16. Theoretical Relation amongst the Higgs Mass and the Isospin-Up and Isospin-Down Quark vevs; and the Two-Minimum, Two Maximum Lagrangian Potential for Quarks

When we first introduced the postulate of a second vev for the isospin-down quarks, this was speculative. But because this postulate led to the connection $\theta_{I\Downarrow 21} \equiv \theta_{C12} = 12.975 \pm 0.026\,°$ with observed empirical data at (15.6) in addition to those connections already found for θ_{C23} and θ_{C13} at (14.13), this provided confirming evidence of this second vev. So, now that the empirical data apparently indicates that these two vevs do exist, it is important to understand how they are theoretically tied together. This brings us to the Lagrangian potential.

In the standard model for a U(1) gauge symmetry with a complex scalar field ϕ_h, the Lagrangian potential is written as $V = \mu^2 \phi_h{}^* \phi_h + \lambda\left(\phi_h{}^* \phi_h\right)^2 + \ldots$ with higher-order terms above $\left(\phi_h{}^* \phi_h\right)^2$ neglected. But if there is now to be a second minimum at $v_\Downarrow$, we can no longer neglect these higher order terms, because they will need to be responsible for providing this second minimum. Thus, it is desirable to start by briefly reviewing how the Fermi vacuum is used to establish the vev at $v_\Uparrow = v$ in the standard model, as laid out, e.g., in sections 14.6 through 14.8 including Figure 14.3 of [**ref**]. Then we will take on the task of introducing a second vev at $v_\Downarrow$.

In the standard model, with a gauge-covariant derivative $D_\mu \equiv \partial_\mu + ieA_\mu / \hbar c$ and potential $V = \mu^2 \phi_h{}^* \phi_h + \lambda\left(\phi_h{}^* \phi_h\right)^2 + \ldots$ for complex scalar field ϕ_h, one starts with a Lagrangian density:

$$\begin{aligned}\mathcal{L} = &\left(\left(\hbar c \partial_\mu + ieA_\mu\right)\phi_h\right)^* \left(\left(\hbar c \partial^\mu + ieA^\mu\right)\phi_h\right) - \mu^2 \phi_h{}^* \phi_h - \lambda\left(\phi_h{}^* \phi_h\right)^2 - \tfrac{1}{4} F^{\mu\nu} F_{\mu\nu} \\ &+ \tfrac{1}{2} m_A{}^2 c^4 A^\mu A_\mu - \tfrac{1}{2} m_{\phi h}{}^2 c^4 \phi_h{}^2\end{aligned} \quad . \tag{16.1}$$

In the above, for illustration only, in the lower line we have also included hand-added terms $+\tfrac{1}{2} m_A{}^2 c^4 A^\mu A_\mu$ for a massive gauge boson of mass m_A and $-\tfrac{1}{2} m_{\phi h}{}^2 c^4 \phi_h{}^2$ for a massive scalar boson of mass $m_{\phi h}{}^2$. In four spacetime dimensions, $\mathcal{L}$ has a mass dimensionality of +4, i.e., it has dimensions of energy to the fourth power. We then use $\phi_h = \tfrac{1}{\sqrt{2}}\left(\phi_{1h} + i\phi_{2h}\right)$ as reviewed at (13.4) to introduce the real and imaginary parts of ϕ_h, and then break symmetry in favor of ϕ_{1h} by setting $\phi_{2h} = 0$. Moreover, we also remove the hand-added vector and scalar boson mass terms entirely by setting $m_A = 0$ and $m_{\phi h} = 0$ in the above, in favor of revealing these terms by other means. This is exactly how at (10.6) we set $\hat{\mathcal{L}}_{\mathrm{M}} = 0$ in (10.4) to require that the matter Lagrangian must emerge from the DKK geometry, as it did from the reassignment $\hat{\mathcal{L}}_{\mathrm{M}} \equiv \left(1/2\mathrm{K}\right)\hat{R}^5{}_5$ in (10.5). This is also exactly how at (11.3) we set $m = 0$ in (11.1) to likewise require that the fermion mass must emerge entirely from DKK geometry embodied in Γ_5 and the fifth energy-momentum component cp^5, as it did via the reassignment $-mc^2U_0 \equiv \Gamma_5 cp^5 U_0 = \left(\gamma_5 cp^5 + \phi\gamma_0 cp^5\right)U_0$ in (11.2).

The upshot of all of these steps that that we end up setting $\phi_h = \frac{1}{\sqrt{2}}\phi_{1h}$ in (16.1) while removing the illustrative mass terms, to next arrive at:

$$\mathcal{L} = \tfrac{1}{2}\hbar^2 c^2 \partial_\mu \phi_{1h} \partial^\mu \phi_{1h} + \tfrac{1}{2}e^2 {\phi_{1h}}^2 A_\mu A^\mu - \tfrac{1}{2}\mu^2 {\phi_{1h}}^2 - \tfrac{1}{4}\lambda {\phi_{1h}}^4 - \tfrac{1}{4}F^{\mu\nu}F_{\mu\nu}. \tag{16.2}$$

The potential contained in the above has now become $V = \frac{1}{2}\mu^2 {\phi_{1h}}^2 + \frac{1}{4}\lambda {\phi_{1h}}^4$, expressed with ϕ_{1h} rather than ϕ_h as the domain points. It should also be cross-noted, and will become important shortly, that this very same ϕ_{1h} is the range variable in (13.9) and Figure 1 for the top quark, and likewise for the other quarks in the manner reviewed near the end of the previous section.

The next step is to find the stationary points of *V*, i.e., those domain points at which $V' = \partial V / \partial \phi_{1h} = \phi_{1h}\left(\mu^2 + \lambda {\phi_{1h}}^2\right) = 0$. Clearly, these points occur where $\phi_{1h} = 0$ and $\mu^2 + \lambda {\phi_{1h}}^2$, the latter of which means that ${\phi_{1h}}^2 = -\mu^2 / \lambda$. We assign the energy of this latter stationary point to the Fermi vev by definition, by setting ${\phi_{1h}}^2 = -\mu^2 / \lambda \equiv v^2$, thus $\mu^2 = -\lambda v^2$. Although the square root of ${\phi_{1h}}^2$ can be taken with either of the two possible sign choices $\phi_{1h} = \pm v$, because we are breaking symmetry we choose only one, customarily the positively signed $\phi_{1h} = +v$.

Next, we introduce the Higgs field as also reviewed at (13.4) by expanding ϕ_{1h} in (16.2) about the Fermi vev using $\phi_{1h} = v + h$, with this $\phi_{1h} = +v$ choice. We then substitute $\phi_{1h} = v + h$ into (16.2) along with $\mu^2 = -\lambda v^2$ just deduced. After consolidating terms, this produces:

$$\begin{aligned} \mathcal{L} = {} & \tfrac{1}{2}\hbar^2 c^2 \partial^\mu h \partial_\mu h - \lambda v^2 h^2 - \lambda v h^3 - \tfrac{1}{4}\lambda h^4 + \tfrac{1}{4}\lambda v^4 \\ & + \tfrac{1}{2}e^2 v^2 A^\mu A_\mu + \tfrac{1}{2}e^2 h^2 A^\mu A_\mu + e^2 v h A^\mu A_\mu - \tfrac{1}{4}F^{\mu\nu}F_{\mu\nu} \end{aligned}. \tag{16.3}$$

Comparing with (16.1) which makes us mindful that the Lagrangian term for a massive vector boson is expected to take the form $+\frac{1}{2}{m_A}^2 c^4 A^\mu A_\mu$, we identify $m_A = ve$ as the mass of this boson, so as to now replace the hand-added mass term with one that emerged naturally from the Higgs field expansion. In non-Abelian gauge theory, for example weak interaction SU(2), an additional factor of ½ emerges so that the natural unit mass identification becomes $m_W = \frac{1}{2}vg$ in relation to the coupling strength *g*. More importantly for present purposes, because the term for a massive scalar boson is expected to take the form $-\frac{1}{2}{m_{\phi h}}^2 c^4 {\phi_h}^2$ as also illustrated by this hand-added mass term in (16.1), from the term $-\lambda v^2 h^2$ above we also identify *h* with the Higgs boson itself, and $\frac{1}{2}{m_h}^2 c^4 = \lambda v^2$ with the energy equivalent of the Higgs boson mass. Thus, we have also replaced the hand-added scalar mass with a mass that likewise arises naturally from the Higgs expansion.

Restructured, $\lambda = m_h^{\,2}c^4 / 2v^2$ informs us that the parameter λ is undetermined unless and until we know the mass of the Higgs boson.

As a result of the foregoing, focusing now on the potential $V = \frac{1}{2}\mu^2\phi_{1h}^{\;2} + \frac{1}{4}\lambda\phi_{1h}^{\;4}$ and its derivative $V' = \partial V / \partial\phi_{1h} = \phi_{1h}\left(\mu^2 + \lambda\phi_{1h}^{\;2}\right)$, and using $v_{\Uparrow} \equiv v$ to start distinguishing the Fermi vev from the $v_{\Downarrow}$ vev, we employ $\mu^2 = -\lambda v_{\Uparrow}^{\;2}$ and $\lambda = m_h^{\,2}c^4 / 2v_{\Uparrow}^{\;2}$ to rewrite these as:

$$\begin{aligned}
V\left(\phi_{1h}\right) &= \frac{1}{2}\mu^2\phi_{1h}^{\;2} + \frac{1}{4}\lambda\phi_{1h}^{\;4} = \lambda\left(-\frac{1}{2}v_{\Uparrow}^{\;2}\phi_{1h}^{\;2} + \frac{1}{4}\phi_{1h}^{\;4}\right) = -\frac{1}{4}m_h^{\,2}c^4\phi_{1h}^{\;2} + \frac{1}{8}\frac{m_h^{\,2}c^4}{v_{\Uparrow}^{\;2}}\phi_{1h}^{\;4} \\
&= m_h^{\,2}c^4\left(-\frac{1}{4}\phi_{1h}^{\;2} + \frac{1}{8}\frac{1}{v_{\Uparrow}^{\;2}}\phi_{1h}^{\;4}\right) \\
V' &= \frac{\partial V}{\partial\phi_{1h}} = \phi_{1h}\left(\mu^2 + \lambda\phi_{1h}^{\;2}\right) = \lambda\phi_{1h}\left(\phi_{1h}^{\;2} - v_{\Uparrow}^{\;2}\right) = \frac{m_h^{\,2}c^4}{2v_{\Uparrow}^{\;2}}\phi_{1h}\left(\phi_{1h}^{\;2} - v_{\Uparrow}^{\;2}\right)
\end{aligned} \quad . \qquad (16.4)$$

If we calculate the second derivative then ascertain its value at each of the stationary points $\phi_{1h} = 0$ and $\phi_{1h} = v_{\Uparrow}$, we obtain $V''\left(\phi_{1h} = 0\right) = -m_h^{\,2}c^4 / 2$ and $V''\left(\phi_{1h} = v_{\Uparrow}\right) = +m_h^{\,2}c^4$, from which we discern that the potential has a maximum at $\phi_{1h} = 0$ and a minimum at $\phi_{1h} = v_{\Uparrow}$. This *V*, of course, is the customary "Mexican hat" potential of the standard model Higgs sector.

Moving on from this review, empirically, $v_{\Uparrow} = v = 246.2196508 \pm 0.0000633\text{ GeV}$ is obtained from the Fermi constant G_F. We calculated $v_{\Downarrow} = 6.0491^{+0.0571}_{-0.0430}\text{ GeV}$ at (15.11) from the sum $\frac{1}{\sqrt{2}}v_{\Downarrow} = m_d c^2 + m_s c^2 + m_b c^2 = 4.2773^{+0.0404}_{-0.0304}\text{ GeV}$ of the isospin-down quarks, with two extra decimal places shown to display the impact of the strange and down masses which are known more precisely then the bottom mass, see (15.2), then (15.11) and the discussion following. And while over four decades passed between when the Higgs boson was first postulated and when it was finally observed, today we have experimental data showing the Higgs boson to have a rest energy $m_h c^2 = 125.18 \pm 0.16\text{ GeV}$, see PDG's [i]. It is noteworthy, and will momentarily become important, that $m_h c^2$ is just a touch larger than half the Fermi vev, and to be precise, that $m_h c^2 = v_{\Uparrow} / 2 + 2.07 \pm 0.16\text{ GeV}$. Also, because we now know the Higgs mass empirically, we may deduce that the undetermined parameter $\lambda = m_h^{\,2}c^4 / 2v_{\Uparrow}^{\;2} = 0.1292 \pm 0.0003$. Were the Higgs $m_h c^2$ to be exactly equal to half the Fermi vev, we would have $\lambda = 1/8$. The consequences of this slight deviation from $\lambda = 1/8$ are important, and will drive many of the results now to be reviewed. Finally, using the center values of the data for m_h and $v_{\Uparrow}$, the upper (16.4) yields the range value $V\left(\phi_{1h}^{\;2} = v_{\Uparrow}^{\;2}\right) = -\frac{1}{8}m_h^{\,2}c^4 v_{\Uparrow}^{\;2} = -\left(104.39\text{ GeV}\right)^4$ for the potential at the minimum $\phi_{1h} = v_{\Uparrow}$.

Now, as noted just above, the Higgs mass in $m_h c^2 = v_{\Uparrow}/2 + 2.07 \pm 0.16\ \text{GeV}$ is slightly above the halfway point between zero and the Fermi vev $v_{\Uparrow} = 246.2196508 \pm 0.0000633\ \text{GeV}$. Another way to say this is that twice the Higgs mass is $2m_h c^2 = v_{\Uparrow} + 4.14 \pm 0.32\ \text{GeV}$, exceeding this vev by $4.14 \pm 0.32\ \text{GeV}$. Comparing $\frac{1}{\sqrt{2}} v_{\Downarrow} = m_d c^2 + m_s c^2 + m_b c^2 = \frac{1}{\sqrt{2}} v_{\Downarrow} = 4.28^{+0.04}_{-0.03}\ \text{GeV}$ from (15.11) with the two extra decimal places removed and the error range now set by the bottom quark mass which is least-tightly-known, we see that these two numbers match up *within experimental errors*. This means that within experimental errors, *the Higgs mass is exactly halfway between* $\frac{1}{\sqrt{2}} v_{\Downarrow} = 4.28^{+0.04}_{-0.03}\ \text{GeV}$ *and* $v_{\Uparrow} = 246.2196508 \pm 0.0000633\ \text{GeV}$, with the errors set by the former. Or, put differently, *if we now theoretically define the Higgs mass to be the average of* $v_{\Uparrow} = v = \sqrt{2}\left(m_u c^2 + m_c c^2 + m_t c^2\right)$ *and* $\frac{1}{\sqrt{2}} v_{\Downarrow} = m_d c^2 + m_s c^2 + m_b c^2$ *using the data from (15.11), we find that this relation* $m_h c^2 = \left(v_{\Uparrow} + \frac{1}{\sqrt{2}} v_{\Downarrow}\right)/2$, *expressed as:*

$$\boxed{\begin{aligned} 125.18 \pm 0.16\ \text{GeV} = m_h c^2 &\equiv \tfrac{1}{2}\left(v_{\Uparrow} + \tfrac{1}{\sqrt{2}} v_{\Downarrow}\right) = 125.25 \pm 0.02\ \text{GeV} \\ &= \tfrac{1}{2}\left(\sqrt{2}\left(m_u c^2 + m_c c^2 + m_t c^2\right) + m_d c^2 + m_s c^2 + m_b c^2\right) \end{aligned}}, \tag{16.5}$$

is true within experimental errors. The question now becomes whether $m_h c^2 = \frac{1}{2}\left(v_{\Uparrow} + \frac{1}{\sqrt{2}} v_{\Downarrow}\right)$ above really is a relation of genuine physical significance, or is just a coincidence. There are a number of good reasons we shall now review, why this is likely a real relation:

First, if $V(\phi_h)$ is to have a second (local, shallower) minimum at $\phi_{1h} = v_{\Downarrow}$ to provide a "nest" for isospin-down quarks along with its first (global, deeper) minimum at $\phi_{1h} = v_{\Uparrow}$ where isospin-up quarks are "nested," as well as its usual maximum at $\phi_{1h} = 0$, then it *must* now also have a *second maximum* at some definitive $v_{\Downarrow} < \phi_{1h} < v_{\Uparrow}$ in between the two minimum points. This is not optional: elementary calculus demands that if a function has two minima, it inexorably must have a maximum somewhere between these two minima.

Second, given this required $v_{\Downarrow} < \phi_{1h} < v_{\Uparrow}$ domain for the second maximum, it is not insensible in the present context for the maximum to be reasonably close to the halfway point between $v_{\Downarrow}$ and $v_{\Uparrow}$.

Third, given the requirement for a maximum in the domain $v_{\Downarrow} < \phi_{1h} < v_{\Uparrow}$, just as $v_{\Uparrow}$ and $v_{\Downarrow}$ are physically meaningful numbers, we expect that the energy of ϕ_{1h} at this second maximum should have some physical meaning, for example, that it should be, or should at least be "based on," the rest mass or mass sum of an elementary particle or particles. The empirical rest masses of significance between $v_{\Downarrow}$ (about 6 GeV) and $v_{\Uparrow}$ (about 246 GeV) are the top quark mass, the masses M_W and M_Z of the electroweak vector bosons, and the Higgs mass. The top mass and the electroweak bosons are theoretically accounted for in other ways, so we will make an educated guess that the second maximum is based on the Higgs mass itself.

Fourth, if this maximum is to be close to the halfway point between $v_{\Downarrow}$ and $v_{\Uparrow}$, and is to be based on the Higgs mass, then $m_h c^2 = \frac{1}{2}\left(v_{\Uparrow} + \frac{1}{\sqrt{2}} v_{\Downarrow}\right)$ in (16.5) is indeed a good halfway point, because $v_{\Downarrow} \ll v_{\Uparrow}$. So, we infer that the Higgs itself mass may provide one suitable halfway point, whereby the maximum occurs at $\phi_{1h} = m_h c^2$, just above halfway. Another suitable halfway point would be at $\phi_{1h} = v_{\Uparrow} - m_h c^2$, just below halfway. Both of these are clearly "based on" the Higgs mass. We will momentarily asses which of these options makes better physical sense.

