Fitting the ^{2}H, ^{3}H, ^{3}He, ^{4}He Binding Energies and the Neutron minus Proton Mass Difference to Parts-Per-Million based Exclusively on the Up and Down Quark Masses

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We report a method for expressing the ^{2}H, ^{3}H, ^{3}He and ^{4}He binding energies and the neutron minus proton mass difference, each independently and each to about parts-per-million accuracy, exclusively as a function of the up and down current quark masses. In the process, the precision with which these quark masses are predicted is improved by a factor of at least six orders of magnitude beyond the best presently-known data.

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1. Introduction

The Koide mass formula [1], [2] provides an extremely precise relationship among the electron (e), muon (μ) and tauon (τ) lepton masses, even though its origins are not fully understood even three decades later. If one defines a diagonalized "Koide matrix" K as:

$$K_{AB} \equiv \begin{pmatrix} \sqrt{m_1} & 0 & 0 \\ 0 & \sqrt{m_2} & 0 \\ 0 & 0 & \sqrt{m_3} \end{pmatrix} \tag{1}$$

and assigns $m_1 = m_e$, $m_2 = m_\mu$ and $m_3 = m_\tau$ to this mass triplet, then Koide's relationship may be written using products of traces $(\mathrm{Tr}K)^2$ and traces of products $\mathrm{Tr}K^2$, as:

$$R = \frac{\left(\sqrt{m_e}+\sqrt{m_\mu}+\sqrt{m_\tau}\right)^2}{m_e+m_\mu+m_\tau} = \frac{(\mathrm{Tr}K)^2}{\mathrm{Tr}K^2} = \frac{K_{AA}K_{BB}}{K_{AB}K_{BA}} \cong \frac{3}{2}. \tag{2}$$

Using $m_e = 0.510998928 \pm 0.000000011 MeV$, $m_\mu = 105.6583715 \pm 0.0000035 MeV$ and $m_\tau = 1776.82 \pm 0.16 MeV$ from the 2012 PDG data [3], we find using mean experimental mass values that this ratio $R = 1.500022828$, which differs from 3/2 by just over two parts per hundred thousand.

Protons and neutrons and other baryons are known to contain what is also a triplet of quarks, each of which is understood to have an associated "current quark mass." For the up (u) and down (d) quarks, PDG most recently values these masses at $m_d = 4.8^{+.7}_{-.3}$ MeV and $m_u = 2.3^{+.7}_{-.5}$ MeV. [4]

This letter reports that the Koide matrix (1) can also be used to formulate relationships for the binding (and related fusion-release) energies of the ^{2}H, ^{3}H, ^{3}He and ^{4}He (1s shell) light nuclides as well as for the neutron (*N*) minus proton (*P*) mass difference which all comport extremely closely to what is observed experimentally, each independently, and all *exclusively* as a function of the up and down current quark masses. In all cases, the accuracy attained is even better than that of Koide's original relationship (2). In the process of fitting together all these mass / energy data points, the prediction of these two quark masses is improved by at least six orders of magnitude beyond what is best-known at present. While the author has described what he believes are the theoretical origins and consequences of these findings in four recent papers [5], [6], [7], [8], the purpose of this letter is strictly to report the objective numeric relationships among phenomenological masses and energies while foregoing any theoretical assertions. This is strictly an evidence report intended to leave latitude for others to independently form modified or alternate conceptions of the physics underlying these multiple, independent, strikingly-close relationships.

2. Mass / Energy Relationships

To use a Koide matrix K_P akin to (1) for a proton (*duu*), we simply assign the Koide masses to the quark masses via $m_1 = m_d$, $m_2 = m_3 = m_u$. For the neutron (*udd*) we make a like assignment $m_1 = m_u$, $m_2 = m_3 = m_d$ to form a K_N. Thus:

$$K_{PAB} \equiv \begin{pmatrix} \sqrt{m_d} & 0 & 0 \\ 0 & \sqrt{m_u} & 0 \\ 0 & 0 & \sqrt{m_u} \end{pmatrix}; \qquad K_{NAB} \equiv \begin{pmatrix} \sqrt{m_u} & 0 & 0 \\ 0 & \sqrt{m_d} & 0 \\ 0 & 0 & \sqrt{m_d} \end{pmatrix}. \tag{3}$$

