are raised and lowered with the Kronecker delta:

$$\delta_{\underline{uv}} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \tag{2.237}$$

see the symmetric structure relationship (2.25)(a). Thus, (2.236) looks a little different that the similar (2.220) used in the discussion of spin. Note That it is the definition of the Coulomb charge Q, written here as :

$$Q \equiv (1 \;\; 0) \begin{pmatrix} t^{\underline{0}} + t^{\underline{3}} & t^{\underline{1}} - it^{\underline{2}} \\ t^{\underline{1}} + it^{\underline{2}} & t^{\underline{0}} - t^{\underline{3}} \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = t^{\underline{0}} + t^{\underline{3}} \tag{2.238}$$

which in essence serves to break the electroweak SU(2)xU(1) gauge symmetry, and results simultaneously in the isospin polarization definition (2.234)(c) for the photon:

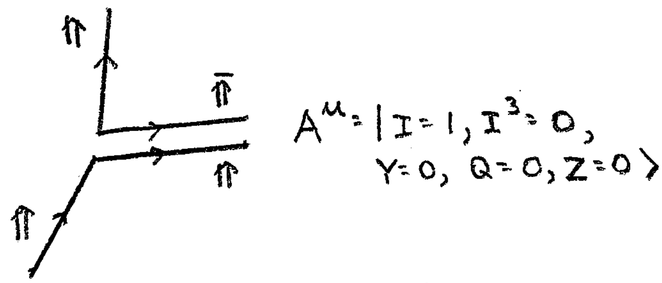
$$A^u = B^u\,(\bar{\chi} Q \chi) = B^u(\bar{\Uparrow}\Uparrow) \tag{2.239}$$

while, using the usual construction (2.233)(e) of the Z generator, which was already introduced in eq. (1.8) of the introductory section, one arrives as the Z^u isospin polarization decomposition: $(I^3 = t^3)$

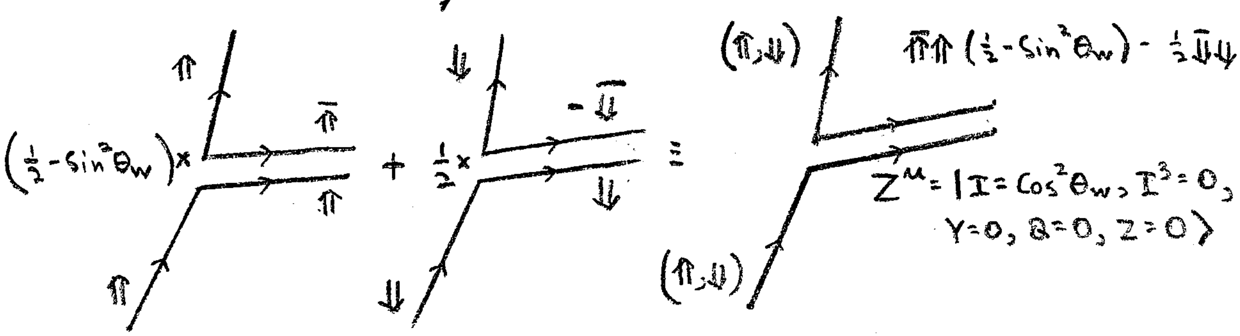
$$Z^u = B^u(\bar{\chi}\,(I^3 - Q\,\mathrm{Sin}^2\theta_W)\,\chi\,) = B^u\left((\tfrac{1}{2} - \mathrm{Sin}^2\theta_W)\bar{\Uparrow}\Uparrow - \tfrac{1}{2}\bar{\Downarrow}\Downarrow\right) \tag{2.240}$$

equation (2.234)(d).

Therefore, the flavor vertex diagrams, which in the "neutral sector" are somewhat different from the neutral current spin vertex diagrams (2.228)(c) and (d), are given by:



(2.241)(a)



(2.241)(b)

Finally, as was done for spin, it is very useful to summarize the $2 \otimes \bar{2} = 3 \oplus 1$ composition of the real Bose particles (W^{+u}, W^{-u}, A^{u}, Z^{u}) out of the preonic states ($\Uparrow, \Downarrow$), as follows:

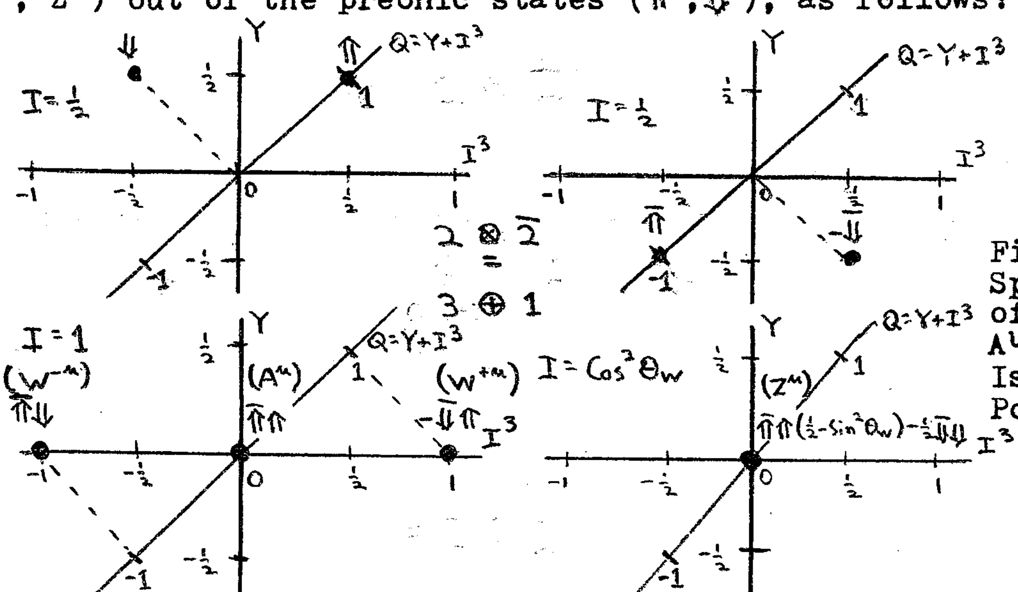


Figure 2.2 - Isospin Space Decomposition of Real W^{+u}, W^{-u}, A^{u}, Z^{u} into Complex Isospin $\Uparrow, \Downarrow$ Polarizations

This should be contrasted with Figure 2.1 for ordinary spin.

It is this specific decomposition that is referred to, when we assert that it is possible to decompose the four real Bose particles of Electroweak theory into two complex preonic states; in a manner which, aside from certain minor modifications in the neutral current sector, is quite similar to the way in which four real spacetime dimensions may be decomposed into two complex spinor states, ie., Fig. 2.1.

The question that now becomes particularly intriguing, given the above manner of constructing the four electroweak Bose particles out of two complex isospin preons, is whether it is possible, through the introduction of additional preons, to construct the various flavors of fermion, ie., quark and lepton, as well? This will comprise the central focus of the discussion in the sections to follow.