Fifth, the Higgs mass itself and the related parameter $\lambda = m_h{}^2 c^4 / 2v_{\Uparrow}{}^2$ have long been entirely unexplained as a theoretical matter. Given that we now have good empirical data for the Higgs mass, and that $m_h c^2 = \frac{1}{2}\left(v_{\Uparrow} + \frac{1}{\sqrt{2}} v_{\Downarrow}\right)$ is confirmed by that data within experimental errors, regarding (16.5) as a new, correct theoretical relation of physical significance would allow us after more than four decades to finally place the value of λ on an entirely theoretical basis, as we shall further detail momentarily.

Sixth, the empirical data on the right in (16.5) has a tighter error bound than the data on the left: $m_h c^2 = 125.25 \pm 0.02 \text{ GeV}$ is tighter then the presently-known $m_h c^2 = 125.18 \pm 0.16 \text{ GeV}$ by a factor of almost 10, and raises the center value by $.07 \text{ GeV}$. Thus, if we regard (16.5) as true, this contains a *prediction* that when the Higgs mass becomes measured more tightly than at present, it will fit in the range $m_h c^2 = 125.25 \pm 0.02 \text{ GeV}$. So, this sixth reason to regard (16.5) as a true physical relation on at least a provisional basis, is that it can likely be *experimentally tested* in the foreseeable future.

Seventh and finally, following (15.13) we noted using the parameterization $m_d, m_s, m_b = F\left(m_d, m_s, \theta_{C21}\right)$ or alternatively $m_d, m_s, m_b = F\left(m_d, m_b, \theta_{C21}\right)$ that we had squeezed one degree of freedom from the isospin-down quark masses via the first relation (15.12) for the CKM mixing angle θ_{C21} and the Yukawa couplings for these masses. With the discovery of (16.5), we now have a basis for expressing the previously-undetermined number $v_{\Downarrow}$ as a function $v_{\Downarrow} = F\left(v_{\Uparrow}, m_h\right)$. In other words, given the Higgs mass and the Fermi vev, we may deduce $v_{\Downarrow} = \sqrt{2}\left(m_d c^2 + m_s c^2 + m_b c^2\right)$ from $v_{\Uparrow}$ and m_h via (16.5). This means that if we choose to regard the Higgs mass as a "given" number, related to the two vevs by (16.5), we can squeeze yet another unexplained energy number out of the parameters which drive the natural world. This would enable us to remove m_s or m_b from the above parameterizations and now write $m_d, m_s, m_b = F\left(m_d, m_h, \theta_{C21}\right)$ for the isospin-down quark masses. Together with $m_u, m_c, m_t = F\left(v, \theta_{C31}, \theta_{C23}\right)$, this would mean that *we can now eliminate five (5) out of the six unexplained quark masses* and "explain" these as they relate to θ_{C21}, θ_{C23}, θ_{C31}, v, and m_h, leaving only m_d now unexplained. Of course, this would not explain why the five parameters θ_{C21}, θ_{C23}, θ_{C31}, v, and m_h have the empirical values that they have. But this would explain how these are related to the quark masses and so render five of these six quark mass numbers into the status of "redundant" data.

Accordingly, for all the reasons just reviewed, we shall now regard $m_h c^2 = \tfrac{1}{2}\left(v_{\Uparrow} + \tfrac{1}{\sqrt{2}} v_{\Downarrow}\right)$ in (16.5) to be a true theoretical physical relation for the Higgs mass, and will use the tighter, raised-center value $m_h c^2 = 125.25 \pm 0.02$ GeV for this mass from here on.

Next, turning to the square roots of masses, if we write (16.5) as:

$$\left(\sqrt{v_{\Uparrow}} / c\right)^2 + \left(\sqrt{v_{\Downarrow}} / \sqrt[4]{2}c\right)^2 = \left(\sqrt{2m_h}\right)^2, \tag{16.6}$$

we see a Pythagorean relation amongst $\sqrt{v_{\Uparrow}} / c$, $\sqrt{v_{\Downarrow}} / \sqrt[4]{2}c$ and $\sqrt{2m_h}$, with the former two on the legs of a right triangle and the latter on the hypotenuse. This can be used to define an angle:

$$\sin\theta_v \equiv \frac{\sqrt{v_{\Downarrow}} / \sqrt[4]{2}c}{\sqrt{2m_h}}; \quad \cos\theta_v = \frac{\sqrt{v_{\Uparrow}} / c}{\sqrt{2m_h}}; \quad \tan\theta_v = \frac{\sqrt{v_{\Downarrow}}}{\sqrt[4]{2}\sqrt{v_{\Uparrow}}}, \tag{16.7}$$

wherein θ_v effectively measures the magnitude of each of the two vevs in relation to one another and the Higgs mass. Using the data from (15.11) and (16.5) we calculate that the central value for this angle is $\theta_v = 6.3085°$. This can all be represented in the rather simple geometric Figure below:

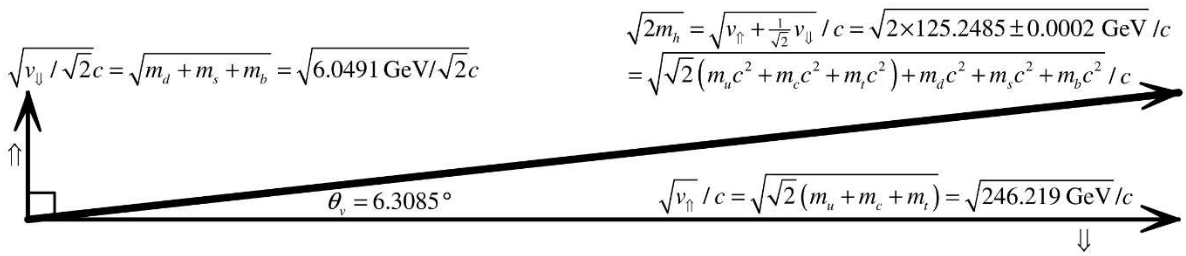

Figure 4: Vacuum and Higgs Mass Mixing in Quark Rest Mass Space

Then, as noted in the fifth reason reviewed above, by advancing (16.5) to a meaningful relation, we also can deduce that the long-undetermined parameter λ in $V = \tfrac{1}{2}\mu^2 \phi_{1h}{}^2 + \tfrac{1}{4}\lambda \phi_{1h}{}^4 + \ldots$ is theoretically given, also using (16.7), by:

$$\lambda = \frac{m_h{}^2 c^4}{2 v_{\Uparrow}{}^2} = \frac{\left(v_{\Uparrow} + \tfrac{1}{\sqrt{2}} v_{\Downarrow}\right)^2}{8 v_{\Uparrow}{}^2} = \frac{1}{8}\left(1 + \frac{1}{\sqrt{2}} \frac{v_{\Downarrow}}{v_{\Uparrow}}\right)^2 = \frac{1}{8}\left(1 + \tan^2\theta_v\right)^2 = 0.12938 \pm 0.0004. \tag{16.8}$$

So physically, using Figure 4 and (16.8), λ and $\theta_v = \tan^{-1}\left(\sqrt{v_\Downarrow} / \sqrt{v_\Uparrow}\sqrt[4]{2}\right)$ are now understood as measures of the ratio $v_\Downarrow / v_\Uparrow$ of the two vevs. In the limiting case where $\theta_v \to 0$, we also have $\lambda \to 1/8$, $m_h c^2 \to \frac{1}{2} v_\Uparrow$ and $v_\Downarrow \to 0$. With $v_\Downarrow \to 0$, this also causes all of the isospin-down quark masses $m_\Downarrow c^2 = \frac{1}{\sqrt{2}} v_\Downarrow G_\Downarrow \to 0$ to approach zero, allocating all mass to the isospin-up quarks.

Given this, it is important to understand how Figures 2, 3 and 4 all tie together, wherein the Higgs rest energy is distributed into the two quark vevs in accordance with Figure 4, with these two vevs then parceling out their energies into the rest energies for each quark in their sector as illustrated in Figures 2 and 3, via the bi-unitary mass rotations that we started to develop at (14.9). Specifically: In Figure 2, $\left(\sqrt{v_\Uparrow / c}\right) / \sqrt[4]{2}$ was the hypotenuse projected into each of three isospin-up mass roots and in Figure 3, $\left(\sqrt{v_\Downarrow / c}\right) / \sqrt[4]{2}$ was the hypotenuse projected into each of three isospin-up mass roots. This means that (16.6) and Figure 4 are the bridge between the two spaces in Figures 2 and 3, in a mass square root space that is overall *six dimensional*, as also seen from the bottom line of (16.5). So, with the coefficients and square roots as shown, geometrically, one starts with the Higgs mass m_h which is placed along the hypotenuse in Figure 4. This Higgs mass hypotenuse is then projected onto the two orthogonal axes, represented with $\Uparrow$ and $\Downarrow$ for isospin-up and isospin-down, to arrive at the related vevs $v_\Uparrow$ and $v_\Downarrow$. Then, in three of the six dimensions $v_\Uparrow$ is further projected into the masses for the top, charm and up quarks as shown in the not-to-scale Figure 2, and in the other three of six dimensions $v_\Downarrow$ is projected into the bottom, strange and down masses as shown in Figure 3. The azimuthal and polar angles in the former, and the azimuthal angle in the latter, simultaneously are the three real CKM mixing angles.

Now we come to the point touched upon in the second through fourth reasons why (16.5) should be regarded as a true physical relation. From these, we discerned that $\phi_{1h} = m_h c^2$ and alternatively $\phi_{1h} = v_\Uparrow - m_h c^2$ can suitably serve as the domain points for the Lagrangian potential maximum, because each is approximately halfway between $v_\Downarrow$ and $v_\Uparrow$, and because each is clearly based upon the Higgs mass. In the former, $\phi_{1h} = m_h c^2$ is at the energy equivalent of the Higgs mass. Because $\phi_{1h} = v_\Uparrow + h$, the latter implies $h = -m_h c^2$, so that the energy of the Higgs field at the maximum domain point is equal to the negative energy equivalent of Higgs mass. Note from (13.5) and Figure 1, that the Higgs field energy is always negative close to a fermion. Now the question is: which of these two alternatives makes the best physical sense?

Using $m_h c^2 = \frac{1}{2}\left(v_\Uparrow + \frac{1}{\sqrt{2}} v_\Downarrow\right) = 125.25 \pm 0.02 \text{ GeV}$ from (16.5), and mindful of $\phi_{1h} = v_\Uparrow + h$, for the former alternative there would be a Lagrangian potential maximum at the domain point:

$$\begin{aligned} &\phi_{1h}\left(x^{\mathrm{M}}\right) \equiv m_h c^2 = \tfrac{1}{2}\left(v_\Uparrow + \tfrac{1}{\sqrt{2}} v_\Downarrow\right) = 125.25 \pm 0.02 \text{ GeV}; \\ &\text{i.e. } h\left(x^{\mathrm{M}}\right) = \phi_{1h}\left(x^{\mathrm{M}}\right) - v_\Uparrow = m_h c^2 - v_\Uparrow = \tfrac{1}{2}\left(\tfrac{1}{\sqrt{2}} v_\Downarrow - v_\Uparrow\right) = -120.97 \pm .02 \text{ GeV} \end{aligned} . \tag{16.9a}$$

But again, while the calculus demands that there be a maximum *somewhere* in the domain $v_{\Downarrow} < \phi_{1h} < v_{\Uparrow}$, it does not tell us exactly where this maximum must be. The precise location is to be decided by physics. So, given the sensibility of this location being based on the Higgs boson mass, we also consider the alternative where the maximum is at:

$$\begin{aligned} &h\left(x^{\mathrm{M}}\right) = -m_h c^2 = -\tfrac{1}{2}\left(v_{\Uparrow} + \tfrac{1}{\sqrt{2}} v_{\Downarrow}\right) = -125.25 \pm 0.02 \text{ GeV}; \\ &\text{i.e. } \phi_{1h}\left(x^{\mathrm{M}}\right) = v_{\Uparrow} + h\left(x^{\mathrm{M}}\right) = v_{\Uparrow} - m_h c^2 = \tfrac{1}{2}\left(v_{\Uparrow} - \tfrac{1}{\sqrt{2}} v_{\Downarrow}\right) = 120.97 \pm .02 \text{ GeV} \end{aligned} . \tag{16.9b}$$

In effect, (16.9b) this shifts the maximum hypothesized in (16.9a) to the left, toward the isospin-down vev, by $\frac{1}{2}\left(\frac{1}{\sqrt{2}} v_{\Downarrow} - v_{\Uparrow}\right) - \frac{1}{2}\left(v_{\Uparrow} - \frac{1}{\sqrt{2}} v_{\Downarrow}\right) = \frac{1}{\sqrt{2}} v_{\Downarrow} = m_d c^2 + m_s c^2 + m_b c^2 = 4.28^{+0.04}_{-0.03}$ GeV, which is the sum of the charged lepton masses related to (15.11) with the error bar set by the bottom quark. Now, we are called upon to determine which of (16.9a) versus (16.9b) is the better hypothesis, and this is a physics question, not a mathematics question.

Because standard model electroweak theory teaches that the *W* and *Z* bosons draw their rest energies from the Fermi vacuum, we anticipate the Higgs boson *h* draws its rest energy out of the vacuum in a similar way. This is especially so, because as reviewed in sections 11 and 12, the relation (12.16) applies to both fermions and bosons. Focusing on the upper relation (12.16), we noted following (13.1) that while fermions couple to the Fermi vacuum via $m_f c^2 = \frac{1}{\sqrt{2}} G_f v$, bosons couple via $m_B c^2 = \frac{1}{2} g_B v$, with a constant coefficient that is diminished from that for the fermions by a factor of $\frac{1}{\sqrt{2}}$. Referring, for example, to Figure 1, this likewise means that for a given Yukawa coupling G_f, a fermion draws energy out of the vacuum to acquire its rest masses at an amplified draw rate of $\sqrt{2}$ times the rate at with a boson with a given coupling g_B draws its rest energy from the vacuum. This $\sqrt{2}$ amplifier is why in Figure 1, the top quark draws almost all of the energy out of a vacuum with a vev $v \cong 246.22$ GeV, while having a rest energy that is only just shy of $\frac{1}{\sqrt{2}} v$. And it is this relation which, at (13.8), provided the first clue which subsequently allowed us in sections 14 and 15 to fit all of the fermion masses to the CKM mixing angles using bi-unitary transformations acting on fermion mass matrixes.

Now let's consider the Higgs boson and how it connects to the Higgs field to draw its rest energy out of the Fermi vacuum. Here, starting with the upper relation (12.16) we set $m \mapsto m_h$ and ${p_+}^5 \mapsto {p_{+h}}^5$ to have this apply specifically to the Higgs boson, so that with $V_{(5)} = x^0 x^1 x^2 x^3 x^5$ the upper (12.16) becomes $c{p_{+h}}^5 \phi_1 \cong \pm\sqrt{2} m_h c^2 \exp\left(-V_{(5)} / {L_+}^5\right)$. Now, at (13.5) we used $c{p_{+f}}^5 \phi_1\left(x^{\mathrm{M}}\right) \equiv h\left(x^{\mathrm{M}}\right)$ to connect the Higgs field $h\left(x^{\mathrm{M}}\right)$ with the symmetry-broken Kaluza-Klein scalar $\phi_1\left(x^{\mathrm{M}}\right)$ for a *fermion* with a fifth-dimensional momentum component ${p_{+f}}^5$. But now we need to connect $h\left(x^{\mathrm{M}}\right)$ to $c{p_{+h}}^5 \phi_1\left(x^{\mathrm{M}}\right)$ which is for a Higgs *boson*, not $c{p_{+f}}^5 \phi_1\left(x^{\mathrm{M}}\right)$ for a fermion.

Because the connection to $h\left(x^{\mathrm{M}}\right)$ determines the rate at which energy is drawn from the vacuum for rest mass, and because bosons with a given g are coupled less strongly to the vacuum than fermions with a given G by a factor of $\frac{1}{\sqrt{2}}$, this means that boson energy draws will be diminished by the same factor. Therefore, for bosons generally, the appropriate Higgs field connection is $\frac{1}{\sqrt{2}} cp_{+B}{}^{5}\phi_1\left(x^{\mathrm{M}}\right) \equiv h\left(x^{\mathrm{M}}\right)$, which is diminished by this $\frac{1}{\sqrt{2}}$ in relation to the fermion connection. Therefore, for the Higgs boson specifically, with the above multiplied through by $\sqrt{2}$, we obtain $\boxed{cp_{+h}{}^{5}\phi_1\left(x^{\mathrm{M}}\right) \equiv \sqrt{2}h\left(x^{\mathrm{M}}\right)}$. Combined with $cp_{+h}{}^{5}\phi_1 \cong \pm\sqrt{2}m_h c^2 \exp\left(-V_{(5)} / L_+{}^5\right)$ above, this yields:

$$h\left(x^{\mathrm{M}}\right) \equiv \frac{1}{\sqrt{2}} cp_{+h}{}^{5}\phi_1 = -m_h c^2 \exp\left(-\frac{V_{(5)}}{L_+{}^5}\right). \tag{16.10}$$

This is in contrast to the upper (13.5) for fermions. With $h \mapsto B$ for the subscript, this likewise applies to other massive bosons, most notably the W and Z bosons.