The non-zero components of the (3x3)(3x3) outer products $K_P \otimes K_P = K_{PAB} K_{PCD}$ and $K_N \otimes K_N = K_{NAB} K_{NCD}$ are m_u , m_d and $\sqrt{m_u m_d}$. It is easily deduced as well that the product of traces:

$$\left(\mathrm{Tr} K_P\right)^2 = K_{PAA} K_{PBB} = m_d + 4\sqrt{m_u m_d} + 4m_u \,, \tag{4}$$

$$\left(\mathrm{Tr} K_N\right)^2 = K_{NAA} K_{NBB} = m_u + 4\sqrt{m_u m_d} + 4m_d \,, \tag{5}$$

and also that the trace of the products:

$$\mathrm{Tr} K_P{}^2 = K_{PAB} K_{PBA} = m_d + 2m_u \,, \tag{6}$$

$$\mathrm{Tr} K_N{}^2 = K_{NAB} K_{NBA} = m_u + 2m_d \,. \tag{7}$$

The latter (6) and (7) specify the sum of current quark masses inside a proton and a neutron and are akin to the denominator in Koide's (2). The former (4) and (5) are akin to the numerator in (2). The only difference is the index summation.

It is fruitful to start by subtracting proton trace product (4) from neutron trace product (5), all divided by $(2\pi)^{1.5}$ *, and to then substitute the PDG values $m_d = 4.8^{+.7}_{-.3}$ MeV and $m_u = 2.3^{+.7}_{-.5}$ MeV. We find:

$$\left((\mathrm{Tr}K_N)^2 - (\mathrm{Tr}K_P)^2\right)/(2\pi)^{1.5} = 3(m_d - m_u)/(2\pi)^{1.5} = 0.476^{+.228}_{-.190} \text{ MeV}. \tag{8}$$

The electron rest mass m_e = 0.510998928 MeV [9] differs from the above by only about 3%. This is well within the wide experimental errors which are just over 20% for the down mass and just over 50% for the up mass. Also, the above expresses a difference between some energy number $(\mathrm{Tr}K_N)^2$ associated with a neutron and a like-energy number $(\mathrm{Tr}K_P)^2$ associated with a proton. Also, neutrons undergo β^- decay into protons by emitting an electron and a virtually-massless antineutrino. Given all of the foregoing, we now introduce a *first postulate*, with no claims attached for the moment, that (8) is actually an *exact* meaningful relationship among the electron, up and down masses, i.e., that (we also show m_e in atomic mass units (AMU)):

$$0.510998928 \text{ MeV}=0.000548579909 \text{ u} = m_e \equiv 3(m_d - m_u)/(2\pi)^{1.5}. \tag{9}$$

We will now proceed to employ this postulate in other relationships which will offer it either contradiction or support.

Next, we note that the lightest mass in the outer products $K_P \otimes K_P$ and $K_N \otimes K_N$ mentioned following (3) is $m_u = 2.3^{+.7}_{-.5} MeV$. We simultaneously note that the deuteron binding energy B (calculated from nuclide masses in [10]) is $B(^2H) = 2.224566$ MeV, which is equal to the up quark mass well within PDG's $^{+.7}_{-.5} MeV$ experimental errors. As a *second postulate* (also to be tested momentarily, making no present claims), we regard the up quark mass to be either identical to the deuteron binding energy, i.e.:

$$m_u \equiv B(^2H) = 2.224566 \text{ MeV}=0.002388170100 \text{ u}, \tag{10}$$

or to be very close thereto (we shall in the end show why these actually appear to differ, but by less than 1 part per million AMU). In making this postulate, we are actually introducing a broader hypothesis that the binding energies of individual nuclides are directly related to the current masses of the quarks which they contain, and that these binding energies can be constructed solely and exclusively from the outer products $K_P \otimes K_P$ and $K_N \otimes K_N$, and

* We note $(2\pi)^{1.5}$ is a natural number resulting from the basic Gaussian integral $(2\pi)^{.5} = \int \exp(-x^2/2)\,dx$ taken over three space dimensions, and that energies in general are calculated from an energy tensor via a three-space integral $E = \iiint T^{00} d^3\mathbf{x}$. This $(2\pi)^{1.5}$ divisor was in fact deduced from such an energy integral and first appeared in (11.16) of [5]. But for purposes of this letter which eschews theoretical discussion in favor of simply reporting data, this may be simply regarded as a natural number which causes various mass and energy data points to all fit together properly, and which could be found to have significance other than that of the context from which it first originated.

specifically, from their traces (4) to (7), their components m_u, m_d and $\sqrt{m_u m_d}$, and in some instances a $(2\pi)^{1.5}$ divisor.