With (16.10), the counterpart to the upper (13.6), now for the Higgs boson, also multiplied through by $\sqrt{2}$, and also defining a coupling g_h by $\frac{1}{2} g_h v \equiv m_h c^2$ in the usual form for bosons, is:

$$\sqrt{2}\phi_h\left(x^{\mathrm{M}}\right) = \phi_{1h}\left(x^{\mathrm{M}}\right) = v + h\left(x^{\mathrm{M}}\right) = v - m_h c^2 \exp\left(-\frac{V_{(5)}}{L_+{}^5}\right) = v - \frac{1}{2} g_h v \exp\left(-\frac{V_{(5)}}{L_+{}^5}\right). \tag{16.11}$$

Likewise, the energy draw which gives the Higgs boson its mass is a (13.7) counterpart, namely:

$$\frac{1}{L_+{}^5}\int_0^\infty h\left(x^{\mathrm{M}}\right) dV_{(5)} = -\frac{1}{L_+{}^5} m_h c^2 \int_0^\infty \exp\left(-\frac{V_{(5)}}{L_+{}^5}\right) dV_{(5)} = m_h c^2 \exp\left(-\frac{V_{(5)}}{L_+{}^5}\right)\Bigg|_0^\infty = -m_h c^2. \tag{16.12}$$

The only difference from the parallel energy conservation relation (13.7) is the absence of a $\sqrt{2}$ in front of the first integral, and again, all of this extends to bosons generally by the subscript replacement $h \mapsto B$. Additionally, combining the newly-defined $\frac{1}{2} g_h v_\Uparrow \equiv m_h c^2$ with the positive square root of (16.8), as well as using $m_h = 125.25 \pm 0.02\ \mathrm{GeV}$ from (16.16) with $v = 246.2196508 \pm 0.0000633\ \mathrm{GeV}$, for the Higgs coupling g_h we obtain:

$$\frac{1}{2} g_h = \frac{m_h c^2}{v_\Uparrow} = \sqrt{2\lambda} = \frac{1}{2}\left(1 + \tan^2\theta_v\right) = 0.50869 \pm 0.00008 = \frac{1}{2}\left(1.01738 \pm 0.00016\right), \tag{16.13}$$

thus $g_h = 1.01738 \pm 0.00016$. Continuing the discussion following (16.8), this also means that when $\theta_v \to 0$, this $g_h \to 1$. Finally, writing (16.11) for $\phi_{1h}\left(x^{\mathrm{M}}\right)$ in the form of (13.9) we obtain:

$$\frac{\phi_{1h}\left(x^{\mathrm{M}}\right)}{v}=1+\frac{h\left(x^{\mathrm{M}}\right)}{v}=1-\frac{m_h c^2}{v}\exp\left(-\frac{V_{(5)}}{L_+{}^5}\right)=1-\frac{1}{2}g_h\exp\left(-\frac{V_{(5)}}{L_+{}^5}\right). \tag{16.14}$$

Then, analogously to (13.9) and Figure 1, we use this to draw a plot for how the Higgs boson extracts the energy for its mass from the vacuum, as shown below:

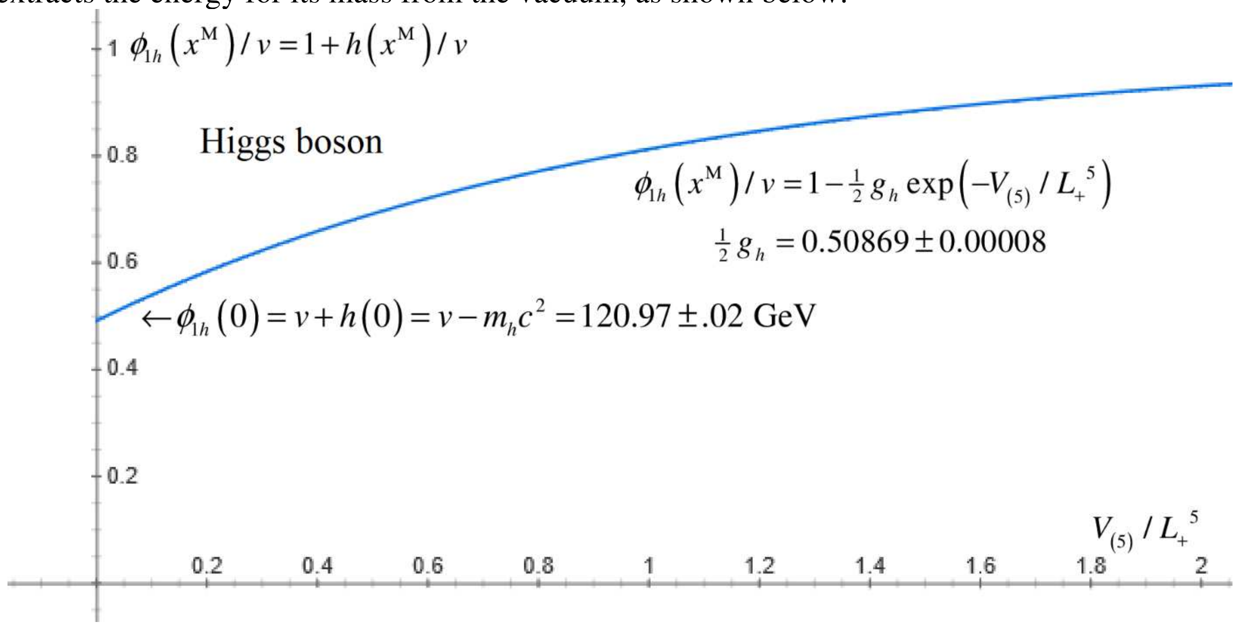

Figure 5: Higgs field extraction of rest energy from the Fermi vacuum, for the Higgs boson

As noted, the other massive bosons, namely the *Z* and the *W*, have plots similarly governed by (16.11), merely with the subscript replacement $h\mapsto B$. So, at its origin, the plot for the *Z* boson will not dip down quite as far as this Higgs boson plot above, and the plot for the *W* boson will have a slightly-shallower dip than that for the *Z* boson, because of the respective masses $m_hc^2=125.25\pm0.02$ GeV from (16.5), along with $m_Zc^2=91.1876\pm0.0021$ GeV and $m_Wc^2=$ 80.379 ± 0.012 GeV from [i]. Also, for massless bosons – presently known to comprise photons, gravitons and gluons – this becomes the flatline plot for $\phi_{1h}\left(x^{\mathrm{M}}\right)=v$ and $h\left(x^{\mathrm{M}}\right)=0$, illustrating how the Higgs "molasses" does nothing to add a rest mess, and thus nothing to slow these parties from luminous to sub-luminous propagation.

So, considered from the viewpoint of Figures 1 and 5 and equations (13.7) and (16.17) which show how the energy of the vacuum is conserved while fermions and bosons acquire the rest energy for their masses, we see that the Higgs boson field $h\left(x^{\mathrm{M}}\right)$ is a measure of how much energy has been *removed from the vacuum* in order to bestow a rest energy upon a particle, and that $\phi_{1h}\left(x^{\mathrm{M}}\right)$ is conversely a measure of how much energy is *retained by the vacuum* after the particle has acquired its rest energy. This means if we choose $h\left(x^{\mathrm{M}}\right)=-m_hc^2$ which is option (16.9b), then the *V* maximum will be based on the amount of energy *extracted* from the vacuum (with the minus sign indicating "extraction" or "removal"). Conversely, if we choose

$\phi_{1h}\left(x^{\mathrm{M}}\right)=m_h c^2$ which is option (16.9a), then the V maximum will be based on the amount of energy *retained* by the vacuum (with an implicit plus sign indicating "retention.") *So, the physics question is whether the domain point of the V maximum should be based upon energy removed from the vacuum, versus upon energy retained by the vacuum.* Figure 5 clearly points toward (16.9b) and energy removal, but let's make sure we examine all considerations to confirm.

Next, we turn to (15.11) which teaches that for quarks the Lagrangian potential V has *two minima*, one for isospin-up and one for isospin-down quarks. This central to what we are presently studying, and is why we are needing to pinpoint a V maximum in the first place. We anticipate that $v_{\Downarrow}$ will establish energetically-favored "nests" for isospin-down quarks and that $v_{\Uparrow}$ will establish energetically-favored nests for isospin-up quarks. As we shall shortly examine in detail, we also anticipate that weak beta decays between isospin-up and isospin-down quarks will require the incoming quark at a beta-decay vertex to pass over or through the V maximum between $v_{\Uparrow}$ and $v_{\Downarrow}$ in order to decay into the outgoing quark. (We shall see that the top quark is an exception to what was just stated, because of its exceptionally-large rest mass.) So, if (16.9b) was the physically-correct alternative used to establish the maximum V between these two minima as we presently suspect it is, then the *range* point $\phi_{1h}(0)=v+h(0)=v-m_h c^2=120.97\pm.02\ \text{GeV}$ which is illustrated in Figure 5 would establish the *domain* point at which V has its inter-vev maximum. This would mean that while quarks (or at least the less-massive quarks) are expected to "nest" near the minima of V, the Higgs bosons themselves would be "rooted" precisely at this V maximum.

This all means that a V maximum established using the alternative (16.9b) would give rise to three further physical characteristics for the Higgs boson: First, as an energy maximum at which the Higgs boson is rooted, this V maximum would cause the Higgs boson to be an unstable particle that decays quickly toward more stable energy configurations – which it is. Second, having the Higgs boson rooted at the maximum of V would mean that in addition to its large rest mass $m_h c^2=125.25\pm0.02\ \text{GeV}$ from (16.5), the Higgs boson would also have a very high *Lagrangian potential energy* in the vacuum. This in turn would make available energy which can be used – for example – to give rise to the also-large masses $m_Z c^2=91.1876\pm0.0021\,\text{GeV}$ and $m_W c^2=\ 80.379\pm0.012\ \text{GeV}$ of the Z and W bosons [i], and for any fermion mass increases which need to occur during weak beta decay (which depends, of course, on the pair of fermions involved). But most importantly, as we shall see when we study beta decay more closely, this would provide the requisite energy for a fermion undergoing beta decay at a vertex with a W boson to climb out from the vev minimum of its potential well and pass over or through this V maximum. Third, by the Higgs boson sitting precisely atop the maximum which separates the isospin-up from the isospin-down wells, the Higgs boson has no bias toward either well, and so can readily generate masses for the isospin-up and isospin down quarks with equal facility.

Therefore, in view of all the foregoing, and especially the clear quantitative support from Figure 5, it makes the most physical sense for the V maximum to be defined at $h\left(x^{\mathrm{M}}\right)=-m_h c^2$ by the energy drawn out of the vacuum to bestow an equivalent-magnitude mass $+m_h$ upon the Higgs boson, and not at $\phi_{1h}=m_h c^2$ by the energy retained by the vacuum after the energy draw.

Using $\phi_{1h} = m_h c^2$ would place the V maximum about $4.28\text{ GeV} \cong m_d c^2 + m_s c^2 + m_b c^2$ shy of the energy draw needed to give the Higgs boson its rest mass which is illustrated in Figure 5, and so would be close to the V maximum, but not right at the V maximum. Accordingly, recognizing that the vacuum field must give up an energy $-m_h c^2 \cong -125.25\text{ GeV}$ to provide a mass $m_h c^2 \cong 125.25\text{ GeV}$ to a Higgs boson in accordance with the energy conservation principles illustrated by (13.7) and (16.12), we now formally make the hypothesis as between (16.9a) and (16.9b), that *the maximum of the Lagrangian potential V between* $v_\Downarrow$ *and* $v_\Uparrow$ *is situated at the domain point where* $h\left(x^{\mathrm{M}}\right) = -m_h c^2$, that is, at the point where the Higgs field energy is equal to minus the Higgs mass times c^2, with the negative sign representing energy which has been drawn out of the Fermi vacuum to briefly provide mass to the high-potential thus energetically unstable and short-lived Higgs boson.

Now, we have all ingredients needed to revise the potential in (16.4) with the higher-order terms necessary to provide the usual first minimum at $\phi_{1h} = v_\Uparrow = v$ and the usual first maximum at $\phi_{1h} = 0$, as well as a second minimum at $\phi_{1h} = v_\Downarrow$ and, via (16.9b) a second maximum at $\phi_{1h} = v_\Uparrow - m_h c^2$. We start with $V' = dV / d\phi_{1h}$ and build in these minima and maxima by *defining*:

$$\begin{aligned} V' &\equiv A \frac{m_h{}^2 c^4}{2 v_\Uparrow{}^2} \phi_{1h} \left(\phi_{1h}{}^2 - v_\Uparrow{}^2\right)\left(\phi_{1h}{}^2 - \left(v_\Uparrow - m_h c^2\right)^2\right)\left(\phi_{1h}{}^2 - v_\Downarrow{}^2\right) \\ &= A \frac{m_h{}^2 c^4}{2 v_\Uparrow{}^2} \begin{pmatrix} -v_\Downarrow{}^2 v_\Uparrow{}^2 \left(v_\Uparrow - m_h c^2\right)^2 \phi_{1h} + \left(v_\Downarrow{}^2 v_\Uparrow{}^2 + \left(v_\Uparrow{}^2 + v_\Downarrow{}^2\right)\left(v_\Uparrow - m_h c^2\right)^2\right)\phi_{1h}{}^3 \\ -\left(v_\Uparrow{}^2 + v_\Downarrow{}^2 + \left(v_\Uparrow - m_h c^2\right)^2\right)\phi_{1h}{}^5 + \phi_{1h}{}^7 \end{pmatrix}. \end{aligned} \tag{16.15}$$

This is constructed so that the leading terms $\left(m_h{}^2 c^4 / 2 v_\Uparrow{}^2\right)\phi_{1h}\left(\phi_{1h}{}^2 - v_\Uparrow{}^2\right)$ in the top line above precisely match the usual V' in (16.4). We also include an overall coefficient A which we will use to make certain that when we momentarily integrate (16.15), the leading $\phi_{1h}{}^2$ term of V in (16.4) will continue to be $-\frac{1}{4} m_h{}^2 c^4 \phi_{1h}{}^2$, with all changes to V introduced at higher order. This leading term we are matching stems from the "mass" term $\frac{1}{2}\mu^2 \phi_{1h}{}^2$ in $V = \frac{1}{2}\mu^2\phi_{1h}{}^2 + \frac{1}{4}\lambda\phi_{1h}{}^4$, see (16.2). It will be seen by inspection that the top line in the above will become zero at all four of $\phi_{1h} = 0$, $\phi_{1h} = v_\Uparrow - m_h c^2$, $\phi_{1h} = v_\Downarrow$ and $\phi_{1h} = v_\Downarrow$. We will then need to choose the overall sign in A so that the first two provide maxima and the latter two provide minima for V itself.

Next, we easily integrate the above. For the leading term to match $-\frac{1}{4} m_h{}^2 c^4 \phi_{1h}{}^2$ in (16.4) we must set $A = 1/ v_\Downarrow{}^2 \left(v_\Uparrow - m_h c^2\right)^2$. Also based on the "initial condition" of matching (16.4), we discard any integration constant. We then consolidate and reduce to obtain:

$$V\left(\phi_{1h}\right)=m_h{}^2c^4\begin{pmatrix} -\frac{1}{4}\phi_{1h}{}^2+\frac{1}{8}\frac{1}{v_\Uparrow{}^2}\phi_{1h}{}^4+\frac{1}{8}\left(\frac{1}{v_\Downarrow{}^2}+\frac{1}{\left(v_\Uparrow-m_hc^2\right)^2}\right)\phi_{1h}{}^4 \\ -\frac{1}{12}\left(\frac{1}{v_\Uparrow{}^2v_\Downarrow{}^2}+\frac{1}{\left(v_\Uparrow-m_hc^2\right)^2}\frac{v_\Uparrow{}^2+v_\Downarrow{}^2}{v_\Uparrow{}^2v_\Downarrow{}^2}\right)\phi_{1h}{}^6+\frac{1}{16}\frac{1}{\left(v_\Uparrow-m_hc^2\right)^2}\frac{1}{v_\Uparrow{}^2v_\Downarrow{}^2}\phi_{1h}{}^8\end{pmatrix}. \quad (16.16)$$

Comparing with V in (16.4), we indeed see the original $\phi_{1h}{}^2$ and $\phi_{1h}{}^4$ terms. But there are some new additions to the $\phi_{1h}{}^4$ term, and brand new $\phi_{1h}{}^6$ and $\phi_{1h}{}^8$ terms. These new terms, of course, are the ones we expect will deliver the second maximum and minimum as specified via (16.15).

To simplify calculation, it is very useful to restructure the above to separate terms which do not and which do have a $1/\left(v_\Uparrow-m_hc^2\right)^2$ coefficient, and to then explicitly apply $m_hc^2=\left(v_\Uparrow+\frac{1}{\sqrt{2}}v_\Downarrow\right)/2$ from (16.5), thus $v_\Uparrow-m_hc^2=\frac{1}{2}\left(v_\Uparrow-\frac{1}{\sqrt{2}}v_\Downarrow\right)$, as follows:

$$\begin{aligned} V\left(\phi_{1h}\right)&=m_h{}^2c^4\left(-\frac{1}{4}\phi_{1h}{}^2+\frac{1}{8}\frac{v_\Uparrow{}^2+v_\Downarrow{}^2}{v_\Uparrow{}^2v_\Downarrow{}^2}\phi_{1h}{}^4-\frac{1}{12}\frac{1}{v_\Uparrow{}^2v_\Downarrow{}^2}\phi_{1h}{}^6\right) \\ &\quad+\frac{m_h{}^2c^4}{\left(v_\Uparrow-m_hc^2\right)^2}\left(\frac{1}{8}\phi_{1h}{}^4-\frac{1}{12}\frac{v_\Uparrow{}^2+v_\Downarrow{}^2}{v_\Uparrow{}^2v_\Downarrow{}^2}\phi_{1h}{}^6+\frac{1}{16}\frac{1}{v_\Uparrow{}^2v_\Downarrow{}^2}\phi_{1h}{}^8\right) \\ &=\frac{\left(v_\Uparrow+\frac{1}{\sqrt{2}}v_\Downarrow\right)^2}{4}\left(-\frac{1}{4}\phi_{1h}{}^2+\frac{1}{8}\frac{v_\Uparrow{}^2+v_\Downarrow{}^2}{v_\Uparrow{}^2v_\Downarrow{}^2}\phi_{1h}{}^4-\frac{1}{12}\frac{1}{v_\Uparrow{}^2v_\Downarrow{}^2}\phi_{1h}{}^6\right) \\ &\quad+\frac{\left(v_\Uparrow+\frac{1}{\sqrt{2}}v_\Downarrow\right)^2}{\left(v_\Uparrow-\frac{1}{\sqrt{2}}v_\Downarrow\right)^2}\left(\frac{1}{8}\phi_{1h}{}^4-\frac{1}{12}\frac{v_\Uparrow{}^2+v_\Downarrow{}^2}{v_\Uparrow{}^2v_\Downarrow{}^2}\phi_{1h}{}^6+\frac{1}{16}\frac{1}{v_\Uparrow{}^2v_\Downarrow{}^2}\phi_{1h}{}^8\right) \end{aligned}. \quad (16.17)$$

So, the behavior of $V\left(\phi_{1h}\right)$ is entirely driven by the two energy-dimensioned numbers in (15.11): The first is the Fermi vev $v_\Uparrow=v$ which establishes the usual minimum, and which we have learned is related to the sum of the isospin-up quark masses via $v_\Uparrow=\sqrt{2}\left(m_uc^2+m_cc^2+m_tc^2\right)$. The second is the second vev $v_\Uparrow$ which establishes a second minimum and is related to the sum of the isospin-down quark masses via $v_\Downarrow=\sqrt{2}\left(m_dc^2+m_sc^2+m_bc^2\right)$. Additionally, the Higgs mass itself establishes a second maximum via (16.9b), but the new relation $m_hc^2=\left(v_\Uparrow+\frac{1}{\sqrt{2}}v_\Downarrow\right)/2$ discovered in (16.5) means that only two of these energy numbers are truly independent of one another.

It is pedagogically-useful to graph the potential $V\left(\phi_{1h}\right)$ in (16.17) using the numerical values of $v_\Uparrow$ and $v_\Downarrow$ in (15.11), and / or the Higgs mass in (16.5). Substituting these into (16.17), reconsolidating terms at each order, and rounding the coefficient at each order to four significant digits, with ϕ_{1h} expressed in GeV thus $V(\phi_{1h})$ in GeV^4, we obtain:

$$V(\phi_{1h})\left[\mathrm{GeV}^4\right] = -3922\phi_{1h}^{\ 2} + 53.76\phi_{1h}^{\ 4} - 0.003032\phi_{1h}^{\ 6} + 3.020\times10^{-8}\phi_{1h}^{\ 8}. \tag{16.18}$$

Keeping in mind from (16.1) through (16.4) that $V(\phi_{1h})$ is part of the Lagrangian density and so has physical dimensions of quartic energy, and that ϕ_{1h} is linear in energy, (16.18) can be easily graphed to produce the following plot:

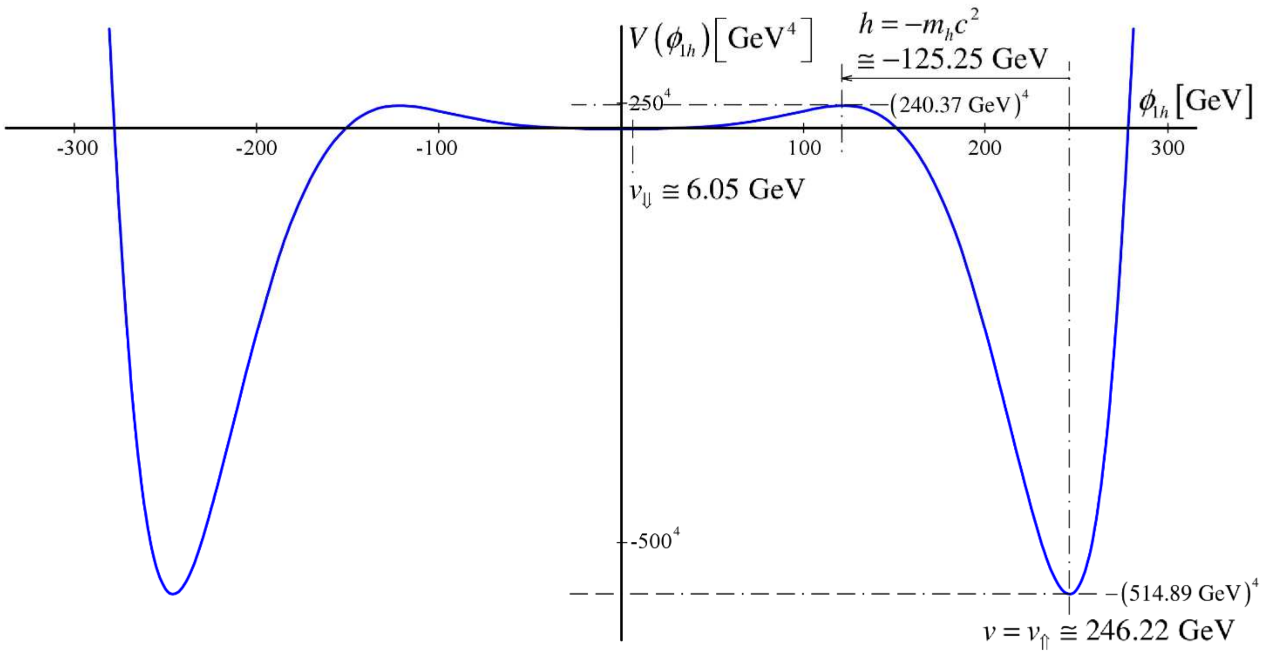

Figure 6: Lagrangian Potential for Quarks – Wide View

Above we see the usual minimum at $\phi_{1h} = v_\Uparrow \cong 246.22\ \mathrm{GeV}$, where along the *y* axis we have an energy "well" with a depth of $V(\phi_{1h}) \cong -(514.89\ \mathrm{GeV})^4$. But we now have a new maximum at $\phi_{1h} = v_\Uparrow - m_h c^2 \cong 120.97\ \mathrm{GeV}$ based on (16.9b), and at this maximum there is an energy "barrier" with a height of $V(\phi_{1h}) \cong (240.37\ \mathrm{GeV})^4$. Closer to the origin is the usual maximum at $\phi_{1h} = 0$ and the new minimum at $\phi_{1h} = v_\Downarrow \cong 6.05\ \mathrm{GeV}$. But comparatively to the foregoing, these are extremely small, and are impossible to see in Figure 6. So, it is also useful to magnify the domain from $-10\ \mathrm{GeV} < \phi_{1h} < 10\ \mathrm{GeV}$ in Figure 6, while also magnifying the range, to obtain the magnified view of the center of Figure 6, as shown below:

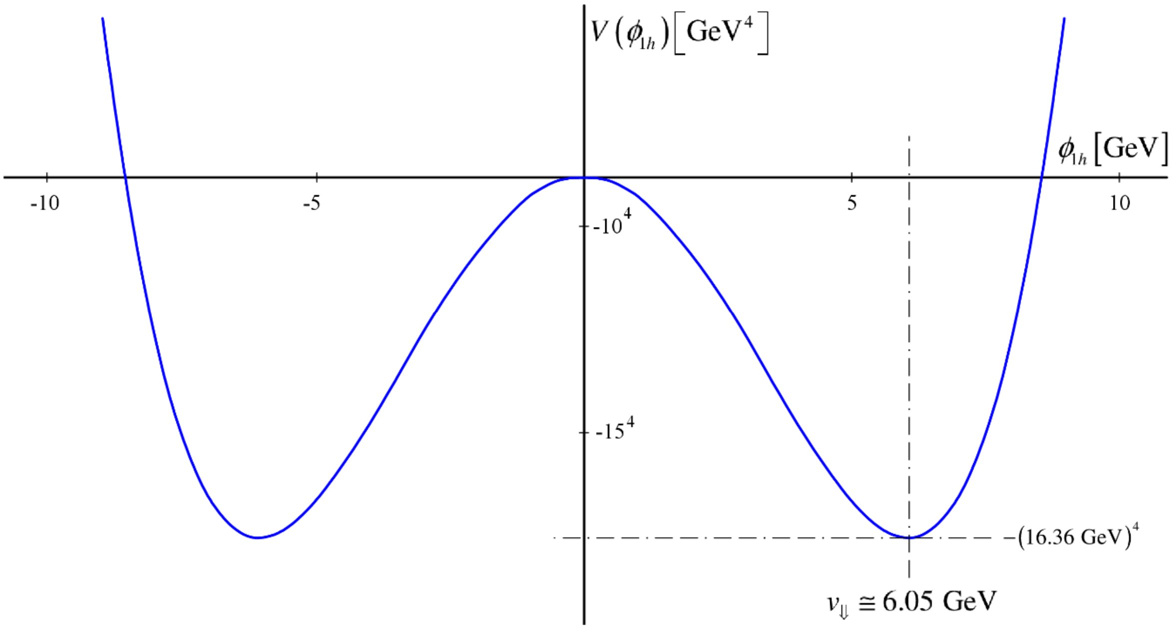

Figure 7: Lagrangian Potential for Quarks – Magnified Center View

Here, the usual maximum at $V(\phi_{1h}=0)=0$ is readily apparent, as is the new minimum at $\phi_{1h}=v_{\Downarrow}\cong 6.05\ \mathrm{GeV}$ where there is a second energy well of depth $V(v_{\Downarrow})\cong -(16.36\ \mathrm{GeV})^4$. The above is simply an extremely magnified view of the region in Figure 6 close to the origin.

As reviewed following (16.4), the well depth for the usual $V=\frac{1}{2}\mu^2\phi_{1h}{}^2+\frac{1}{4}\lambda\phi_{1h}{}^4$ in (16.4) at $\phi_{1h}=v_{\Uparrow}$ was $V(v_{\Uparrow}{}^2)=-(104.39\ \mathrm{GeV})^4$. But of course, this was based only on square and quartic field terms. Now, in Figure 6, at the same $\phi_{1h}=v_{\Uparrow}$ we have $V(v_{\Uparrow}{}^2)\cong -(514.89\ \mathrm{GeV})^4$ which is deeper by a factor of almost 5 in linear energy dimensions. This substantially-increased depth is driven by the combination of setting $A=1/v_{\Downarrow}{}^2(v_{\Uparrow}-m_hc^2)^2$ going from (16.15) to (16.16) to preserve the leading $-\frac{1}{4}m_h{}^2c^4\phi_{1h}{}^2$ mass term in $V=-\frac{1}{4}m_h{}^2c^4\phi_{1h}{}^2+\frac{1}{8}(m_h{}^2c^4/v_{\Uparrow}{}^2)\phi_{1h}{}^4$ from (16.4) without change, and from the new minimum at $v_{\Downarrow}$ and new maximum at $\phi_{1h}=v_{\Uparrow}-m_hc^2$. That is, this increased depth is driven entirely by the new higher-order $\phi_{1h}{}^4$, $\phi_{1h}{}^6$ and $\phi_{1h}{}^8$ terms, in combination with maintaining the existing $\phi_{1h}{}^2$ and $\phi_{1h}{}^4$ terms as is. Given the substantially-greater depth of $V(v_{\Uparrow}{}^2)\cong -(514.89\ \mathrm{GeV})^4$ versus $V(v_{\Downarrow})\cong -(16.36\ \mathrm{GeV})^4$, we see that the minimum at $\phi_{1h}=v_{\Downarrow}$ is merely a *local* minimum, while that at $\phi_{1h}=v_{\Uparrow}$ is a *global* minimum, as we previewed following (16.5). Given that quarks situated in these wells will seek out the lowest available energy states, this means that will all else being equal, a quark will find it energetically-favorable to maintain an isospin-up state over and isospin-down state. As we shall later see, this is part of why free neutrons decay into free protons, rather than vice versa.

Even with Figures 6 and 7, however, the energetic behavior of quarks in these wells and the impact of the new maximum are not brought out as much as they could be, because ϕ_{1h} is linear in energy while V is quartic in energy. So, it is also useful to reproduce Figures 6 and 7 by taking the fourth root $\sqrt[4]{V(\phi_{1h})}$, and also by scaling the energies along the x ordinate and y abscissa at 1:1, to match one another precisely. Of course, the fourth root of +1 has the quartic roots 1, –1, i, and $-i$. So below the x axis, to connect everything together, we display what is really $-\sqrt[4]{-V(\phi_{1h})}$ using 1 for the quartic root. So, taking the fourth root along the vertical axis in Figure 6 and scaling what are now linear energy numbers along each axis to one another, we obtain the plot below:

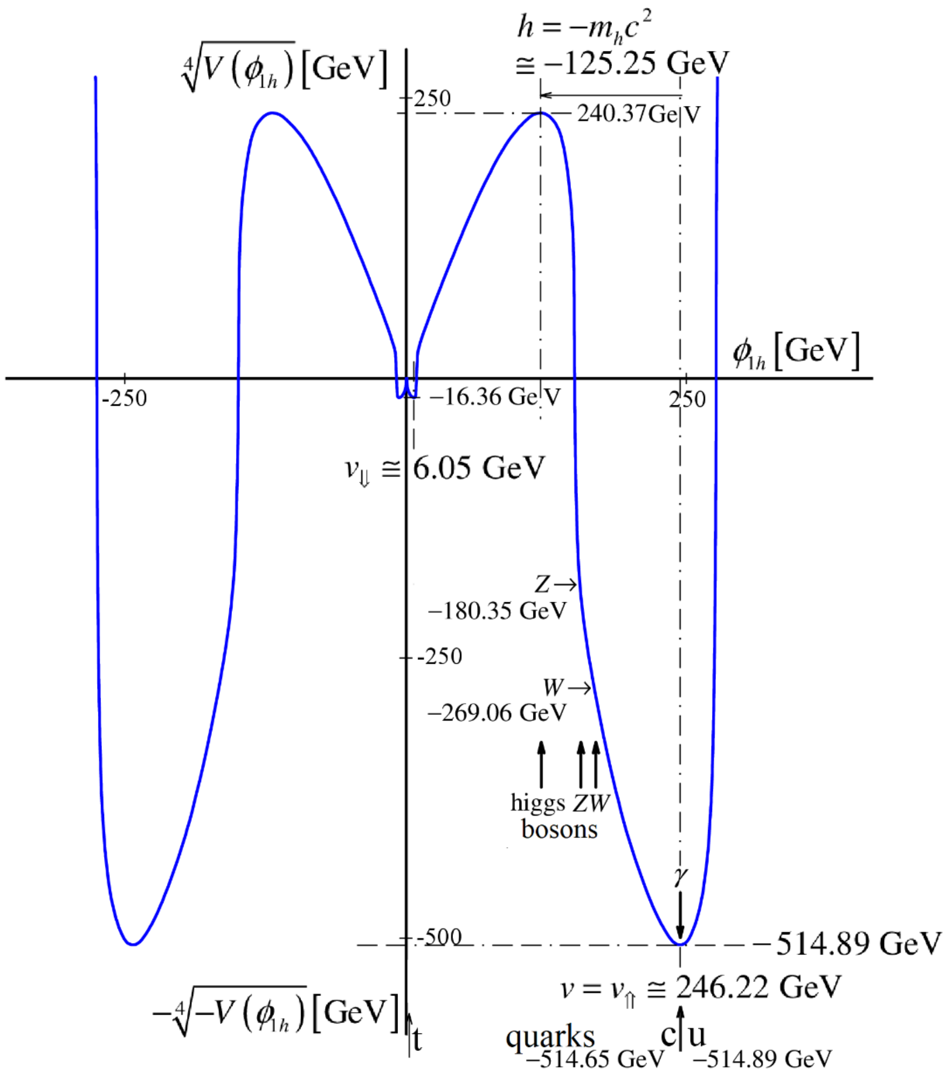

Figure 8: Lagrangian Potential for Quarks and Bosons, Fourth Root – Wide View

It is important to note the upward-pointing arrows designating the u, c and t quarks, as well as the Higgs, Z and W bosons and a downward-pointing arrow for the photon γ at the very bottom of the $v = v_{\Uparrow}$ well, which will momentarily be reviewed. Above, we are able to see both minima and maxima in the same plot, although the central region is still rather small. Therefore, in Figure

9 below, we magnify Figure 8 over the domain $-10\ \text{GeV} < \phi_{1h} < 10\ \text{GeV}$, and again scale energies on a 1:1 basis along the vertical and horizontal axes. Figure 9 is equivalent to the fourth root of the magnified potential view in Figure 7.

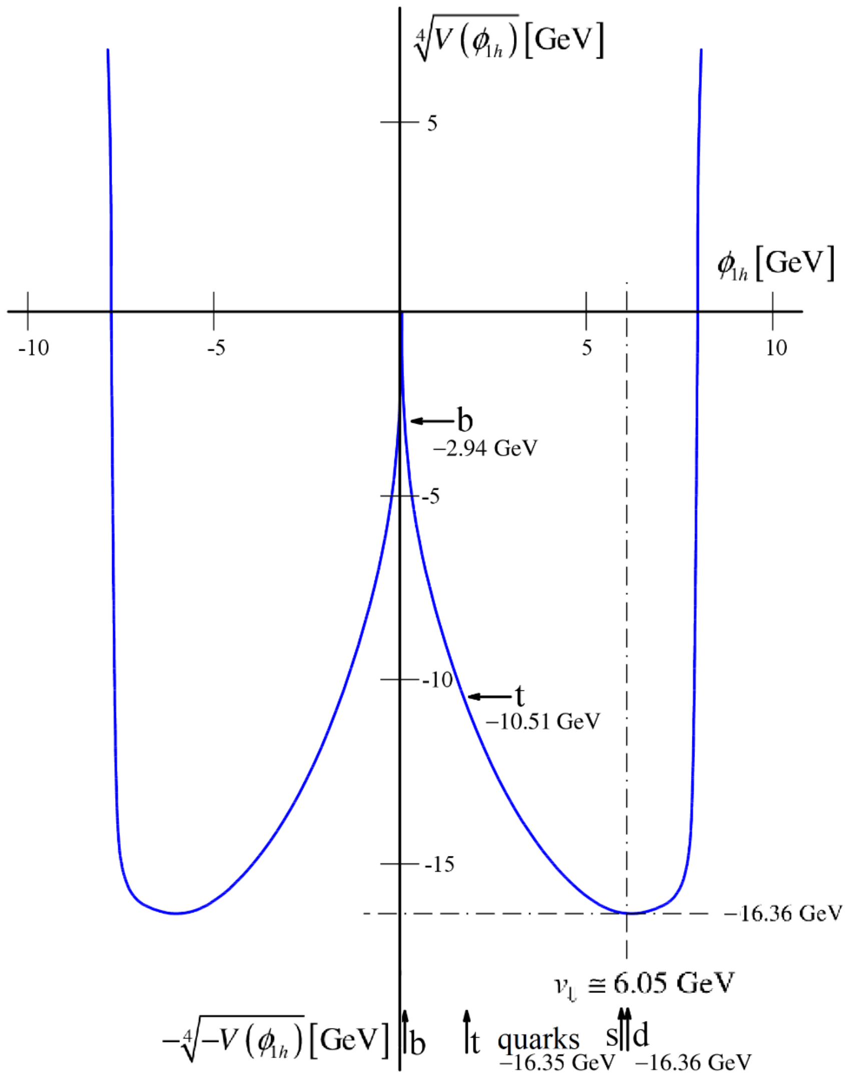

Figure 9: Lagrangian Potential for Quarks, Fourth Root – Magnified Center View

Here, also note the upward-pointing arrows designating the b, s and d quarks as well as the t quark, which will also be reviewed momentarily.