If both of these postulates are true, then (9) and (10) may be combined to deduce a down quark mass valued at:

$$m_d = (2\pi)^{\frac{3}{2}} m_e / 3 + m_u = 4.907244 \text{ MeV} = 0.005268143299 \text{ u} , \tag{11}$$

well within PDG's $m_d = 4.8^{+.7}_{-.3}$ MeV error range. This, together with (10), provides us with up and down quark masses specified at least a million times more accurately than those which are presently-listed by PDG. But are these reliable mass values? Specifically, can we interconnect these two postulated masses, which are well within the PDG error ranges, with other energies or masses which are *empirically-known* on an *independent* basis?

First, using the more precise up and down masses (10), (11) emerging from postulates (9), (10), let us calculate the differences ΔE between the energies represented by $\mathrm{Tr}K^2$ in (6), (7), and those represented by $(\mathrm{Tr}K)^2$ in (4), (5) divided by $(2\pi)^{1.5}$. The results are:

$$\begin{aligned}\Delta E_P \equiv \mathrm{Tr}K_P{}^2 - (\mathrm{Tr}K_P)^2 / (2\pi)^{1.5} &= m_d + 2m_u - \left(m_d + 4\sqrt{m_u m_d} + 4m_u\right) / (2\pi)^{1.5} \\ &= 7.640679 \; MeV = 0.008202607332 \text{ u}\end{aligned} , \tag{12}$$

$$\begin{aligned}\Delta E_N \equiv \mathrm{Tr}K_N{}^2 - (\mathrm{Tr}K_N)^2 / (2\pi)^{1.5} &= m_u + 2m_d - \left(m_u + 4\sqrt{m_u m_d} + 4m_d\right) / (2\pi)^{1.5} \\ &= 9.812358 \text{ MeV} = 0.010534000622 \text{ u}\end{aligned} . \tag{13}$$

We note that the average of these two energies is 8.726519 MeV, and that the binding energies of all but the very lightest and heaviest nuclides are in the range between 8 and 9 MeV per nucleon. From here, we will carry out calculations in AMU rather than MeV to obtain better experimental precision, due to the "relatively poorly known electronic charge." [11] In general, we use empirical data drawn from [11] or [12] or, if not available at these sources, from [13].

First we consider the alpha particle, which is the ^{4}He nucleus. This has *Z*=2 protons and *N*=2 neutrons. If we calculate *Z*=2 times ΔE_P in (12) plus N=2 times ΔE_N in (13) and subtract off $2\sqrt{m_u m_d}$, and if we then compare the result to the empirical binding energy *B* of the alpha particle, we find that:

$$\begin{aligned} 2 \cdot \Delta E_P + 2 \cdot \Delta E_N - 2\sqrt{m_u m_d} &= \mathbf{0.030379212155 \; u} \\ B\left({}^4 He\right) &= \mathbf{0.030376586499 \; u} \\ \text{Difference:} \quad & \mathbf{2.625656 \times 10^{-6} \; u} \end{aligned} , \tag{14}$$

These energies differ from one another by less than 3 parts per million AMU. Keeping in mind that the alpha contains two protons and two neutrons, which together in turn house six up and six down quarks, it is also to be noted that (14) is fully symmetric under both $P \leftrightarrow N$ and $u \leftrightarrow d$ interchange.

Next, consider the ^{3}He nucleus, the helion. Here, we form $\mathrm{Tr}K_P = \sqrt{m_d} + 2\sqrt{m_u}$, multiply this by $\sqrt{m_u}$, and compare to the empirical binding energy *B*. The result is:

$$\begin{array}{ll} \sqrt{m_u}\,\mathrm{Tr}K_P = 2m_u + \sqrt{m_u m_d} & = \mathbf{0.008323342076\ u} \\ B\left({}^3He\right) & = \mathbf{0.008285602824\ u} \\ \text{Difference:} & \mathbf{3.7739252 \times 10^{-5}\ u} \end{array} \quad . \tag{15}$$

These differ by less than 4 parts in 10^5.

Next, we examine the triton, which is the ^{3}H nucleus. Making use of a $(2\pi)^{1.5}$ divisor, here we find that:

$$\begin{array}{ll} 4m_u - 2\sqrt{m_\mu m_d} \,/\, (2\pi)^{\frac{3}{2}} & = \mathbf{0.009102256308\ u} \\ B\left({}^3H\right) & = \mathbf{0.009105585412\ u} \\ \text{Difference:} & \mathbf{-3.329104 \times 10^{-6}\ u} \end{array} \quad . \tag{16}$$

These differ by less than 4 parts in one million.