These two plots in Figures 8 and 9 help provide a deeper understanding of how quarks behave in the Lagrangian potential, and as we shall explore in the next section, of how they beta decay. First, it will be seen with energies linearized along both axes and scaled at 1:1, that the potential wells are very deep and steep. Moreover, it will be seen that the maximum at $V(\phi_{1h}=0)=0$ is not smooth as one might conclude looking at Figures 6 and 7. Rather, when comparing energies to energies at a 1:1 scale, this maximum comes to a sharp upward point with a slope that is infinite at the origin. Second, it is apparent, most clearly from Figure 8, that the $v_{\Downarrow}$ potential well establishes a *local* minimum while the $v_{\Uparrow}$ potential well presents a *global* minimum, as already noted. As shown, the $v_{\Downarrow}$ local minimum has an energy depth of –16.36 GeV and the $v_{\Downarrow}$ global minimum has a depth of –514.89 GeV, about 31.47 times as large. Third, we see most clearly from Figure 8 that there is high barrier between the two wells set by the new maximum which was built around the Higgs boson, which barrier has a height of +240.37 GeV. All this will be important shortly, to better understand the role of the Higgs field and boson in weak beta decay. First, however, let's carefully review how all of Figures 1 through 9 now tie together.

To fully understand Figures 6 through 9 and what they teach about the behavior of individual quarks, *it is essential to understand that* ϕ_{1h} *plotted along the* <u>*horizontal*</u> *axis in all of Figure 6 through 9 is the very same* ϕ_{1h} *which is plotted along the* <u>*vertical*</u> *axis of Figures 1 and 5*. Although Figure 1 applies to the up quark, we discussed following (15.17) how similar plots with the exact same character can be drawn for all of the other quarks, and just reviewed how this extends to the Higgs and *Z* and *W* bosons. It is most helpful to refer to Figure 8 to see all this:

For isospin-up quarks, far from the charm and up quarks where $V_{(5)} / L^5 \to \infty$, the Figure 1-type plots will level off at $\phi_{1h} = v = v_{\Uparrow} \cong 246.22 \text{ GeV}$, just like Figure 1. But right at $V_{(5)} / L^5 = 0$ which defines the energetic minimum at which a quark is most likely to nest, there will be far less energy taken out of the vacuum. Referring to (14.7), this is because at $V_{(5)} / L^5 = 0$, while the dimensionless Yukawa coupling for the top quark $G_t = 0.99266^{+0.00021}_{-0.00014}$ removes about 99.3% of the energy from the Fermi vacuum with a global vev minimum $v = v_{\Uparrow}$ shown on the horizontal axis of Figure 8, in contrast, for the charm quark $G_c = 0.00732^{+0.00014}_{-0.00020}$ removes only about 0.7% from the vacuum and for the up quark $G_u = 0.000013^{+0.000003}_{-0.000002}$ removes a scant 0.001% from the vacuum.

For the isospin down quarks the vev itself is different, namely $v_{\Downarrow} \cong 6.05 \text{ GeV}$. But this is also plotted on the horizontal axis of Figure 8, albeit far closer to the origin and much shallower than $v_{\Uparrow}$. Now referring to (15.7), for $V_{(5)} / L^5 \to \infty$ the Figure 1 analogs for each of the b, s, d quarks will level off at $\phi_{1h} = v_{\Downarrow} \cong 6.05 \text{ GeV}$, because of the second local vev minimum. But right at $V_{(5)} / L^5 = 0$ we have $G_b = 0.97725^{-0.00016}_{+0.00021}$ which shows that the bottom quark takes an approximate 97.7% energy bite out of this diminished-magnitude vacuum, while $G_s = 0.02161^{+0.00015}_{-0.00020}$ indicates a 2.1% bite from the strange quark and $G_d = 0.00115 \pm 0.00001$ a 0.1% bite from the down quark. For both isospin-up and isospin-down quarks, we have the relations $G_u + G_c + G_t = 1$ from (14.3) and $G_b + G_s + G_d = 1$ from (15.1), which establish the hypotenuses in Figures 2 and 3, respectively. These two hypotenuses are then tied together by the Higgs mass in Figure 4 which also via (16.9b) sets the peak between the isospin-up and isospin-down vevs, which peak is in all of Figures 6 through 9, albeit most visually-clearly in Figure 8.

Finally, we just showed in Figure 5 how to draw this plot for the Higgs boson, noting that similar plots albeit with shallower origin dips may be drawn for the *Z* and *W* bosons. For all of the bosons, as also reviewed, the energy draws are damped by a factor of $\sqrt{2}$ relative to fermions for any given coupling because of the constant coefficients in the coupling relations $m_B = \frac{1}{2} v_{\Downarrow} g_B$ versus $m_f = \frac{1}{\sqrt{2}} v_{\Downarrow} g_f$. For the massless photon, the boson relation means that $g_\gamma = 0$, which is what places this at the bottom of the $v_{\Uparrow}$ well in Figure 8.

So with this understanding in place, we see that the upward-pointing arrows in Figure 8 and 9 for the individual quarks and boson simply represent, for each particle, the value of ϕ_{1h} at which their Figure 1-type plots bottom out at $V_{(5)} / L^5 = 0$ where the particle is most likely to nest, i.e., the value of ϕ_{1h} at their "expected centers." But of course, in Figures 8 and 9 ϕ_{1h} is plotted on the horizontal axis, not the vertical axis as in Figure 1. So, in Figures 8 and 9 (and 6 and 7) the removal of energy from the vacuum and its reappearance in the rest energy of an individual quark or bosons via the respective energy conservation relations (13.7) and (16.17) (the latter extended to the *Z* and *W*), is represented by right-to-left displacement along the $+\phi_{1h}$ horizontal axis, rather than by the downward displacement of a Figure 1-type plot the along $+\phi_{1h}$ vertical axis. So in Figure 8 it is the small $G_c = 0.00732^{+0.00014}_{-0.00020}$ for the charm quark and the even-smaller $G_u = 0.000013^{+0.000003}_{-0.000002}$ for the up quark that places their nest-center arrows very-slightly to the left of the global minimum at $\phi_{1h} = v = v_{\Uparrow} \cong 246.22 \text{ GeV}$. But the very large $G_t = 0.9937 \pm 0.0023$ causes the top quark not only to nest far to the left of $v_{\Uparrow}$, but even to the left of $v_{\Downarrow} \cong 6.05 \text{ GeV}$, butting right up against the *y* axis. *This crossover of the nest-center from one vev minimum to the other is unique to the top quark.* Then, in Figure 9 we have a magnified view of the $v_{\Downarrow}$ region. Here, we see the small $G_s = 0.02161^{+0.00015}_{-0.00020}$ for the strange quark and the even smaller $G_d = 0.00115 \pm 0.00001$ for the down quark causing these nest-center arrows to situate very-slightly to the left of the local minimum at $\phi_{1h} = v_{\Downarrow} \cong 6.05 \text{ GeV}$. Bu the very large $G_b = 0.97725^{-0.00016}_{+0.00021}$ moves the bottom quark nest-center arrow far to the left of $v_{\Downarrow}$, and indeed, as we can see in this magnified view, also well to the left of the top quark.

Turning to the bosons, we see from its upward-pointing arrow in Figure 8 that the Higgs boson has its $V_{(5)} / L^5 = 0$ expected center precisely at the new maximum between the two vev minima, with a height of +240.37 GeV. Again, this was established *by definition* when we determined that (16.9b) was the appropriate relation for the precise placement of the Lagrangian potential maximum. This means that during the brief lifetime of a Higgs boson which is the field quantum of the Higgs field, this Higgs boson not only has a very large mass $m_h c^2 = 125.25 \pm 0.02 \text{ GeV}$, but it also has a very large (linearized) Lagrangian potential energy of approximately +240.37 GeV. Again, this energy will be very important when we review the weak beta decay of quarks in the next section. We also see the $V_{(5)} / L^5 = 0$ expected center placements of the *Z* and the *W* bosons. The former has a Lagrangian potential of about –180.35 GeV and the latter about –269.06 GeV, both of which are negative energies, albeit still well-above the global minimum of with a depth of –514.89 GeV where $\phi_{1h} = v_{\Uparrow} \cong 246.22\ \mathrm{GeV}$. And again, the displacement of any fermion to the left of its vev minimum is amplified by a $\sqrt{2}$ factor relative to boson displacement, because of the coupling relation $m_f = \frac{1}{\sqrt{2}} v_{\Downarrow} g_f$ versus $m_B = \frac{1}{2} v_{\Downarrow} g_B$. Finally, the photon (and by extension the graviton and the gluons), being massless, it situated precisely at the bottom of the $v_{\Uparrow}$ well at a linearized potential depth of –514.89 GeV.

Having now tied together Figures 6 through 9 with Figures 1 and 5 by understanding that the vertical axes in Figures 1 and 5 (and their analogues for other quarks and massive bosons) are synonymous with the horizontal axes in Figures 6 through 9, let is represent all of this in consolidated form. After (15.7) we pointed out that $\phi_{h1}\left(x^{\mathrm{M}}\right) = v_{\Uparrow} - \sqrt{2} m_{\Uparrow} c^2 \exp\left(-V_{(5)} / L_+^{\ 5}\right)$ in (13.6) for isospin-up quarks becomes $\phi_{h1}\left(x^{\mathrm{M}}\right) = v_{\Downarrow} - \sqrt{2} m_{\Downarrow} c^2 \exp\left(-V_{(5)} / L_+^{\ 5}\right)$ for isospin-down quarks, with a $\Uparrow \mapsto \Downarrow$ replacement. And at (16.11), generalized with $h \mapsto B$, we learned that $\phi_{1h}\left(x^{\mathrm{M}}\right) = v_{\Uparrow} - m_B c^2 \exp\left(-V_{(5)} / L_+^{\ 5}\right)$ for the Higgs and electroweak gauge bosons, including the photon. We can consolidate this for an elementary particle *p* which can be quark *q* or a boson *B*, by writing $\phi_{1hp}\left(x^{\mathrm{M}}\right) = v_p - C_p m_p c^2 \exp\left(-V_{(5)} / L_+^{\ 5}\right)$ where $v_p = v_{\Uparrow}$ for the u, c, t quarks and for all the bosons including the Higgs, $v_p = v_{\Downarrow}$ for the d, s, b quarks, with a numeric coefficient $C_p = \sqrt{2}$ for all the quarks and with $C_p = 1$ for all the bosons, and where m_p is the particle mass for all of the quark and massive bosons. Then, if we wish, we can start with the relation (16.17) for $V\left(\phi_{1h}\right)$ and substitute this consolidated $\phi_{h1p}\left(x^{\mathrm{M}}\right)$ to obtain Lagrangian potentials $V_p\left(-V_{(5)} / L_+^{\ 5}\right)$ as a function of spacetime-plus-one Poincare-invariant volumetric separation ratio $V_{(5)} / L_+^{\ 5}$ from the center of the particle "nests," for each particle *p*, to find that that:

$$V_p\left(\frac{V_{(5)}}{L_+^{\ 5}}\right) = \frac{\left(v_{\Uparrow} + \frac{1}{\sqrt{2}} v_{\Downarrow}\right)^2}{4} \begin{pmatrix} -\frac{1}{4}\left(v_p - C_p m_p c^2 \exp\left(-\frac{V_{(5)}}{L_+^{\ 5}}\right)\right)^2 + \frac{1}{8} \frac{v_{\Uparrow}^{\ 2} + v_{\Downarrow}^{\ 2}}{v_{\Uparrow}^{\ 2} v_{\Downarrow}^{\ 2}} \left(v_p - C_p m_p c^2 \exp\left(-\frac{V_{(5)}}{L_+^{\ 5}}\right)\right)^4 \\ -\frac{1}{12} \frac{1}{v_{\Uparrow}^{\ 2} v_{\Downarrow}^{\ 2}} \left(v_p - C_p m_p c^2 \exp\left(-\frac{V_{(5)}}{L_+^{\ 5}}\right)\right)^6 \end{pmatrix} + \frac{\left(v_{\Uparrow} + \frac{1}{\sqrt{2}} v_{\Downarrow}\right)^2}{\left(v_{\Uparrow} - \frac{1}{\sqrt{2}} v_{\Downarrow}\right)^2} \begin{pmatrix} \frac{1}{8}\left(v_p - C_p m_p c^2 \exp\left(-\frac{V_{(5)}}{L_+^{\ 5}}\right)\right)^4 - \frac{1}{12} \frac{v_{\Uparrow}^{\ 2} + v_{\Downarrow}^{\ 2}}{v_{\Uparrow}^{\ 2} v_{\Downarrow}^{\ 2}} \left(v_p - C_p m_p c^2 \exp\left(-\frac{V_{(5)}}{L_+^{\ 5}}\right)\right)^6 \\ +\frac{1}{16} \frac{1}{v_{\Uparrow}^{\ 2} v_{\Downarrow}^{\ 2}} \left(v_p - C_p m_p c^2 \exp\left(-\frac{V_{(5)}}{L_+^{\ 5}}\right)\right)^8 \end{pmatrix} . \tag{16.19}$$

Right at the center of each nest where $V_{(5)} / L_+^{\ 5} = 0$ thus $\exp\left(-V_{(5)} / L_+^{\ 5}\right) = 1$ this reduces to:

$$V_p\left(\frac{V_{(5)}}{L_+^{\,5}}=0\right)=\frac{\left(v_{\Uparrow}+\frac{1}{\sqrt{2}}v_{\Downarrow}\right)^2}{4}\left(-\frac{1}{4}\left(v_p-C_pm_pc^2\right)^2+\frac{1}{8}\frac{v_{\Uparrow}^{\,2}+v_{\Downarrow}^{\,2}}{v_{\Uparrow}^{\,2}v_{\Downarrow}^{\,2}}\left(v_p-C_pm_pc^2\right)^4-\frac{1}{12}\frac{1}{v_{\Uparrow}^{\,2}v_{\Downarrow}^{\,2}}\left(v_p-C_pm_pc^2\right)^6\right)$$

$$+\frac{\left(v_{\Uparrow}+\frac{1}{\sqrt{2}}v_{\Downarrow}\right)^2}{\left(v_{\Uparrow}-\frac{1}{\sqrt{2}}v_{\Downarrow}\right)^2}\left(\frac{1}{8}\left(v_p-C_pm_pc^2\right)^4-\frac{1}{12}\frac{v_{\Uparrow}^{\,2}+v_{\Downarrow}^{\,2}}{v_{\Uparrow}^{\,2}v_{\Downarrow}^{\,2}}\left(v_p-C_pm_pc^2\right)^6+\frac{1}{16}\frac{1}{v_{\Uparrow}^{\,2}v_{\Downarrow}^{\,2}}\left(v_p-C_pm_pc^2\right)^8\right). \quad (16.20)$$

These $V_p(0)$ in (16.20) are the potentials pinpointed by the arrow pointers in Figures 8 and 9 for the six quarks and the Higgs, *Z* and *W* bosons. Because they are calculated at $V_{(5)}/L_+^{\,5}=0$, these are the potential energies of the interaction of each of these quarks and bosons with the Fermi vacuum, at their "expected centers."

Now, let's study what Figures 8 and 9 in particular, teach about the energetic behaviors of quarks in the Lagrangian potential, and about the role of the Higgs field not only in the acquisition of rest mass by quarks, but in the weak beta decays of quarks.

17. Quantization of the Lifetime and the Breit-Wigner Probability Rest Energy Distribution, for the *Z* Boson

The so-called "Mexican hat" Lagrangian potential $V=\frac{1}{2}\mu^2\phi_{1h}^{\;2}+\frac{1}{4}\lambda\phi_{1h}^{\;4}$ reviewed in (16.1) through (16.4) and associated with the Higgs mechanism is well known. Figures 6 and 7 together show a modified "two-dip" Mexican hat potential *V* based on (16.17) and numerically on (16.18), and Figures 8 and 9 show this potential in linear rather than quartic dimensions of energy with a 1:1 scaling of energies along the horizontal and vertical axes, together with arrows designating center-placement of the quarks and the Higgs and *Z* and *W* bosons. The placement of these arrows, and the depth (or height for the Higgs boson) of this potential is crucial to understanding the role of the Higgs boson in weak beta decays between quarks, and will later be similarly crucial for understanding lepton beta decay. Consequently, this will be explicated in detail in this section. To start, it is important to carefully review the actual *physical meaning* of the Lagrangian potential *V* in (16.17) and (16.18) which is graphed in its native energy-to-the-fourth-power dimensionality in Figures 6 and 7, and linearized in energy in Figures 8 and 9.

Physically, a potential always represents a field in the vicinity of a first material body, which field will give rise to a potential energy of interaction once a second material body is introduced into the field of the first body. For example, the gravitational potential $-GM/r$ of a massive body has dimensions of energy / per mass, so that once a second *m* mass is placed into the potential at a given center-separation *r*, the construct $-GMm/r$ represents the potential energy from the gravitational interaction of these two bodies as a function of the separation *r* between the centers of the two bodies. Likewise, the Coulomb potential k_eQ/r has dimensions of energy-per-charge, so that when a second charge *q* is placed into this potential, k_eQq/r represents the potential energy owing to the electrical interaction between two charges, also as a function of center-separation *r*. So, when we now talk about *V* being a Lagrangian "potential," albeit with dimensions of energy to the fourth power, we must clearly answer four questions: 1) What is the first material body analogous to *M* and *Q* which gives rise to this potential? 2) What is the second material body which gets places into this potential to give rise to a potential energy of interaction? 3) What is the physical meaning of this potential energy of interaction? And 4) What is the analogue to the center separation *r*?

The respective answers for the Lagrangian potential *V* are as follows: 1) The first material body is the Fermi vacuum itself. 2) The second material body is any fermion or boson which is placed into or exists in that vacuum. 3) The potential energy is the interaction energy between that fermion or boson and the vacuum. 4) The analogue of *r* is the Poincare invariant volumetric measure $V_{(5)} / L_+^{\,5}$ in (16.19) and (16.20). But because $V_{(5)} / L_+^{\,5} = 0$ not only represents the excepted center position of each particle type in spacetime-plus-one, but also establishes a different $\phi_{1h\,p}(0)$ for each particle *p* at its expected center, it is more revealing to think about $\phi_{1h\,p}(0)$ – the value of $\phi_{1h\,p}(x^{\mathrm{M}})$ at the expected particle center – as the analogue of the radial *r* between expected centers of particles in gravitational and electromagnetic potentials. As a result, the potential energy of interaction between the vacuum and any particular fermion or boson depends upon the center-valued $\phi_{1h}(0)$ of that particular fermion or boson. Therefore, *the physical meaning of the arrows in Figures 8 and 9, is that these point to the potential energies of the six quarks and the Higgs and Z and W bosons at their expected-centers, arising from their interactions with the vacuum.*

If we now denote these potential energies of particles at their expected centers interacting with the vacuum by E_V, denote the depth of the isospin-up well by $E_{v\Uparrow} = -514.890 \text{ GeV}$, denote that of the isospin-down well by $E_{v\Downarrow} = -16.362 \text{ GeV}$, and then for the isospin up quarks calculate $E_V - E_{v\Uparrow}$ which measures how much each quark's expected-center potential energy is elevated above the global minimum at $v_{\Uparrow} = v = 246.2196508(633)\,\text{GeV}$, while also showing the center value of each particle's rest energy $E_0 = mc^2$, we obtain:

$$\begin{array}{llll} & mc^2 & E_V & E_V - E_{v\Uparrow} \\ u: & 0.002 \text{ GeV} & -514.890 \text{ GeV} & +0.002 \text{ MeV} \\ c: & 1.275 \text{ GeV} & -514.653 \text{ GeV} & +0.237 \text{ GeV} \\ t: & 172.826 \text{ GeV} & -10.507 \text{ GeV} & +504.383 \text{ GeV} \end{array} . \tag{17.1a}$$

For the isospin-down quarks, against the local minimum at $v_{\Downarrow} = 6.0491^{+0.0571}_{-0.0430} \text{ GeV}$, we find:

$$\begin{array}{llll} & mc^2 & E_V & E_V - E_{v\Downarrow} \\ d: & 0.005 \text{ GeV} & -16.362 \text{ GeV} & +0.021 \text{ MeV} \\ s: & 0.092 \text{ GeV} & -16.355 \text{ GeV} & +0.007 \text{ GeV} \\ b: & 4.180 \text{ GeV} & -2.935 \text{ GeV} & +13.427 \text{ GeV} \end{array} . \tag{17.1b}$$

And for the three bosons and the photon relative to $E_{v\Uparrow}$ we have:

	mc^2	E_V	$E_V - E_{v\Uparrow}$	$E_V - E_{v\Downarrow}$
h:	125.250 GeV	+240.374 GeV	+755.264 GeV	+256.736 GeV
Z:	91.188 GeV	−180.347 GeV	+334.543 GeV	
W:	80.379 GeV	−269.058 GeV	+245.832 GeV	
γ:	0 GeV, exactly	−514.890 GeV	0 GeV, exactly	

. (17.1c)

The above simply summarize numerically, what is graphically illustrated in Figures 8 and 9. For all but the up and down quarks this is calculated to three digits in GeV. For the up and down quarks, we calculate six digits, which converts to three digits in MeV. For the Higgs boson which sits precisely atop the peak between the two wells, we show both $E_V - E_{v\Uparrow}$ and $E_V - E_{v\Downarrow}$. And the photon, because it is massless, sits exactly at the bottom of the $v_{\Uparrow}$ well. Because the Lagrangian potential $V(\phi_{1h})$ is a course invariant under the Poincare group (as well as having other well-known symmetries), and because $V_p\left(V_{(5)} / L_+{}^5 = 0\right)$ in (16.20) is determined by the invariant rest mass of each particle type, and because $V_{(5)} / L_+{}^5$ is likewise designed via the initial conditions on the integration constant $L_+{}^5$ have Poincare symmetry as reviewed at the end of section 13, the vacuum potential energies listed in (17.1) are all invariant scalar numbers having a similar character in this regard to the rest masses themselves, as opposed to being simply a time / energy *component* (not invariant scalar) of an energy-momentum vector $cp^\mu = (E, c\mathbf{p})$.

We see that the up quark has the lowest potential energy relative to the vacuum of any quark in the isospin-up well, that the down quark has the lowest in the isospin-down well, and that both are elevated above their well bottoms by $0.002\ \text{MeV}$ and $0.021\ \text{MeV}$ respectively. These are very tiny elevations: the former is about 250 times smaller and the latter about 25 times smaller than the electron rest mass. Of course, with the isospin-up well establishing a global and the isospin-down well only a local minimum, this means that the up quark has the lowest vacuum potential energy of all the quarks. The top quark by virtue of its crossover into the isospin-down well has an exceptionally-high elevation of 504.383 GeV above its native isospin-up well, though its still-negative potential energy of –10.507 GeV is exceeded by the –2.935 GeV potential energy of the bottom quark. The *Z* and *W* bosons sit on the side wall of, and are all substantially-elevated above the bottom of, the isospin-up well, though their potential energies relative to the vacuum are nonetheless still negative. Finally, and uniquely, the Higgs boson – which, recall, was used via (16.9b) to *define* the peak between the two wells – is the only one of the particles in (17.1c) with a positive potential energy from its interaction with the vacuum, namely, about +240.374 GeV. Because the Higgs sits directly atop the peak – again by definition – in (17.1c) we show its elevation from the bottom of each well, namely, about 755.265 GeV above the bottom of the isospin-up well and 256.736 GeV above the bottom of the isospin-down well.

If we now apply least action principles whereby a particle will always seek to move from higher-energy to a lower-energy state, then with the potential energies of the quarks increasing from the first to the second and again from the second to the third generations for each of the isospin-up and isospin-down quarks, we see that higher generation quarks (and thus mesons and baryons) will generally decay to lower-generation quarks when there is an available energy path to do so. The third-generation top and bottom quarks have potentials well-above those of the first- and second-generation quarks, so are clearly the least energetically-stable and thus most subject to rapid decay. Additionally, with the *Z* and *W* bosons sitting on the side wall of the isospin-up well and having elevated potentials relative to the bottom of this well, and with the Higgs boson sitting right atop the peak between the isospin-up and isospin-down wells, all three of these bosons are likewise energetically unstable, short-lived particles subject to rapid decay. And in fact, all of these energetically-predicted decay behaviors are obliged by the observed physical behaviors of the quarks and the *Z*, *W* and Higgs bosons. Moreover, real photons can travel indefinitely without decay, which is supported by these being situated at the very bottom of the global minimum.

Most importantly, as between the lowest-potential energy thus most-stable up and down quarks of the first generation, the down quark nests essentially at the bottom of the isospin-down well at a depth of about –16.362 GeV while the up quark nests essentially at the bottom of the isospin-up well at a much greater depth of about –514.890 GeV. Therefore, when an energetic path is available to do so, and with all else being equal, down quarks will decay into up quarks. This explains, for example, why free neutrons beta decay into free protons with an extra up quark versus down quark, and not vice versa. However, it is extremely important – and will be central to properly understanding weak beta decay – to observe that the +240.374 GeV peak at $h=-m_h c^2$ in Figure 8 establishes a high energetic barrier which must be cleared before a down quark (or any isospin-down quark) can decay into an up (or charm) quark. (Note that beta decays between the top quarks and any of the isospin-down quarks is an exception, because of the top quark's crossover position in the isospin-down well, see Figures 8 and 9.) And this brings us to the Higgs boson and its heretofore-unappreciated role in weak beta decay and weak interactions generally. We shall start with the photon, then progress to the *Z* boson, then finally to the *W* boson which is well-known to be the mediator of weak beta decay.

To being, consider the very simple gamma reaction $f\gamma \to f$ in which a propagating fermion *f* with a charge *Q* absorbs a single photon γ while propagating. The Feynman diagram for this, which we shall not draw, simply shows a fermion line, along with a vertex at which a photon is incoming to and interacts with the fermion. The interaction of the photon with the fermion is one of the innumerable quantum interactions which, taken together, become the interaction of the fermion with other charged material bodies by its placement in and movement through a potential a.k.a. voltage. Using $p_{i\sigma}$ and $p_{f\sigma}$ to denote the initial and final energy-momentum vectors of this fermion on either side of the vertex, the incoming fermion wavefunction at the reaction vertex is $\psi = u\exp\left(-ip_{i\sigma}x^\sigma/\hbar\right)$, a second incoming adjoint wavefunction which translates to an outgoing ordinary wavefunction is $\overline{\psi}=\overline{u}\exp\left(ip_{f\sigma}x^\sigma/\hbar\right)$, and the current density $J^\mu=\overline{\psi}Q\gamma^\mu\psi$. Using (2.11a) the photon $A_\mu = A\varepsilon_\mu\exp\left(-iq_\sigma x^\sigma\right)$, with energy-momentum q_σ, and is also incoming to the vertex. So, the Lagrangian density of this gamma decay with the photon incoming, is formally given by:

$$\begin{aligned}\mathcal{L} = A_\mu J^\mu = A_\mu\overline{\psi}\gamma^\mu\psi = A\varepsilon_\mu\overline{u}\gamma^\mu\psi u\exp\left(-iq_\sigma x^\sigma/\hbar\right)\exp\left(ip_{f\sigma}x^\sigma/\hbar\right)\exp\left(-ip_{i\sigma}x^\sigma/\hbar\right)\\ = A\varepsilon_\mu\overline{u}\gamma^\mu\psi u\end{aligned}. \tag{17.2}$$

At the last equality, the spacetime-dependent x^σ terms are removed by setting

$$\begin{aligned}&\exp\left(-iq_\sigma x^\sigma / \hbar\right)\exp\left(ip_{f\sigma} x^\sigma / \hbar\right)\exp\left(-ip_{i\sigma} x^\sigma / \hbar\right)=\exp i\left(\left(p_{f\sigma}-q_\sigma-p_{i\sigma}\right)x^\sigma / \hbar\right)\\&=1=\exp i\left(2\pi n\right)\end{aligned} \tag{17.3}$$

where $n=0,\pm1,\pm2,\pm3...$ is a positive or negative integer or zero. By writing this 1 as $1=\exp i\left(2\pi n\right)$ (which is similar to how Dirac-Wu-Yang magnetic monopoles and the Dirac quantization of electric charge may be uncovered, see [**ref**] pages 220-221), introduce a quantum number that will turn out to will provide an entirely-natural genesis for the so-called "$+i\varepsilon$ prescription" which is used to address the poles that appear when the denominator of the propagator for any particle is zero, $c^2 p_\sigma p^\sigma - mc^2 = 0$ (see, e.g., section 6.17 of [**ref**]) and the propagator thereby becomes singular (infinite), and that will provide a much deeper understanding of the lifetimes of virtual particles. Removing exponentials and multiplying through by c to provide energy dimensionality throughout, this means that:

$$cp_{f\sigma}x^\sigma - cp_{i\sigma}x^\sigma - cq_\sigma x^\sigma = 2\pi n\hbar c = nhc \tag{17.4}$$

The energy momentum vectors in the above have the spacetime components $cp^\sigma = \left(E, c\mathbf{p}\right)$ for the fermion and $cq^\sigma = \left(E_\gamma, c\mathbf{q}\right)$ for the photon. For the fermion, $E = mc^2 + E_K + E_\phi$ is the total energy, which is the respective sum of the rest energy, the kinetic energy, and any potential energy arising from interaction with some material body. For the photon, $E_\gamma = h\nu = \hbar\omega$, see (2.11c). However, because (17.2) is *implicitly* the Lagrangian density for a fermion in an electromagnetic potential dissected into one quantum interaction at a time, and because $f\gamma \to f$ contain no other boson vertexes and thus no other interactions in a potential, we may set $E_\phi = 0$, so that the fermion energies are simply $E = mc^2 + E_K$. Were the photon to have a rest mass (which we account for as a placeholder because we shall soon turn to the *Z* and *W* bosons which do have a rest masses), and with no other lines coming into or out of the photon (thus no potential energies other than those expressly represented on a quantum basis by the Feynman diagram), then the time component of the photon's energy momentum would be $E_\gamma = m_\gamma c^2 + E_{\gamma K}$, that is, its rest-plus-kinetic energy. Applying all of the foregoing in Minkowski space with $\mathrm{diag}\left(\eta_{\mu\nu}\right) = \left(1,-1,-1,-1\right)$ to (17.4), and dividing out the *c*, we next expand with $x^\sigma = \left(ct, \mathbf{x}\right)$ to:

$$\begin{aligned}&nh = E_f t - \mathbf{p}_f\cdot\mathbf{x} - E_i t + \mathbf{p}_i\cdot\mathbf{x} - E_\gamma t + \mathbf{q}\cdot\mathbf{x}\\&=\left(m_f c^2 - m_i c^2 - m_\gamma c^2\right)t + \left(E_{fK} - E_{iK} - E_{\gamma K}\right)t - \left(\mathbf{p}_f - \mathbf{p}_i - \mathbf{q}\right)\cdot\mathbf{x}\\&=-\left(m_\gamma c^2 + E_{\gamma K}\right)t + \left(E_{fK} - E_{iK}\right)t = -\left(m_\gamma c^2 + h\nu_\gamma\right)t + \left(E_{fK} - E_{iK}\right)t\end{aligned} \tag{17.5}$$

To get to the final line, we have set $m_f = m_i$ because the fermion never changes its rest mass (this will not be the case when we get to weak beta decays mediated by W bosons), and applied $\mathbf{p}_f = \mathbf{p}_i + \mathbf{q}$ because of momentum conservation. The expression $E_{fK} - E_{iK}$ which does remain, together with the remaining presence of the photon's $E_{\gamma K} = h\nu_\gamma$ and placeholder rest mass $m_\gamma c^2 (=0)$, shows how the energy of the photon – which again is a field quantum of the electromagnetic potential – causes the fermion to change its kinetic energy at the vertex.

Customarily, one sets $n=0$ in the above (that is, one applies $1=\exp i(0)$ in (17.3) and ignores the mathematically-more-general $1=\exp i(2\pi n)$). For this customary $n=0$, and setting $m_\gamma c^2 = 0$ to accord with the actual photon being luminous, the above reduces and restructures to:

$$E_{fK} = E_{iK} + E_{\gamma K} = E_{iK} + h\nu_\gamma . \tag{17.6}$$

This states the energy conservation principle that the kinetic energy of the outgoing fermion is equal to its incoming kinetic energy, plus the kinetic energy $E_{\gamma K} = h\nu_\gamma$ delivered by the incoming photon. In a classical context, this illustrates how the fermion converts electromagnetic potential energy to kinetic energy, one quantum interaction at a time, using the kinetic energy $E_{\gamma K} = h\nu_\gamma$ of the incoming photon which is the field quantum and mediator of electromagnetic interactions. This is why we do not need to expressly show potential energies anywhere in (17.5) and (17.6), because these are implicit in the photon-carried energies. Put another way: at the quantum level, there really are no "potential" energies. There are simply bosons mediating interactions between fermions by carrying their own rest and potential energies which they impart to or remove from the fermions.

But the question remains for how the huge negative potential energy $E_{\gamma V} = -514.890$ GeV of interactions between the photon and the vacuum becomes physically observed. Were we to try to include this in $cq^\sigma = (E_\gamma, c\mathbf{q})$ and specifically in E_γ for the photon, this would have the effect of adding a large negative photon mass to the photon energy in (17.6). Because (17.6) is the correct, observed conservation law at the vertex, we must conclude that $E_{\gamma V} = -514.890$ GeV is not directly observed in the form of an energy, and so must be observed in some other way. This now raises the more general question: *how are the E_V of the various particles in (17.1), which are the heights and depths of these particles in the Lagrangian potential as seen in Figures 8 and 9, actually observed in the physical world?*

Now let's turn from the photon to the Z boson and use the reaction $fZ \to f$ as an example. The Feynman diagram is the same as that for $f\gamma \to f$, but for the fact that the Z boson is substituted for the photon. From (17.2c), this boson has a rest energy $m_Z c^2 = 91.188$ GeV and a potential energy $E_{ZV} = -180.347$ GeV from its interactions with the vacuum. The relations (17.2) still apply. However, because the Z boson is massive, the polarization vector will contain a third, longitudinal degree of freedom, and the energy (time) component E_Z in $cq_Z{}^{\mu} = (E, c\mathbf{q})_Z$ will include this Z rest energy. Moreover, while real photons have unlimited lifetimes while virtual photons may have finite, brief lifetimes, Z always bosons have very short lifetimes.

So, let us return to (17.5), and write this for the Z boson as:

$$nh = -\left(m_Z c^2 + E_{ZK}\right)t + \left(E_{fK} - E_{iK}\right)t = -\left(m_Z c^2 + h\nu_Z\right)t + \left(E_{fK} - E_{iK}\right)t , \tag{17.7}$$

where we have also used $E_{ZK} = h\nu_Z$ for the Z boson kinetic energy. If we then divide through by t and restructure, this becomes:

$$E_{fK} = E_{iK} + h\nu_Z + m_Z c^2 + nh/t . \tag{17.8}$$

For $n = 0$ this reverts to $E_{fK} = E_{iK} + h\nu_Z + m_Z c^2$ which is the energy conservation relation (17.6), merely changed to reflect the non-zero mass of the Z boson. But with $n \neq 0$ there is now an extra energy-dimensioned term nh/t, and we need to decipher the physical meaning of this term.