Thus far we have been examining binding energies, but let's look at fusion-release energies to see if similar close results obtain. First, consider $2P \rightarrow {}^2H$, the fusion of two protons into a deuteron via ${}^1_1H + {}^1_1H \rightarrow {}^2_1H + e^+ + \nu + \text{Energy}$. Here, with *E* representing the *empirical* fusion-release energy, we find that:

$$\begin{array}{ll} 2\sqrt{m_\mu m_d} \,/\, (2\pi)^{\frac{3}{2}} & = \mathbf{0.000450424092\ u} \\ E\left(2P \rightarrow {}^2H\right) & = \mathbf{0.000451141003\ u} \\ \text{Difference:} & \mathbf{-7.16911 \times 10^{-7}\ u} \end{array} \quad . \tag{17}$$

The difference here is less just over 7 parts in ten million.

Now consider ${}^2H + P \rightarrow {}^3H$, which entails fusing a deuteron and proton into a triton via ${}^1_1H + {}^2_1H \rightarrow {}^3_1H + e^+ + \nu + \text{Energy}$. Here, we find:

$$\begin{array}{ll} 2m_u & = \mathbf{0.004776340200\ u} \\ E\left({}^2H + P \rightarrow {}^3H\right) & = \mathbf{0.004780386215\ u} \\ \text{Difference:} & \mathbf{-4.046015 \times 10^{-6}\ u} \end{array} \quad . \tag{18}$$

This is a difference just over 4 parts per million.

In fact, the ^{3}H binding energy (16) is not independent from (17) and (18); rather it is derived from (17) and (18) as shown in the Appendix of [6]. But the other very crucial relationship derived from (17) and (18), which we compare to the observed *neutron minus proton mass difference* $M_N - M_P$ is:

$$\begin{aligned} & m_u - \left(3m_d + 2\sqrt{m_\mu m_d} - 3m_u\right)/\left(2\pi\right)^{\frac{3}{2}} = \mathbf{0.001389166099\ u} \\ & M_N - M_P \qquad = \mathbf{0.001388449188\ u} \\ & \text{Difference:} \qquad \mathbf{7.16911 \times 10^{-7}\ u} \end{aligned} \quad . \tag{19}$$

This inherits the accuracy of what we found in (17), and *appears to describe the neutron minus proton mass difference to just over 7 parts in ten million!*

Given these close relations for the light nuclides, let us also sample a heavier nuclide, $^{56}\mathrm{Fe}$ which has *Z*=26 protons and *N*=30 neutrons, just to gain some confidence that we can also express heavier nuclide binding energies *exclusively* as a function of up and down quark masses. Similarly to the top line of (14), we now calculate $Z \cdot \Delta E_P + N \cdot \Delta E_N$ using (12) and (13), compare this to the empirical ^{56}Fe binding energy in MeV, and then calculate the percentage of the latter over the former, to obtain:

$$\begin{aligned} & 26 \cdot \Delta E_P + 30 \cdot \Delta E_N = \mathbf{493.028394\ MeV} \\ & B\left({}^{56}Fe\right) = \mathbf{492.253892\ MeV} \\ & B\left({}^{56}Fe\right)/\left(26 \cdot \Delta E_P + 30 \cdot \Delta E_N\right) = \mathbf{99.842909\%} \end{aligned} \quad . \tag{20}$$

This is closely related to the observation after (13) that the average of (12) and (13) is 8.726519 MeV, which is also very close to the binding energies per nucleon of many nuclides in the middle of the periodic table. Clearly then, the binding energies of heavier nuclides can also be closely expressed as functions of the up and down current quark masses.

It turns out after thorough examination that ^{56}Fe has the highest $B/\left(Z \cdot \Delta E_P + N \cdot \Delta E_N\right)$ percentage of *all* the nuclides in the periodic table and that *there is no nuclide which exceeds 100%*. It is also worth keeping in mind that the contribution of each neutron to any calculation of an energy number $\mathrm{E} \equiv Z \cdot \Delta E_P + N \cdot \Delta E_N$ via (12) and (13), is greater than each proton contribution by about 28.4%, i.e., by a factor of:

$$\frac{\Delta E_N}{\Delta E_P} = \frac{0.010534000622\,u}{0.008202607332\,u} = 1.284225880325\,, \tag{21}$$

and to juxtapose this with the fact that above ^{4}He, all stable nuclides either have equal numbers of protons and neutrons, or are neutron-rich.