Given that a frequency $\nu = 1/t$ has dimensions of inverse time (e.g., Hertz = cycles-per-second), one might be inclined rewrite (17.8) as $E_{fK} = E_{iK} + h\nu_Z + m_Z c^2 + nh\nu$, then try to decipher the physical meaning of $nh\nu$ which, of course, is spot-on equivalent in form to the historical Planck relation $E = nh\nu$ that opened the door to quantum physics at the start of the 20th century. One might then think to regard this new $h\nu$ as the kinetic energy of the Z boson, but there are two reasons why this is problematic: First, there is already a term $h\nu_Z$ for the kinetic energy of the Z boson, which would render the new $h\nu$ redundant. Second, more importantly, (17.8) is the energy conservation relation at the vertex $fZ \to f$ of a *single* fermion interacting with a *single* Z boson. In contrast, the quantum number n in $nh\nu$ represents the existence of multiple field quanta, that is n field quanta, each with an individual energy $h\nu$. So, there is no clear way to understand nh/t in (17.8) as the Planck-style $nh\nu$, in the context of the particular decay vertex $fZ \to f$ to which (17.8) actually applies.

Instead, we take the vantage point that in (17.8), whenever $n = 0 \to n \neq 0$, the Z boson rest energy $m_Z c^2 \to m_Z c^2 + nh/t$, simultaneously. If we square this relation, this also means that $m_Z{}^2 c^4 \to m_Z{}^2 c^4 + 2m_Z c^2 nh/t + n^2h^2/t^2$. Next, we consider that $c^2 p_\sigma p^\sigma - m^2c^4$ appears in the propagator denominator for any and all field quanta (fermions and bosons). In order to resolve the on-shell poles where $c^2 p_\sigma p^\sigma = m^2c^4$ (which is the relativistic energy-momentum relation), one employs the $+i\varepsilon$ prescription $-m^2c^4 \to -m^2c^4 + i\varepsilon$ to displace the poles above and below the real energy axis along an orthogonal imaginary energy axis. This also brings about a correspondence with the *finite lifetimes* of virtual particles. Then, we view both of these circumstances together, and introduce the *hypothesis* that these two situations are not disconnected, but are in fact synonymous. That is, we view $m_Z{}^2 c^4 \to m_Z{}^2 c^4 + 2m_Z c^2 nh/t + n^2h^2/t^2$ when $n = 0 \to n \neq 0$ to be completely synonymous with setting $m^2c^4 \rightarrow m^2c^4 - i\varepsilon$ to deal with the propagator poles, and give precise embodiment to this hypothesis by *defining*:

$$\rho_\sigma p^\sigma - mc^2 + i\varepsilon \equiv \rho_\sigma p^\sigma - mc^2 - 2m_Z c^2 nh/t - n^2h^2/t^2 . \tag{17.9a}$$

i.e.

$$\boxed{-i\varepsilon \equiv 2m_Z c^2 nh/t + n^2h^2/t^2}. \tag{17.9b}$$

If this hypothesis is true, then because $-i\varepsilon \neq 0$ whenever $n \neq 0$, the quantum number *n* which naturally-emerges by using $1 = \exp i(2\pi n)$ in (17.3) provides a completely natural way to introduce the $+i\varepsilon$ prescription and manage the propagator poles by taking the particles off shell with non-zero *n*, and the finiteness of virtual particle lifetimes becomes indicative of this new quantum umber having a $n \neq 0$ value. Now, let's calculate what is implied by (17.9) to see how it relates to observed particle data.

To begin, we easily rewrite (17.9b) as:

$$0 = i\varepsilon t^2 + 2m_Z c^2 nht + n^2h^2 . \tag{17.10}$$

The above is a quadratic in both *t* and nh, so let's solve both of these. For the former:

$$t = \frac{-2m_Z c^2 nh \mp \sqrt{4m_Z{}^2c^4n^2h^2 - 4i\varepsilon n^2h^2}}{2i\varepsilon} = i\frac{nh}{\varepsilon}\left(m_Z c^2 \pm \sqrt{m_Z{}^2c^4 - i\varepsilon}\right), \tag{17.11}$$

and for the latter:

$$nh = \frac{-2m_Z c^2 t \pm \sqrt{4m_Z{}^2c^4t^2 - 4i\varepsilon t^2}}{2} = t\left(-m_Z c^2 \pm \sqrt{m_Z{}^2c^4 - i\varepsilon}\right). \tag{17.12}$$

We may also combine both of these and reduce to obtain the identity relation:

$$i\varepsilon = \left(m_Z c^2 \mp \sqrt{m_Z{}^2c^4 - i\varepsilon}\right)\left(m_Z c^2 \pm \sqrt{m_Z{}^2c^4 - i\varepsilon}\right). \tag{17.13}$$

The commonly-appearing square root $\sqrt{m_Z{}^2c^4 - i\varepsilon}$ can be separated using the mathematical identity $\sqrt{A \pm iB} = \frac{1}{\sqrt{2}}\left(\sqrt{\sqrt{A^2+B^2}+A} \pm i\sqrt{\sqrt{A^2+B^2}-A}\right)$, to find that:

$$\sqrt{m_Z{}^2c^4 - i\varepsilon} = \frac{1}{\sqrt{2}}\left(\sqrt{\sqrt{m_Z{}^4c^8+\varepsilon^2}+m_Z{}^2c^4} - i\sqrt{\sqrt{m_Z{}^4c^8+\varepsilon^2}-m_Z{}^2c^4}\right). \tag{17.14}$$

Using (17.14) in (17.11) and (17.12) then yields, respectively:

$$t = i\frac{nh}{\varepsilon}\left(m_Z c^2 \pm \tfrac{1}{\sqrt{2}}\sqrt{\sqrt{m_Z{}^4 c^8 + \varepsilon^2} + m_Z{}^2 c^4} \mp \tfrac{1}{\sqrt{2}} i\sqrt{\sqrt{m_Z{}^4 c^8 + \varepsilon^2} - m_Z{}^2 c^4} \right), \tag{17.15}$$

$$nh = t\left(-m_Z c^2 \pm \tfrac{1}{\sqrt{2}}\sqrt{\sqrt{m_Z{}^4 c^8 + \varepsilon^2} + m_Z{}^2 c^4} \mp \tfrac{1}{\sqrt{2}} i\sqrt{\sqrt{m_Z{}^4 c^8 + \varepsilon^2} - m_Z{}^2 c^4} \right), \tag{17.16}$$

so that the combination (17.13) now becomes the identity:

$$\begin{aligned} i\varepsilon = &\left(m_Z c^2 \pm \tfrac{1}{\sqrt{2}}\sqrt{\sqrt{m_Z{}^4 c^8 + \varepsilon^2} + m_Z{}^2 c^4} \mp \tfrac{1}{\sqrt{2}} i\sqrt{\sqrt{m_Z{}^4 c^8 + \varepsilon^2} - m_Z{}^2 c^4} \right) \\ &\bullet\left(m_Z c^2 \mp \tfrac{1}{\sqrt{2}}\sqrt{\sqrt{m_Z{}^4 c^8 + \varepsilon^2} + m_Z{}^2 c^4} \pm \tfrac{1}{\sqrt{2}} i\sqrt{\sqrt{m_Z{}^4 c^8 + \varepsilon^2} - m_Z{}^2 c^4} \right) \end{aligned}. \tag{17.17}$$

Now, in propagator theory, we encounter energies which are complex numbers with a real and imaginary part, which is how is how we circumvent the aforementioned poles which cause the propagators to become singular. Here, by definition, the quantum number n is real, and of course, Planck's constant is also real. This means that the time coordinate t in (17.15) must be a complex number with real and imaginary parts. When confronted with an imaginary time, it is perhaps natural to think about Minkowski's original 1909 interpretation of space and time in which space coordinates are thought of as imaginary time coordinates $\mathbf{x} \Leftrightarrow ict$ [ii], which was the first indicator of the Minkowski metric tensor $\eta_{\mu\nu}$ now used to specify a flat spacetime metric. But in the present context, the space coordinates have already been accounted for, and at (17.5) removed via momentum conservation at the vertex.

Therefore, a better understanding of the complex time coordinate in (17.5) starts with the fact that the propagator poles are circumvented by a complex energy with both real and imaginary parts, with the poles displaced off the real axis by the addition of $i\varepsilon$ to the propagator denominator. But energy and time, of course, are complementary variables, for example as exhibited by the energy-time uncertainty relation $\Delta E \Delta t \ge \frac{1}{2}\hbar$. Given that energies for individual field quanta are related to their frequencies by $E = h\nu$, this is merely reflective of Fourier transforms operating between frequency and time, with the minimum at $\Delta E \Delta t = \frac{1}{2}\hbar$ occurring only for Gaussian distributions. So, given this well-settled complementarity of energy and time, it should be unsurprising that with energy taking on an imaginary axis for propagator pole displacement, time itself would also have to take on an imaginary component and become a complex number when dealing with individual particle propagation, as seen in (17.15).

So, if the time coordinate for an individual propagating field quantum is now a complex number, the next step is to calculate the square *magnitude* $|t|^2 = t^* t$ of this time coordinate, then take the square root to obtain the real number $|t|$, then see if this $|t|$ can be made to correspond to observable data for individual field quanta, and particularly, to the observed lifetimes of virtual particles. Staring with (17.15), we can separately calculate $|t|$ for each of the $\pm$ choices permitted by the quadratic solutions to (17.10), then combine these results to obtain:

$$|t| = \frac{nh}{\varepsilon}\sqrt[4]{m_Z{}^2c^4 + \sqrt{m_Z{}^4c^8 + \varepsilon^2}}\sqrt{\sqrt{m_Z{}^2c^4 + \sqrt{m_Z{}^4c^8 + \varepsilon^2}} \pm \sqrt{2}m_Zc^2}\ . \tag{17.18}$$

Note that without the $\pm\sqrt{2}m_Zc^2$ term, this would be $|t| = \frac{nh}{\varepsilon}\sqrt{m_Z{}^2c^4 + \sqrt{m_Z{}^4c^8 + \varepsilon^2}}$. Now we need to closely examine ε itself.

The hypothesis of (17.9) connected the prescription of replacing $-m^2c^4 \to -m^2c^4 + i\varepsilon$ to avoid propagator poles, with the $n \neq 0$ solutions which first appeared at (17.3). Of course, ε must have physical dimensions of energy-squared. And it is widely regarded that for any given particle, the relation between ε for that particle and energies associated with that particle is $\varepsilon = mc^2\Gamma$, where *m* is the rest mass of that particle, and where Γ is the Breit–Wigner decay width of that particle. It is also well known that $\Gamma \ll mc^2$, at least for the *Z* and *W* bosons and the Higgs boson, and that the mean (average) lifetime for a particle with a given Γ is $\langle\tau\rangle = \hbar/\Gamma$. But if Γ specifies the *mean* lifetime a particle generally, and does not represent the *actual* lifetime of *one specific particle*, then it really incongruous to use $-m^2c^4 \to -m^2c^4 + imc^2\Gamma$ in the propagator. This is because a propagator applies to *one individual particle, not to large collections of particles*, while Γ tells us nothing about the particular lifetime of a particular particles, but only the average lifetimes of large numbers of individual particles. So, by placing the statistical average Γ into the propagator for an individual particle, one is truly combining apples with oranges.

A more congruous approach would replace $-m_p{}^2c^4 \to -m_p{}^2c^4 + im_pc^2\Gamma_p$ for an individual particle *p*, with Γ_p denoting a width *of that individual particle* which is related to the actual lifetime *of that individual particle* by $\tau_p = \hbar/\Gamma_p$. This means that $\varepsilon_p = m_pc^2\Gamma_p$. With a particle-dependent Γ_p in the propagator, the means that, for example, the propagator for a first particle p_1 with a lifetime τ_1 would be numerically different from the propagator for a second p_2 of the exact same particle type but with a different lifetime $\tau_2 \neq \tau_1$. And in fact, it is physically sensible that, for example, a short-lived *Z* boson will have a different numerical physical effect on the fermions whose interactions it mediates, from that of a long-lived *Z* boson.

But even this does not fully solve the problem, particularly when we consider virtual photons which are massless, or gluons or gravitons which are also massless. Here, if we set, say, $\varepsilon_\gamma = m_\gamma c^2\Gamma_\gamma$ for the photon, then $\varepsilon_\gamma = 0$ because $m_\gamma = 0$, and we lose the ability to describe virtual photons. In other words, we can describe massive virtual particles using $\varepsilon_\gamma = m_\gamma c^2\Gamma_\gamma$, but not massless virtual particles. So, it is clear that we still need an extra term. For a massive boson, the usual $\varepsilon = mc^2\Gamma$ could be revised to $\varepsilon = mc^2\Gamma + \Gamma^2 \cong mc^2\Gamma$, in view of the knowledge that $\Gamma \ll mc^2$, with the Γ^2 term thereby having a comparatively-small effect. But for a massless boson, this additional term is essential, if we wish to be able to accommodate *virtual* variants of that massless boson. Therefore, adding this extra term to accommodate massless virtual bosons, but having this apply to individual particle lifetimes rather than average lifetimes, we now *hypothesize* the following association between ε_p for an individual particle *p*, and its m_p and its *specific individual lifetime* related to this by $\Gamma_p = \hbar / \tau_p$:

$$\varepsilon_p \equiv m_p c^2 \Gamma_p + \Gamma_p{}^2 \tag{17.19}$$

Now, the remaining question is what to associate with Γ_p for any particular particle type. This Γ_p needs to be energy-dimensioned, it needs to be an invariant scalar in the same way that the rest energy $m_p c^2$ is invariant, yet it needs to be different from the rest energy. Now, we return to (17.1), to find exactly what we are looking for: every particle in (17.1) not only has a rest energy (including a rest energy of zero for the photon), but it also has a potential energy E_{pV} of its interaction with the vacuum. And, the very problem we are trying to solve right at present, is that of how these E_{pV} are observed, knowing that these are not directly observed as energies, but suspecting that they may become manifest through virtual particle lifetimes. So as the final piece of the puzzle, we introduce the further hypothesis that Γ_p in (17.19) is synonymous with E_{pV} in (17.1), via the definition:

$$\Gamma_p \equiv E_{pV} \tag{17.20}$$

When we then combine this with (17.19) containing a $\Gamma_p{}^2$ term to accommodate virtual massless particles, we arrive at:

$$\varepsilon_p \equiv m_p c^2 E_{pV} + E_{pV}{}^2 . \tag{17.21}$$

With this, we use this in (17.18) for the *Z* boson, along with $h = 2\pi\hbar$, to obtain:

$$\boxed{\begin{aligned} |t_Z| &= 2\pi n \bullet \frac{\hbar}{\sqrt{m_Z c^2 E_{ZV} + E_{ZV}{}^2}} \\ &\bullet \frac{\sqrt[4]{m_Z{}^2 c^4 + \sqrt{m_Z{}^4 c^8 + \left(m_Z c^2 E_{ZV} + E_{ZV}{}^2\right)^2}} \bullet \sqrt{\sqrt{m_Z{}^2 c^4 + \sqrt{m_Z{}^4 c^8 + \left(m_Z c^2 E_{ZV} + E_{ZV}{}^2\right)^2}} \pm \sqrt{2} m_Z c^2}}{\sqrt{m_Z c^2 E_{ZV} + E_{ZV}{}^2}} \end{aligned}} . \tag{17.22}$$

Then, we simply insert $m_Z c^2$ and E_{ZV} from (17.1c) obtain time magnitudes corresponding respectively to the $\pm$ sign in front of $\sqrt{2} m_Z c^2$ originating in the quadratic (17.10), designating the quantum numbers respective to each by n^+ and n^-, as such:

$$\left|t_Z\right|_n^{\pm}=\begin{cases}n^+\cdot 5.5983\times 10^{-26}\,\text{s}\\ n^-\cdot 1.9000\times 10^{-26}\,\text{s}\end{cases},\tag{17.23}$$

In this calculation of (17.22), we employ $\hbar = 6.582119514(40)\times 10^{-25}$ GeV s [**ref**] for the reduced Planck constant as widely-used in particle physics to convert between energies and times, to obtain $2\pi\hbar / \sqrt{m_Z c^2 E_{ZV} + E_{ZV}{}^2} = 3.2614\times 10^{-26}\,\text{s}$. The entire bottom line of (17.22) is a dimensionless number of energy-over-energy, and has the numeric values 1.7165 and 0.5826 corresponding with $\pm\sqrt{2} m_Z c^2$. These multiplied together are what produce the time numbers in (17.23). Then, there is the quantum number n^+ or n^- multiplying each of these times.

What we immediately see from this result, that each of these times $\left|t_Z\right|_n^{\pm}$ is quantized in discrete units of time characterized by the $n^{\pm}=1$ results, either $\left|t_Z\right|_1^{+} = 1.9000\times 10^{-26}\,\text{s}$ or $\left|t_Z\right|_1^{-} = 5.5983\times 10^{-26}\,\text{s}$. It is also well-known that the *Z* boson full width is $\Gamma_Z = 2.4952(23)$ GeV [**ref**], and this used via $\langle\tau_Z\rangle = \hbar/\Gamma_Z$ in to calculate a mean lifetime $\langle\tau_Z\rangle = 2.6379(24)\times 10^{-25}$ s. From this we can also calculate that:

$$\left|t_Z\right|_1^{+}/\langle\tau_Z\rangle = 1/4.7119;\quad \left|t_Z\right|_1^{-}/\langle\tau_Z\rangle = 1/13.8836;\quad \left|t_Z\right|_1^{+}/\left|t_Z\right|_1^{-} = 2.9465.\tag{17.24}$$

In other words, $\left|t_Z\right|_1^{+}$ is about 4.71 times shorter than the mean *Z* boson lifetime, and $\left|t_Z\right|_1^{-}$ is about 13.88 times shorter than the mean. This leads us to suspect that the $\left|t_Z\right|_n^{\pm}$ may be understood as the *actual* lifetimes of *individual Z* bosons as opposed to an average lifetime, with a suitable Breit-Wigner probability distribution which will momentarily be reviewed. However, $\left|t_Z\right|_1^{+}$ emerges from the *coordinate time* which is the time component of the coordinates $x^\mu = (ct, \mathbf{x})$ first explicitly introduced at (17.5), whereas $\langle\tau_Z\rangle$ obtained from $\langle\tau_Z\rangle = \hbar/\Gamma_Z$ is a *proper time* which is independent of how the coordinates are chosen and in particular is a Lorentz scalar *independent of the state of motion* of a *Z* boson. So, we need to carefully assess this.