It is also worth noting that as among all of ^{2}H, ^{3}H, ^{3}He and ^{4}He, that the alpha, ^{4}He, is the only nuclide for which the binding energy (14) includes, using *Z*=2 and *N*=2, the energy number $\mathrm{E} \equiv Z \cdot \Delta E_P + N \cdot \Delta E_N$. None of ^{2}H, ^{3}H, ^{3}He contains $\mathrm{E} \equiv Z \cdot \Delta E_P + N \cdot \Delta E_N$, and this fully accounts for why the binding energy is very much higher for ^{4}He than for ^{2}H, ^{3}H and ^{3}He.

Having presented all of the foregoing data, we now return to our second postulate (10) which identified the up quark mass m_u with the deuteron ^{2}H binding energy. We see that the binding energies for all the other 1s nucleons ^{3}H, ^{3}He, ^{4}He, and even the neutron minus proton mass difference itself, as well as the (not independent) $^2H + p \to {}^3H$ and $^2H + p \to {}^3H$ fusion energies and the ^{56}Fe binding energy can also be very closely approximated using only the traces (4) to (7) and components m_u, m_d and $\sqrt{m_u m_d}$ of the outer products $K_P \otimes K_P$ and $K_N \otimes K_N$ formed from Koide matrices (1) to which we assign $m_1 = m_d$, $m_2 = m_3 = m_u$ for the proton and $m_1 = m_u$, $m_2 = m_3 = m_d$ for the neutron, and the divisor $(2\pi)^{1.5}$. These multiple close relationships appear to validate the postulate (10) that nuclear binding energies are in fact directly reflective of the up and down current quark masses confined within the nuclide nucleons, wherein the deuteron, as the very smallest composite nuclide, simply derives its binding energy from the very lightest mass, namely that of the up quark. Because the first postulate (9) for the relationship among the electron, up and down masses was also integrally involved in deducing all of these binding and fusion energy concurrences, this tends to offer retrospective confirmation that (9) does indeed give a correct, physically-meaningful relationship as well. By any objective assessment, the odds against all of these empirical concurrences being wholly coincidental are astronomical.

Retrospectively, noting that the deduced relationships (14) to (19) – while very close – are still not exact within experimental errors, we are now motivated to withdraw the second postulate (10) identifying the up quark mass *exactly* with the deuteron binding energy, and in its place to offer the substitute postulate that the neutron minus proton mass difference is actually the *exact* relationship which drives all the others. That is, we replace (10) with the *substitute postulate* that

$$M_N - M_P = \mathbf{0.001388449188\ u} \equiv m_u - \left(3m_d + 2\sqrt{m_\mu m_d} - 3m_u\right) / \left(2\pi\right)^{\frac{3}{2}} \tag{22}$$

is an exact relationship. We also regard the first postulate in (9) to be confirmed by all of the close relationships (14) through (20), and so now take (9) to be an *exact* relationship among the electron, up and down masses. We then use (9) and (22) to recalibrate the up and down quark masses, and all the binding and fusion-release energy relationships, accordingly.

As a result, the recalibrated quark masses which *by definition* render (22) exact to all decimal places in the empirical $M_N - M_P = 0.001388449188$ u mass difference, are:

$$m_u = 0.002387339327 \text{ u}, \tag{23}$$

$$m_d = 0.005267312526 \text{ u}. \tag{24}$$

As other ways to independently measure quark masses are made more precise beyond the current PDG spreads $m_d = 4.8^{+.7}_{-.3}$ MeV and $m_u = 2.3^{+.7}_{-.5}$ MeV, (23), (24) provide many decimal places at which these quark mass predictions (23), (24) can be strengthened or contradicted.

The recalibrated binding energies, contrast (14), (15) and (16) respectively for ^{4}He, ^{3}He, ^{3}H, now become:

$$2\cdot\Delta E_P+2\cdot\Delta E_N-2\sqrt{m_u m_d}=0.030373002032\text{ u},\tag{25}$$

$$2m_u+\sqrt{m_u m_d}=0.008320783890\text{ u},\tag{26}$$

$$4m_u-2\sqrt{m_\mu m_d}\,/\left(2\pi\right)^{\frac{3}{2}}=0.009099047078\text{ u}.\tag{27}$$