Of course, proper time τ along the *Z* boson worldline in flat spacetime is related to coordinate time by $dt/d\tau = \gamma = 1/\sqrt{1-v^2/c^2}$ where v is the velocity of the *Z* boson. Discarding an integration constant, this integrates to $t=\gamma\tau$. Or, $\tau = t/\gamma$, which tells us that t/γ is a Lorentz scalar, because coordinate time scales with $t \propto \gamma$. Therefore, let us next insert $t=\gamma\tau$ into (17.23):

$$\left|\tau_Z\right|_n^{\pm}=\begin{cases}n^+\cdot 5.5983\times 10^{-26}\,\text{s}/\gamma\\ n^-\cdot 1.9000\times 10^{-26}\,\text{s}/\gamma\end{cases}\tag{17.25}$$

Now, on the left $|\tau_Z|_n^{\pm}$ is Lorentz scalar. On the right, $n^{\pm}$ are just integers which are invariant numbers always, by definition, multiplied by a t/γ which is a Lorentz scalar wherein $t \propto \gamma$ scale together. Because $|\tau_Z|_n^{\pm}$ is a Lorentz scalar, we may choose any state of motion to evaluate this, known that it will then remain unchanged in any other state of motion. So, we choose the rest frame for which $\gamma = 1$, whereby (17.25) now becomes:

$$|\tau_Z|_n^{\pm} = \begin{cases} n^+ \cdot 5.5983\times 10^{-26}\,\mathrm{s} \\ n^- \cdot 1.9000\times 10^{-26}\,\mathrm{s} \end{cases}. \tag{17.26}$$

This now shows an *invariant proper time* on both sides of the equation, following a conversion of the coordinate time in (17.23) to a proper time in (17.26). The difference between (17.23) and (17.26) is that in the former, the numbers $5.5983\times 10^{-26}\,\mathrm{s}$ and $1.9000\times 10^{-26}\,\mathrm{s}$ are coordinate times $t = \gamma\tau$ which increase with motion, whereas in the latter they are *proper times* that remain the same irrespective of motion. With this, the final relation in (17.24), now $|\tau_Z|_1^{+} / |\tau_Z|_1^{-} = 2.9465$, tells us that the $|\tau_Z|$ for any given *n* are about 2.95 times as long for the + solution (denoted below with red dots) as for the – solution (in blue). Finally, with all this, we are in a position to interpret $|\tau_Z|_n^{\pm}$ in (17.26) as the *proper lifetimes* of *individual Z* bosons as measured along their worldlines, which are then plotted graphically in Figure 10 below:

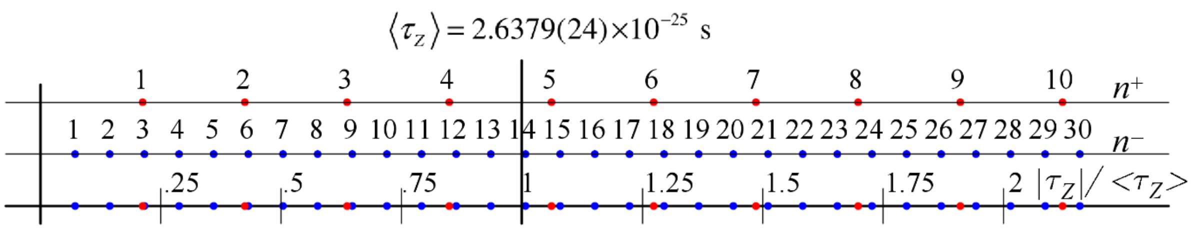

Figure 10: Quantization of *Z* boson lifetimes (n^+ solutions in red, n^- minus in blue)

What we already know about particle lifetimes, including that of the *Z* boson, is that a mean lifetime for each particle is obtained from its full width via $\langle\tau\rangle = \hbar/\Gamma$. The full width, in turn, specifies the width of the Breit-Wigner probability distribution $p(E)$ for the rest energy of a virtual particle at the ordinate where the probability distribution $p(E) = \frac{1}{2} p_{\max}(E)$, see, for example, the online plot at https://commons.wikimedia.org/wiki/File:Breit-Wigner.png. What Figure 10 adds to this knowledge, very consequentially, is that *the Breit-Wigner distribution is not really a continuous, but rather is a* <u>*discrete*</u> *probability distribution*. We see this most clearly by defining an energy-dimensioned inverse for each of the $|t_Z|_n^{\pm}$, such that:

$$\tfrac{1}{2}\Gamma_n^{\pm} \equiv \tfrac{1}{2}\hbar / |\tau_Z|_n^{\pm} = \begin{cases} 5.879\ \mathrm{GeV}/n^+ \\ 17.321\ \mathrm{GeV}/n^- \end{cases}, \tag{17.27}$$

and then using this to plot out the usual Breit-Wigner distribution for the *Z* bosons, but only with the discrete energies $\Gamma_n^{\pm}$ in (17.27) which vary by $1/n^{\pm}$ for $\gamma \cong 1$, as shown below:

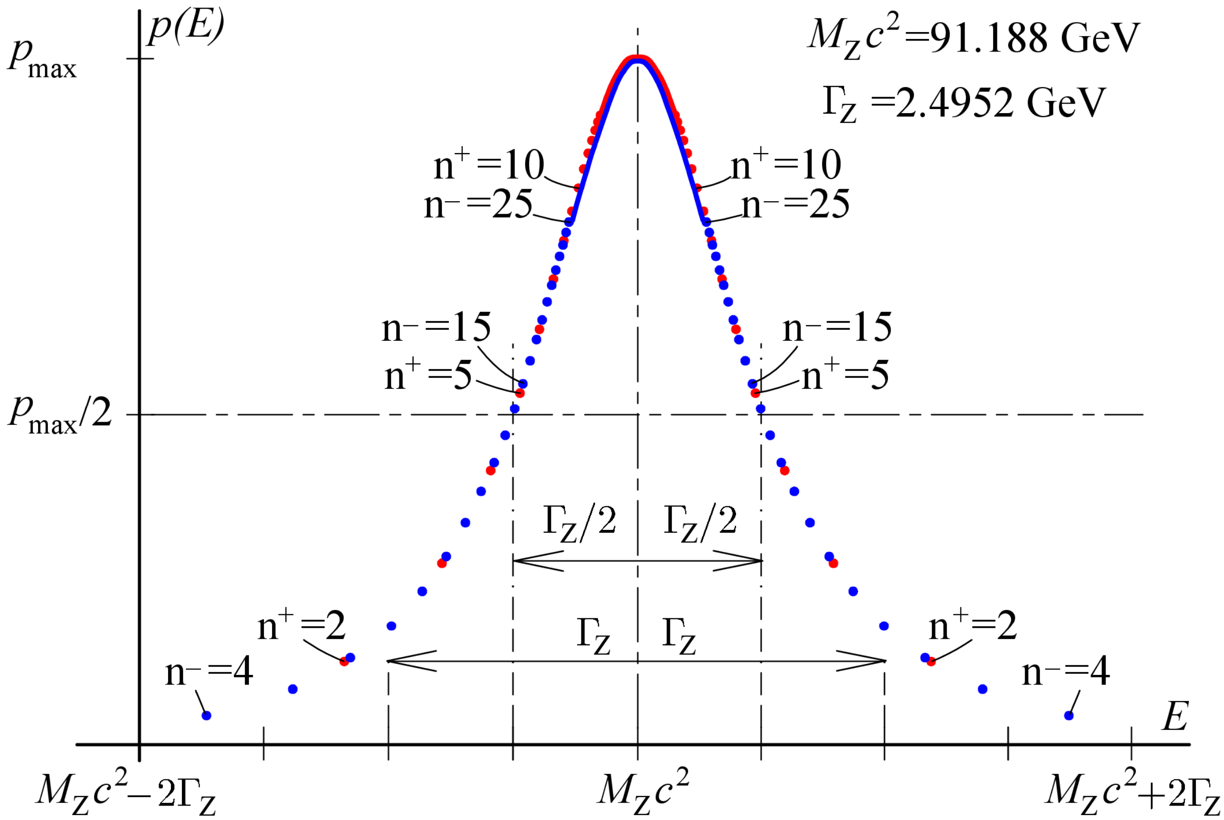

Figure 11: Discrete Breit-Wigner Probability Distribution for the *Z* boson

In addition to the fact that is now discrete rather than continuous, several features of the above plot are particularly noteworthy: First, as with any probability distribution, the probabilities for all of $1 \le n^{\pm} \le \infty$ must sum to unity (1), which corresponds to the requirement that for a continuous distribution the area under the curve must be 1. Second, the placements of the specific $n^{\pm}$ quantum numbered states in Figure 11 along the usual Breit-Wigner curve naturally embody $\frac{1}{2}\hbar$ rather than $\hbar$ without the ½, because of what we learn from Figure 10. Specifically, in Figure 10 we see that the mean lifetime $\langle \tau_Z \rangle = 2.6379(24)\times 10^{-25}$ s is straddled between $n^+ = 4$ and $n^+ = 5$, and also between $n^- = 13$ and $n^- = 14$. Because $\Gamma_Z = \hbar / \langle \tau_Z \rangle$, this means that for $\gamma \cong 1$, $\Gamma_n^+ \equiv \hbar / \left| \tau_n^+ \right|$ for the same $n^+ = 4$ and $n^+ = 5$ will straddle the mean positions $m_Z c^2 \pm \frac{1}{2}\Gamma_Z$, as will $\Gamma_n^- \equiv \hbar / \left| \tau_n^- \right|$ for the same $n^- = 13$ and $n^- = 14$, as is illustrated. In this way, about 50% of the time, a *Z* boson will have either $1 \le n^+ \le 4$ or $1 \le n^- \le 13$ and be shorter-lived than average, and the other 50% of the time, a *Z* boson will have either $5 \le n^+ \le \infty$ or $14 \le n^- \le \infty$ and be longer-lived than average. Because the particular curve in Figure 11 only shows the tails over the domain between $m_Z c^2 \pm 2\Gamma_Z$, the $n^+ = 1$ and the $n^- = 1, 2, 3$ do not appear, because these are outside this domain. Third, and related, with the 1/*n*-dependence of (17.27), the high-*n* states are more-densely packed on the energy curve than the low-*n* states. Therefore, as seen, close to $m_Z c^2$ the curve does approximate to a continuous probability distribution because of the close packing of these states. The blue – states are inherently packed about 2.95 times as densely as the red + states, so that the approximation to a continuous distribution shown by the blue line at the top of the curve begins further from $m_Z c^2$ than does that shown by the red line.

Fourth and finally, the very discreteness of Figure 11 illustrates an important insight about the nature of rest mass for a short-lived virtual *Z* boson, and for virtual particles generally: The Breit-Wigner curve generally illustrates how for short-lived particles, the rest mass is not always the same definite number $M_Z c^2 = 91.188\text{ GeV}$, but rather can be and is observed to be spread out around the center of the Breit-Wigner curve. Indeed, experimentally, particle lifetimes are generally measured not directly by a measurement of elapsed time from the emission to the absorption, but rather, by measurements in this rest energy spread governed by the uncertainty principle, then using the general relation $\langle \tau \rangle = \hbar / \Gamma$ to deduce a mean life. But while such a mass spread is most definitely observed, this does not answer the question whether the universe of possible masses falls along a continuous, or a discrete spectrum. Figure 11 answers that question: although the mass of a *Z* boson varies because of its brief lifetime and quantum uncertainty, the possible masses which can be observed for any given *Z* boson fall along a discrete, not a continuous spectrum. However, there are an infinite number of these discrete masses possible, and these are most-densely packed closest to the mean rest energy at the center of the curve. So, near this center, it is unlikely that sufficient experimental resolution can be achieved to distinguish one discrete mass from the next. But this discreteness is most apparent, and potentially most observable, on the tails of the curve where the particles have the shortest lifetimes.

Thus, using Figure 11 as a guide, it is possible to calculate the exact discrete masses which can be observed on the tails of the curve outside of $m_Z c^2 \pm \frac{1}{2}\Gamma_Z$. For n^+ there are only four such states $1 \le n^+ \le 4$, with the sub-mean lifetimes and the associated rest energies both above (high) and below (low) $M_Z c^2$ shown below:

n^+	$\left\|\tau_Z\right\|_n^+$	$M^+_{Z\,\text{high}}$	$M^+_{Z\,\text{low}}$
1	5.598×10^{-26} s	97.067 GeV	85.309 GeV
2	1.120×10^{-25} s	94.127 GeV	88.249 GeV ,
3	1.680×10^{-25} s	93.148 GeV	89.228 GeV
4	2.239×10^{-25} s	92.658 GeV	89.718 GeV

(17.28a)

Note that the masses for each n^+ are calculated by adding or subtracting $\frac{1}{2}\Gamma_Z$, not Γ_Z, to or from $m_Z c^2$, because their Figure 11 displacement from center is likewise by $\pm\frac{1}{2}\Gamma_Z$ in order to ensure proper straddling of quantum states about the means lifetime and energy width, as reviewed. For n^- there are 13 states outside of $m_Z c^2 \pm \frac{1}{2}\Gamma_Z$, with $1 \le n^- \le 13$, and these are as follows:

n^-	$\left\|\tau_Z\right\|_n^-$	$M^-_{Z\,\mathrm{high}}$	$M^-_{Z\,\mathrm{low}}$
1	$1.900\times10^{-26}\,\mathrm{s}$	108.509 GeV	73.867 GeV
2	$3.800\times10^{-26}\,\mathrm{s}$	99.849 GeV	82.527 GeV
3	$5.700\times10^{-26}\,\mathrm{s}$	96.962 GeV	85.414 GeV
4	$7.600\times10^{-26}\,\mathrm{s}$	95.518 GeV	86.858 GeV
5	$9.500\times10^{-26}\,\mathrm{s}$	94.652 GeV	87.724 GeV
6	$1.140\times10^{-25}\,\mathrm{s}$	94.075 GeV	88.301 GeV
7	$1.330\times10^{-25}\,\mathrm{s}$	93.662 GeV	88.714 GeV
8	$1.520\times10^{-25}\,\mathrm{s}$	93.353 GeV	89.023 GeV
9	$1.710\times10^{-25}\,\mathrm{s}$	93.113 GeV	89.263 GeV
10	$1.900\times10^{-25}\,\mathrm{s}$	92.920 GeV	89.456 GeV
11	$2.090\times10^{-25}\,\mathrm{s}$	92.763 GeV	89.613 GeV
12	$2.280\times10^{-25}\,\mathrm{s}$	92.631 GeV	89.745 GeV
13	$2.470\times10^{-25}\,\mathrm{s}$	92.520 GeV	89.856 GeV

, (17.28b)

These two data tables in (17.28) make clear that the rest energy of any individual *Z* boson is still an exact and discrete scalar number, but that the observed mass of that *Z* boson may and is likely to be different from the observed mass of another *Z* boson. Moreover, as noted, approximately 50% of the observed *Z* bosons will be in one of the seventeen (17) quantum states listed in (17.28), and thus will only have one of the thirty-four (34) distinct masses so-listed. If these masses and particularly their discreteness of value can be resolved with sufficient experimental precision (which is an equipment issue as distinguished from the in-principle energy spread based on quantum uncertainty), then this may provide a path for empirical validation of the discrete quantization of virtual particle lifetimes and rest energies. On average, these all net to the mean lifetime $\langle \tau_Z \rangle = 2.6379(24)\times10^{-25}\ \mathrm{s}$, and observed full width $\Gamma_Z = 2.4952(23)\ \mathrm{GeV}$ about a mean rest mass $\langle M_Z \rangle c^2 = 91.188\ \mathrm{GeV}$ which is the average of an infinite number of discrete rest mass values tightly-packed toward the center but more loosely distributed along the tails.

Stepping back, we note that these quantum numbers arose out of the term nh/t in (17.8) which can written as Planck's $E_n = nh\nu$ using a frequency $\nu = 1/t$ if *t* is regarded to be a oscillatory cycle. We also note that in the end this *t* has been turned into quantized lifetimes $\left|\tau_Z\right|_n^{\pm}$ of the *Z* boson, with analogous relationships presumably occurring for other particle types as well. So, if we regard the lifetime of an individual *Z* boson with $n=1$ as one "lifecycle" during which the boson is emitted, propagates and is absorbed using radial frequencies defined by the $n=1$ states via $\omega_{Z1}^{\pm} \equiv 1/\left|\tau_Z\right|_1^{\pm}$ whereby $\omega_1^{+} = 1/5.5983\times10^{-26}\,\mathrm{s}$ and $\omega_1^{-} = 1/1.9000\times10^{-26}\,\mathrm{s}$, then also with $\nu_1 \equiv 2\pi\omega_1$ following the usual form, (17.26) and (17.27) consolidate together into:

$$\boxed{\Gamma_n = \hbar\omega_1 / n = h\nu_1 / n}. \qquad (17.27)$$

The above then becomes seen as a new incarnation of Planck's $E_n = nh\nu$ *, applied not to particle energies and frequencies a.k.a. inverse periods, but to particle rest energy widths and lifetimes.* While the former Planck relation varies in proportion to *n*, the latter varies with 1/*n*.

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[i] M. Tanabashi et al. (Particle Data Group), Phys. Rev. D 98, 030001 (2018), http://pdg.lbl.gov/2018/tables/rpp2018-sum-gauge-higgs-bosons.pdf
[ii] H. Minkowski , Raum und Zeit, Jahresberichte der Deutschen Mathematiker-Vereinigung, 75-88 (1909)