Additionally, because the up and down masses have now been recalibrated by less than one part per million in AMU, the observed ^{2}H deuteron binding energy $B\left({}^2H\right)$=0.002388170100 u is no longer *exactly* equal to the mass of up quark, but instead differs as shown below:

$$\begin{array}{ll} m_u & =\mathbf{0.002387339327\,u} \\ B\left({}^2H\right) & =\mathbf{0.002388170100\,u} \\ \text{Difference:} & \mathbf{-8.30773\times10^{-7}\,u} \end{array}\quad.\tag{28}$$

Following recalibration, the accuracy to less than one part per million of the originally-derived neutron minus proton mass difference has migrated instead to a difference of less than one part per million between the up quark mass and the deuteron binding energy. The difference between the binding energies "retrodicted" by (25) to (28), and those actually observed empirically, is shown in Table 1 below, (which is Figure 11 of [6]) with diagonal lines representing nuclear isobars of like *A*=*Z*+*N*.

$B_{\text{retrodicted-observed}}$	$_Z$Nuclide	$_0$n	$_1$H	$_2$He	
N				A=1	
0			0.000000000000	A=2	A=3
1		0.000000000000	-0.000000830773	0.000035181066	A=4
2	A=1	A=2	-0.000006538334	-0.000003584467	
3		A=3	A=4		

Table 1: Retrodicted Minus Observed Binding Energies ($^{A}_{Z}\mathbf{B}_0$) of 1s Nuclides (AMU)

This close fitting is what retrospectively validates the quark masses (23), (24), the neutron minus proton mass difference (22), and the up and down and electron mass relationship (9), upon all of which this fitting is based. Any substantial alteration in these four relationships would adversely affect the fit in Table 1.

It is also to be noted that the various relationships set forth throughout this letter can be combined to show that:

$$\begin{aligned}&\text{Energy}\left(4\cdot{}^1_1H+2e^-\rightarrow{}^4_2He+\gamma(12.79MeV)+2\gamma(5.52MeV)+2\gamma(.42MeV)+4\gamma(e)+2\nu\right)\\&=4m_u+6m_d-2\sqrt{m_u m_d}+\frac{2m_d-22m_u-12\sqrt{m_u m_d}}{\left(2\pi\right)^{\frac{3}{2}}}=26.7334\,MeV\end{aligned}\quad.\tag{29}$$

This expresses the 26.73 MeV of energy empirically-observed to be released during a single solar fusion event whereby four protons are fused into an alpha particle, solely as a function of the up and down quark masses, also to parts per million in AMU.

If all of these relationships are in fact meaningful, this means that we now have predicted values (23), (24) for the up and down current quark masses to a precision in AMU which is at least six orders of magnitude more accurate than what is known from present data about these masses, and that the nuclear binding energies and the neutron minus proton mass difference are actually providing us with very clear "signals" as to the quark masses confined inside the various nuclides.

3. Conclusion

This letter simply reports that relationships involving the square roots of the quark masses modeled on what Koide has done for charged leptons: 1) enable the binding energies for all of the ^{2}H, ^{3}H, ^{3}He and ^{4}He nuclides (and related $2P \rightarrow {}^2H$ and ${}^2H + P \rightarrow {}^3H$ fusion-release energies) to be specified to near parts-per-million precision as a function *exclusively* of up and down quark masses; 2) allow derivation of a postulated precise relationship for the neutron minus proton mass difference; 3) retrospectively support another postulated precise relationship among the up, down and electron masses; 4) also appear to work well for heavier nuclides based on the example of ^{56}Fe; 5) seem to suggest that the binding energies of *all* nuclides are definitively related *on an exclusive basis* to the current quark mass contents of those nuclides, to at least through the first several orders of magnitude of precision; and 6) enable the current quark masses themselves to be specified with an extremely high degree of precision which is rooted in and inherits the precision with which the proton, neutron and electron masses are known.

Based on this, it seems clear that Koide-style matrices of the form (1) and relationships built out of these do correctly capture some underlying reality as to a substantial variety of mass / energy relationships. While the author has well-formed views elaborated in [5], [6], [7], [8] as to the theoretical foundations upon which these very accurate empirical retrodictions of nuclear binding and fusion energies and the neutron minus proton mass difference may rest, as well as to some of the possible consequences, he has foregone any discussion of those views in this letter, in favor of simply reporting these results starting from earlier, separate work by Koide in [1], [2] which is also empirically accurate, but which has to date been given no discernible theoretical roots. The author's forbearance from theoretical discussions here, is intended to enable others in the nuclear and particle physics communities to evaluate these results based on the data alone, and perhaps develop modified or alternative theories as to the physics which might be underlying these very accurate empirical retrodictions.

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